

Design of Active Control System for Combustion Instability Using Mixed H^2/H^∞ Algorithm

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Abstract

This paper describes the development of a feedback active combustion control (ACC) system. The system targets at a suppression of combustion oscillation in a lean premixed combustor. As a control method, the system uses the secondary fuel injection. The mixed H^2/H^∞ controller is applied to the ACC system. The controller has both robustness using a H^∞ algorithm and good control performance using a H^2 algorithm. The developed system is adapted to an experimental combustor. As the result, the developed ACC system with a mixed H^2/H^∞ controller is able to suppress strong pressure oscillations successfully with the result of 28 dB reduction (acoustic pressure from 171 dB to 143 dB).

Introduction

Lean premixed combustion is used to reduce NOx emissions. It is well known that the self-excited strong pressure oscillations in the lean premixed combustors cause some problems such as oscillatory combustion during development of gas turbine engines. These oscillations are due to the thermoacoustic instability. In this context, feedback control to interrupt the coupling effect between pressure oscillations and heat release oscillations has been suggested to be effective for the robust operation of gas turbine engines in a wide range of equivalence ratio [e.g., 1-3]. As an active combustion control method, for example, secondary fuel injection has been proposed to suppress pressure oscillations, which is the method to improve the inhomogeneous distribution of equivalence ratio in combustion zone. In this regard, periodic secondary fuel injection has a performance suppressing the pressure oscillation [e.g., 4-6].

Recently, there is a direction to create interdisciplinary research between combustion engineering and control engineering. This direction implies that control theory (algorithm) is adapted to a combustion instability system. As for the algorithm of ACC system, several kinds of control methods have been reported [e.g., 7-10]. For example, MIT group investigated an approach for designing model-based active controller. As the remarkable results of controlling thermoacoustic instability, they developed a physical model-based, finite-dimensional model of a continuous combustion process to simulate a model-based self-tuning controller with a loudspeaker actuate system [7, 8]. Furthermore, in one of their recent reports, a physical model for dynamics of periodic fuel injection was reported to demonstrate the performance of controller designed by LQG/LTR algorithm with simulation results [9]. Research group of Pennsylvania State Univ. (PSU) also mentioned the importance of model-based control for combustion instability [10, 11]. Their approach was based on H^∞ -optimization. In their report, it is mentioned that H^∞ algorithm guarantees robust stability and performance within specified bounds of model. Particularly, for modulating combustion dynamics, H^∞ based linear-parameter-varying (LPV) is introduced to describe the slow behavior of mean flow-field and main fuel injection and the fast behavior of acoustic oscillations and secondary fuel injection into the combustion chamber [11].

In this research project on smart control of turbulent, we have developed an effective ACC system, which is anticipated to be used in practical aircraft and industrial gas turbine engines. Table 1 shows the summary of approach (upper column), achievement (the middle) and assignment (the lower) in this project. With respect to the approach of developing ACC system, thermoacoustic control and flame structure control approach are taken into account. In the case of thermoacoustic control approach, we developed a closed-loop feedback control system for microphone-loudspeaker system using H^2 algorithm (2001-2002) [12]. However, loudspeaker as an actuator was not practical device for the combustor. Furthermore, H^2 controller does not have robust stability. The next challenge was to develop the secondary injection system for flame structure control approach. Last year (2003) [13], an open-loop controller was introduced using pressure transducer as a sensor and servo-valve with quick response as an actuator.

In this paper, closed-loop feedback control system is investigated using secondary fuel injection method based on flame structure control approach for ACC. As for the technology of combustion instability control, a mixed H^2/H^∞ controller is applied to ACC system. This controller is designed targeting at robustness using H^∞

algorithm and an effective performance of controller using H^2 algorithm. Performance tests are carried out in the model combustor developed at Japan Aerospace Exploration Agency (JAXA).

Table 1 Summary of approach, achievement and assignment in the project of combustion controller development research group (AGU, NMRI, JAXA)

	2000	2001	2002	2003	2004
Approach	• Development for experiment system (design of combustor)	• Thermoacoustic control using loudspeaker	• Thermoacoustic control with DSP system	• Flame structure control using secondary injection	• Flame structure control with DSP system
Achievement	• Understanding the fundamental phenomenon	• Verification of <u>loudspeaker</u> as an actuator • Open-loop control system	• Development of feedback control system based on <u>H2 algorithm</u> • Closed loop control system	• Some kinds of effective <u>secondary injection</u> through a trial and error process • Open-loop control system	• Development of feedback control system based on <u>H∞/H2 hybrid algorithm</u> • Closed loop control system
Assignment	• Establishment of the active combustion control system	• Development of <u>closed-loop control system</u> for thermoacoustic approach	• Practicality of actuator device	• Control for near the blow-off limit • Performance of <u>adaptive control</u> to the change of <u>oscillatory mode</u>	• ?

Model-Based Controller

In this section, the design on an ACC system based on the mixed H^2/H^∞ control and the experimental method are described.

System Identification

In this study the experimental modal analysis is applied to the design of the controller. To determine the transfer function of the system, system identification is carried out. The system identification based on the modal analysis is one of the useful methods to obtain a structure model of the considered system. The combustion instability controlled by the secondary fuel injection device as an actuator is considered as a quasi-linear system because the secondary fuel injection (diffusion flame) has an influence on the response of the main flame (premixed flame). Thus, we consider that the equation for the general viscous damping system can describe the thermo-acoustical behavior of the combustion system with heat release modulation caused by the secondary fuel injection.

General behavior of the viscous damping system is described by the following equation.

$$M_S \ddot{x} + C_S \dot{x} + K_S x = f \quad (1)$$

where x is a displacement vector, M_S , C_S , K_S are mass matrix of the system, damping matrix, stiffness matrix, respectively. f is the control input. Defining matrix D_S and E_S as follows:

$$D_S = \begin{pmatrix} C_S & M_S \\ M_S & 0 \end{pmatrix},$$

$$E_S = \begin{pmatrix} K_S & 0 \\ 0 & -M_S \end{pmatrix},$$

$$y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix},$$

$$p = \begin{bmatrix} f \\ 0 \end{bmatrix},$$

Eq. 1 can be rewritten as

$$D_S \dot{y} + E_S y = p \quad (2)$$

The frequency response function of such system is described as

$$G(\omega) = \sum_{r=1}^n \left(\frac{U_r + jV_r}{j(\omega - \omega_r) + \sigma_r} + \frac{U_r - jV_r}{j(\omega + \omega_r) + \sigma_r} \right) \quad (3)$$

where ω_r and σ_r are the damping eigen angular frequency and modal damping ratio, respectively. Both U_r and V_r imply the mode constant. This function can be rewritten in the following form:

$$G(s) = \sum_{r=1}^n \left(\frac{\eta_r s + \gamma_r}{s^2 + \beta_r s + \alpha_r} \right) \quad (4)$$

where,

$$\begin{aligned} \alpha_r &= \sigma_r^2 + \omega_r^2 \\ \beta_r &= 2\sigma_r \\ \gamma_r &= 2(\sigma_r U_r - \omega_r V_r) \\ \eta_r &= 2U_r \end{aligned}$$

The applied system is the model combustor at JAXA. In the system the dominant frequency is about 280 Hz at the equivalence ratio of $\phi = 0.5$, the inlet air temperature of 700 K, and the nominal velocity corresponding to the area of the swirler, 90 m/s. The detailed configuration of the combustor and detailed characteristic of the oscillatory flame are described elsewhere in the present proceedings [14]. Pressure transducer and servo-proportional control valve with high operating frequency (≈ 400 Hz) are used as a sensor and an actuator, respectively.

In the experiment, system input is the signal supplied to the control valve and control quantity is the pressure signal obtained by the pressure transducer. As for the signal analysis, A DS2000 (Ono Sokki Co. Ltd.) and CAT-System (Catec Inc.) are used for the frequency response function measurement. A Multi-Channel Analyzer SA-01 (RION) is also used for the measurement of the data used in the simulation of developed controller. Coefficients for each r in Eq. 4 are determined from the obtained data by differential iteration method [15] with oscillation analysis software Vibrant-GEN (Marubeni Solutions Co.). In Eq. 4 eight modes are used for curving fitting and hence two modes are adopted as the model.

Mixed H^2/H^∞ Controller

As far as the concept of developing the ACC system, robustness is one of the key issues for the performance of a developed controller. In this regard two different meanings are included in robustness. One is the robustness against uncertainty in the model. Considering the uncertainty while determining the transfer function theoretically, control divergence would be avoided. As described in the previous paper [8], a change of the dominant frequency is often observed when the fuel flow is modulated to suppress the pressure fluctuation caused by combustion instability. While, the other meaning of the robustness is the ability to keep an effective performance following the varying dominant oscillatory mode. However it is a challenge to control the state of combustion with a wide range of operating conditions.

As for the H^2 algorithm, its performance is characterized by an excellent transient response. A schematic image of the H^2 controller is described in Fig. 1. The H^2 control has a frequency weighting function such as dynamic weighting in LQG control. The feedback system developed by H^2 controller provides the stable control subject with the H^2 norm provided by the average amount of gain in each frequency as shown in Fig. 1. The feature of H^2 control is its excellent performance on transient response. In our previous study in the project [12, 16], we developed the H^2 controller with a microphone-loudspeaker system to suppress the combustion noise caused by the pressure oscillation. As the result of the study, it was found that the H^2 controller had a good performance of suppressing the combustion instability, while robustness of the system was not considered at that time. Hence robustness is one of the key issues to develop the ACC system as the next challenges in this project.

On the other hand, the H^∞ control is to make the system stable with the performance index (H^∞ norm) characterized by the maximum gain in each frequency as shown in Fig. 2. In other word the H^∞ norm implies an index to measure the robust stability based on uncertainties like modeling errors. If we use H^∞ control algorithm, the robustness is basically assured with theoretical background. Thus, the H^∞ algorithm has been applied to combustion instability control as a robust controller [e.g., 10, 11].

In general, although the controller developed by H^∞ algorithm guarantees the robustness on modeling errors, its control performance will not be so effective comparing to that of H^2 controller for the closed-loop system. If we consider more suitable performance on system transient behaviors with robustness, a hybrid control design which consists of H^2 and H^∞ is a choice to give an excellent performance. In this regard the complex system can be solved adding the H^2 control objectives to the known H^∞ control design. The mixed H^2/H^∞ control may be described as to find a controller $K(s)$ to minimize $\|F(s)\|_2$ which subjects to $\|F(s)\|_\infty < 1$. Here $\|\bullet\|$ means norm,

and $\|F(s)\|_\infty < 1$ is to guarantee a robust stability on uncertainties, $\|F(s)\|_2 \rightarrow \min$ to improve a transient behaviors in the subject, respectively. The method to derive the mixed H^2/H^∞ control is described as follows: 1) determine H^2 controller, $K(s)$; 2) design H^∞ controller according to $\|F(s)\|_\infty < 1$ in assumption that free parameter $S(s)$ is chosen without any inhibition as proper and stable rational function; and 3) check the free parameter $S(s)$ whether it is proper and stable or not. If $S(s)$ is proper and stable, $K(s)$ is identified as a mixed H^2/H^∞ controller [e.g., 17-21].

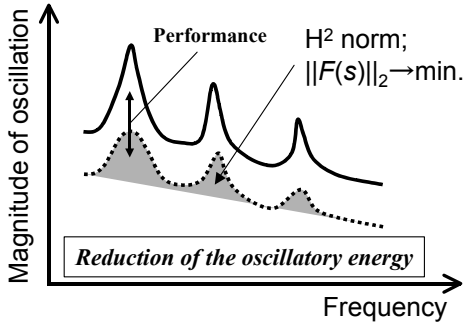


Figure 1 Schematic image of the H^2 controller

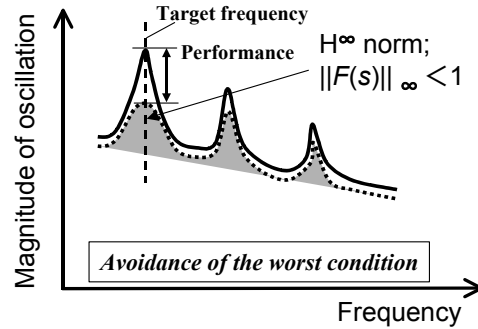


Figure 2 Schematic image of the H^∞ controller

Results and Discussion

Model of the Combustor

The system is identified to obtain the transfer function using the experimental modal analysis (System Identification method). Figure 3 shows the amplitude and phase of the response function in the frequency domain. The system is characterized by the oscillatory flame with the total equivalence ratio at $\phi = 0.5$, inlet air temperature of 700 K , and nominal velocity corresponding to the area of the swirler, 90 m/s . In the results of Fig. 3, the red line shows the identified result and blue one indicates the measured data obtained by the cross-spectrum, $\overline{P_{ij}}(f)$. Figure 3a shows the measured transfer function of the system based on the cross-spectrum analysis. Measured transfer function is given by the ratio of cross-spectrum ($\overline{P_{xy}}(f)/\overline{P_{xx}}(f)$) between the input-output signals ($\overline{P_{xy}}(f)$) and its own signals of the input ($\overline{P_{xx}}(f)$) [15], where \overline{P} is the time average. System identification is carried out using white noise signals from the actuator. In this study we consider the system identification with the frequency ranging from 10 to 400 Hz because of the performance limit of the actuator ($\approx 400\text{ Hz}$). In the region of 260 - 300 Hz , where the target mode (280 Hz) exists, the identified model (red line) represents the real system (blue line) very well. In the case of phase result as shown in Fig. 3b, the discrepancy between the identified model and the real system is noticeable except for the frequency domain characterized in $280 \pm 20\text{ Hz}$. However we expect that this discrepancy does not cause the instability of the system due to the effect of band-stop filter designed with controller. As shown in Eq. 4, we obtained the values of modal parameter as follows: the damping eigen angular frequency $\omega_r = 278\text{ Hz}$; characteristic modal damping ratio $\sigma_r = 3.352\%$; mode constants $U_r = 117.1$; $V_r = 3.053 \times 10^6$.

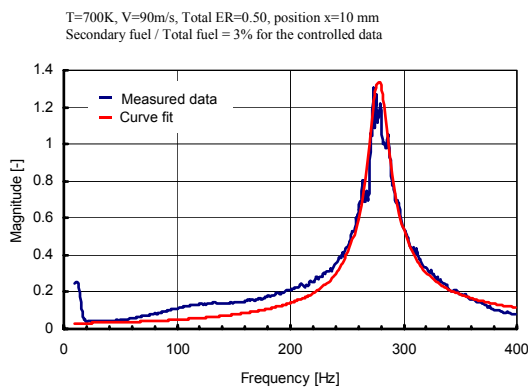


Figure 3(a) System identification (magnitude)

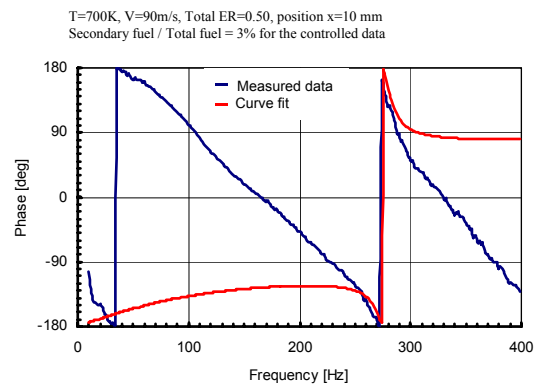


Figure 3(b) System identification (phase)

Design of Mixed H^2/H^∞ Controller for ACC System of the Combustor

In this study the ACC system is applied to a practical size combustor. In the case, it is required that the developed controller suppresses the combustion instability successfully. Furthermore, model uncertainties should be considered to prevent the divergence of thermoacoustic instability. In the previous section, we mentioned the feature of H^2 and H^∞ controllers. To achieve a good performance of the ACC, the mixed H^2/H^∞ controller is adopted. The block diagram of the control system is shown in Fig. 4 where $G(s)$ is the transfer function of the system, $K(s)$ a controller, Z_{21} the performance index of input signal, Z_{22} the performance index of output signal, y the output from the system, u control input, and w a disturbance. First of all we describe the transfer function $G(s)$ in the modal coordinate system to design the controller $K(s)$ by solving a mixed H^2/H^∞ control problem.

Using the modal coordinate system, the r -mode in Eq. 4 is described in the state expression as follows:

$$\begin{aligned} \dot{q}_r &= A_r q_r + B_r u \\ &= \begin{pmatrix} 0 & -\alpha r \\ 1 & -\beta r \end{pmatrix} \begin{Bmatrix} \xi_r \\ \zeta_r \end{Bmatrix} + \begin{Bmatrix} \gamma_r \\ \eta_r \end{Bmatrix} u \end{aligned} \quad (5)$$

$$y_r = C_r q_r = [0, \quad 1] q_r \quad (6)$$

where, $\alpha_r, \beta_r, \gamma_r, \eta_r$ are the coefficients in Eq. 4 and are obtained by the experiments of system identification. With the above expression, the system with n degrees-of-freedom can be described as

$$\begin{aligned} q &= Aq + Bw + Bu \\ &= \begin{pmatrix} A_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & A_n \end{pmatrix} \begin{Bmatrix} q_1 \\ \vdots \\ q_n \end{Bmatrix} + \begin{Bmatrix} B_1 \\ \vdots \\ B_n \end{Bmatrix} w + \begin{Bmatrix} B_1 \\ \vdots \\ B_n \end{Bmatrix} u \end{aligned} \quad (7)$$

$$y = Cq = [C_1, \quad \dots, \quad C_n] q \quad (8)$$

Replacing the variable q with x , the system is described in the following form:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_r \end{bmatrix} &= \begin{pmatrix} A & 0 \\ 0 & A_r \end{pmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w + \begin{bmatrix} B \\ B_w \end{bmatrix} u \\ \begin{Bmatrix} Z_{21} \\ Z_{22} \\ y \end{Bmatrix} &= \begin{pmatrix} 0 & C_w \\ C & 0 \\ C & 0 \end{pmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} w + \begin{bmatrix} D_w \\ 0 \\ 0 \end{bmatrix} u \end{aligned} \quad (9)$$

where A_w, B_w, C_w, D_w are matrices that express W_1 and a_w is a state vector for W_1 .

Using the above state expression, the controller is designed to minimize the H^2 norm of transfer function from w to Z_{22} and to satisfy the condition of the H^∞ norm of transfer function from w to Z_{21} . The H^2 norm of transfer function gives a performance of suppressing the pressure oscillation. Furthermore, the H^∞ norm provides the robustness included in the system identification.

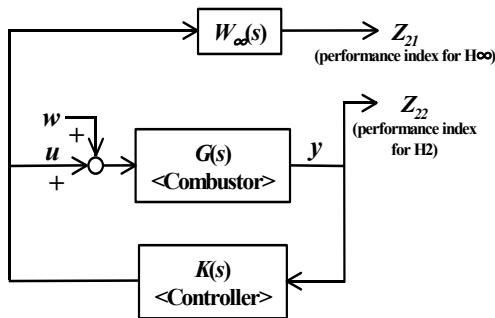


Figure 4 Block diagram of the system

Table 2 Specifications of the developed controller

type ;	mixed H^2 / H^∞ controller
order of controller ;	10
performance index ;	2nd-order mode (280 Hz) with H^2 band-stop filter (50-1000 Hz) with H^∞
model ;	consider the disturbance included in the input signal of the system
remarks ;	<ul style="list-style-type: none"> • oscillatory modes... 1st mode (10 Hz) 2nd mode (280 Hz) 3rd mode (600 Hz) • target mode ... 2nd mode • 1st mode corresponds to the amplitude of 2nd mode • operating frequency of actuator (<400 Hz) • scope of the result for curve fitting (260-300 Hz) • magnitude of input control signal (~0.5 V)

Specifications of the controller are summarized as shown in Table 2. The basic concept to develop controller for the ACC system is to consider the robustness using the H^∞ algorithm with a band-pass filter and also the H^2 algorithm for the performance of suppressing the pressure oscillation. In the case the target frequency is the dominant oscillatory mode characterized with 280 Hz. The dominant frequency region ranging from 260 to 300 Hz is considered as the performance index of robustness. As for the oscillatory modes, 10, 280, and 600 Hz modes exist in the test combustor. In the control system, the pressure fluctuation caused by the oscillatory flame is considered as the disturbance signal included in the input signal of the system (see. Fig. 4). The important parameter to design the controller is that of band-stop filter. The configuration of the filter is an important parameter. If the performance index of H^∞ is used with a setting near the target frequency to control, which implies a narrow band to the dominant frequency, the developed controller would be stable in theory but usually unstable because of the uncertainties between the real system and the model. Hence in this study W_1 described in Fig. 4 is designed to have a property of band-stop filter of 50-1000 Hz.

Performance Evaluation of Mixed H^2/H^∞ Controller Adapted to the Combustor

First of all, the effect of band-stop filter is examined comparing in the range of considered frequencies. Figure 5 shows the result in the case of a narrow band setting. The performance is estimated with comparing the pressure fluctuation results between the open-loop control case and closed-loop one. The oscillatory flame appears at the conditions of total equivalence ratio $\phi = 0.5$, the inlet air temperature of 700 K, and the nominal velocity corresponding to the area of the swirler, 90 m/s. In Fig. 5 the band setting is fixed at 260-300 Hz. The green line indicates the result of open-loop control system, and blue one is that of the closed-loop control system. As for the concept of designing controller, the band-stop condition implies a narrow case in a sense so that the target frequency is oscillatory mode characterized by the 280 Hz. As the result, it is found that the setting of the band-stop frequency domain should be determined in a wide range of the frequency, which implies it is better to set the band-stop condition widely for a good performance. In other word, if we want to design a controller with a strong performance, the controller will be unstable itself because the existence of errors, which are modeling errors, digitalization and else affect the correspondence between the theory and the real system.

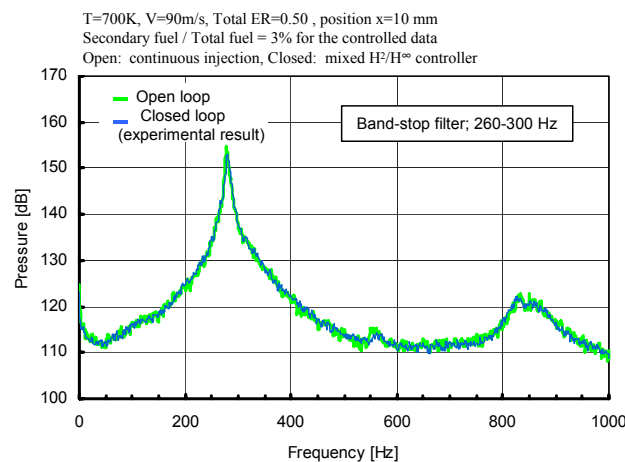


Figure 5 Experimental results of the open- and closed-loop controller with a narrow band-stop filter (260-300 Hz)

From a conclusion of above discussion, the wide-range condition of band-stop filter characterized by 50-1000 Hz is chosen as the performance index on robustness. The performance of designed controller is evaluated by the simulation with MATLAB/Simulink. Time series data obtained by the experiment is used for the simulation. Figure 6 is the simulation result by controller for $\phi = 0.5$ (total equivalence ratio). The target mode (280 Hz) is successfully suppressed, while any divergence behaviors of other modes are not found. In the simulation result, it is found that the dominant mode is suppressed with the magnitude of about 5 dB.

According to the simulation result, the experiments for verification to make sure the performance of developed controller are carried out. Comparison of sound pressure levels among the non-controlled flame, the open-loop controlled flames, and closed-loop controlled ones are shown in Fig. 7. In the figure, the red line is the result of non-controlled flame, the green line the behavior of open-loop controlled flame, and blue one the

closed-loop result. Comparing the results between the non-controlled flame and the open-loop controlled flame, the suppression performance of about 17 dB is achieved successfully. Here the open-loop control corresponds to the steady secondary fuel injection without any modulation of secondary fuel flow rate. Furthermore, comparing the pressure data of the open-loop controlled flame with that of the closed-loop controlled flame, the additional suppression of 10 dB is obtained with the closed-loop control. This result demonstrates an effectiveness of ACC system developed by the mixed H^2/H^∞ control method. In addition, any secondary excited oscillatory modes are not observed in this experiment. Hence we obtained the extra performance to reduce the pressure oscillation with the magnitude of about 28 dB (reduction from 171 dB to 143 dB).

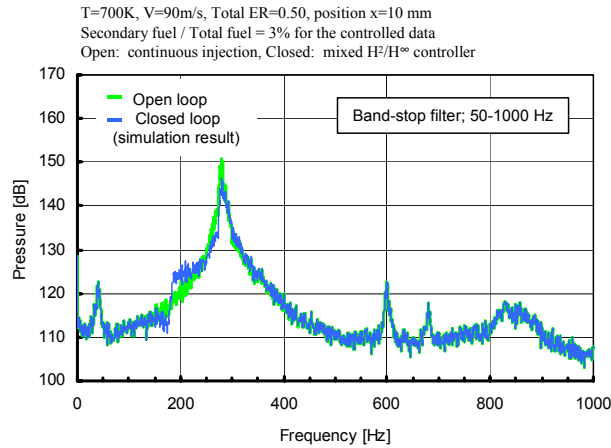


Figure 6 Simulation result of the closed-loop controller with a wide band-stop filter (50-1000 Hz)

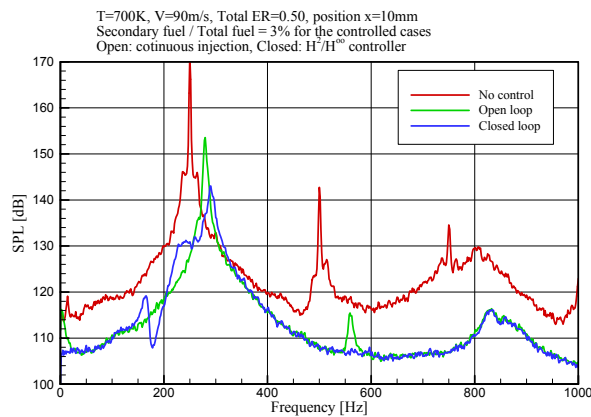


Figure 7 Performance of the developed mixed H^2/H^∞ controller (experimental results)

Concluding Remarks

A mixed H^2/H^∞ controller for a practical active combustion control system was developed to suppress the strong pressure oscillation. The hybrid controller was adopted for the robustness and good performance. The H^∞ algorithm was expected to guarantee the robustness against modeling error, and the H^2 algorithm was expected to achieve good performance. As for the target to control, the oscillatory flame in the model combustor at JAXA was used, which is characterized by the total equivalence ratio at $\phi = 0.5$, the inlet air temperature of 700 K , and the nominal velocity corresponding to the area of the swirler, 90 m/s . The flame conditions are in practical level except for high-pressure conditions. The developed ACC system with the mixed H^2/H^∞ controller indicated a good performance to suppress the strong pressure fluctuation with the result of 28 dB reduction (from 171 dB to 143 dB).

As one of the future challenges, robustness to adapt to the change in instability mode will rate the most concern. A practical and reliable ACC technology using a controller equipped an algorithm such as gain-scheduling is promising candidate.

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