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NONLINEAR SIMULATION OF FLOATING BODY MOTIONS IN WAVES

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ABSTRACT

A time domain simulation method for full nonlinear fluid-body interaction problem is presented. The feature of this method can be found in the formulation of hydrodynamic pressure computation. Introducing the nonlinear acceleration potential, the acceleration field is directly solved to compute the hydrodynamic pressure. The fluid-body interaction is taken into consideration in the implicit form of the body surface boundary condition of the acceleration potential. With this method, accuracy of hydrodynamic pressure computation is significantly improved. In order to demonstrate the capability of this new method, large amplitude motion of a two dimensional midship section body in a rectangle wave basin is simulated and the conservation of volume, momentum and energy are checked. The result shows that this method has excellent accuracy even for the very large amplitude fluid-body interaction problem.

KEY WARDS : Numerical wave basin, Fluid body interaction, Full nonlinear simulation, Acceleration potential, Implicit body surface boundary condition

INTRODUCTION

The numerical treatment for full nonlinear wave simulation was firstly given by Longuet-Higgins in 1976 and his method is well known as Mixed Eulerian and Lagrangian method (MEL). Detonated by this break through, many simulation methods for nonlinear waves and fluid-body interaction problems were developed in the past two decades. But many of them for fluid-body interaction problems are not consistent from hydrodynamic point of view, because the hydrodynamic pressure is computed using the backward finite difference of the velocity potential and consequently the hydrodynamic equilibrium of forces between water and floating bodies are not guaranteed. For consistent time-domain simulations, it is indispensable to solve simultaneous equations of ideal fluid and floating body motions. The first consistent simulation method for two dimensional problem was developed by Vinje & Brevig (1981). They decomposed the acceleration field into four modes corresponding to the unit acceleration of three body motions (heave, sway and roll) and the other accelerations like centripetal acceleration coming from the velocity field, then solved the boundary value problem corresponding to each mode. The solutions of each mode were used with the equation of floating body motions to determine heave, sway and roll acceleration of the body. Since Vinje's method solves the acceleration field four times for the two dimensional problem (seven times if applied to three dimensional case), it is CPU time consuming. The author (Tanizawa,1990) developed a further rational method to solve simultaneous equations in the acceleration field. The author introduced an implicit body surface boundary condition derived from the kinematic body surface boundary condition and the equation of body motions, and showed that simultaneous equations of ideal fluid and floating body motions could be solved without decomposition. Van Daalen (1993) independently came up with the same idea. Recently, the author introduced the nonlinear acceleration potential, extension of Prandtl's idea in the linear wing theory, and derived the exact body surface boundary condition for the acceleration field (Tanizawa,1995a,1995b). With the nonlinear acceleration potential, the physical meaning to solve the acceleration field can be clearly shown.

In this paper, the boundary value problem on the acceleration potential is formulated first, then the formulation is extended to multiple fluid domains and body interaction problem. An alternative formulation for numerical methods is also given and a numerical simulation method is proposed. As demonstrations , simulations of two dimensional floating body motions with fluid cargo is presented and the accuracy of this method is examined.

EULER'S EQUATION OF THE IDEAL FLUID AND THE ACCELERATION POTENTIAL

First of all, let us define the nonlinear acceleration potential from Euler's equation of the ideal fluid. Non-dimensional Euler's equation of the ideal fluid ($\rho = g = 1$) can be written as

$$\boldsymbol{a} = \frac{D\boldsymbol{v}}{Dt} = \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla p - \nabla Z, \qquad (1)$$

where $\boldsymbol{v}$ and $\boldsymbol{a}$ are the velocity and acceleration vectors of the fluid particle respectively. Introducing the velocity potential ϕ, (1) can be written as

$$\begin{aligned}\boldsymbol{a} &= \frac{D\nabla\phi}{Dt} = \frac{\partial\nabla\phi}{\partial t} + (\nabla\phi\cdot\nabla)\nabla\phi \\ &= \nabla\frac{\partial\phi}{\partial t} + \nabla\left(\frac{1}{2}(\nabla\phi)^2\right) = \nabla\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right). \qquad (2)\end{aligned}$$

Here, let us define the acceleration potential $\varPhi$ as

$$\varPhi = \frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2, \qquad (3)$$

then fluid acceleration is expressed as $\boldsymbol{a} = \nabla\varPhi$. This is an extension of Prandtl's acceleration potential to the nonlinear case. The acceleration field described by this acceleration potential is irrotational, but dose not satisfy Laplace's equation ($\nabla^2\varPhi \neq 0$) because of the nonlinearity of the second term of the right side of equation (3). From equations (1),(2) and (3), the acceleration potential is written as

$$\varPhi = -p - Z + const.\ , \qquad (4)$$

where the integral constant can be set to zero. Therefore physical meaning of the acceleration potential is very clear. Despite of this advantage, the acceleration potential has been rarely used to solve hydrodynamic problems. One reason is that the acceleration field neet not be solved in the framework of linear theory. But besides this reason, there exist two unsolved problems. These are (1) the body surface boundary condition of the acceleration potential is not clearly obtained and (2) the acceleration potential is nonlinear and dose not satisfy Laplace's equation. These problems are overcome in this paper.

FORMULATION OF BOUNDARY VALUE PROBLEM ON THE ACCELERATION FIELD

Acceleration of fluid particle on the body surface

In order to get the kinematic body surface boundary condition, let us first study the acceleration of fluid particle sliding on the body surface. As illustrated in Fig.1, the space fixed reference frame $O-XYZ$ and the body fixed reference frame $o-xyz$ are used. The origin of the body fixed reference frame is situated at the center of gravity of the body and moving with translating velocity $\boldsymbol{v}_o$ and angular velocity $\boldsymbol{\omega}$. The relation between $O-XYZ$ and $o-xyz$ frames are described by the fundamental vectors $\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}$. In Fig.1, P is a point fixed to the fluid particle sliding on the body surface. Using position vectors $\boldsymbol{R}$, $\boldsymbol{R}_o$ and $\boldsymbol{r}$ illustrated in Fig.1, the position, velocity and acceleration vectors of point P are expressed as

$$\begin{aligned}\boldsymbol{R} &= \boldsymbol{R}_o + \boldsymbol{r} &(5)\\ \boldsymbol{v} &= \dot{\boldsymbol{R}}_o + \dot{\boldsymbol{r}} = \boldsymbol{v}_o + [\boldsymbol{v}] + \boldsymbol{\omega}\times\boldsymbol{r} &(6)\\ \boldsymbol{a} &= \ddot{\boldsymbol{R}}_o + \ddot{\boldsymbol{r}} &\\ &= \boldsymbol{a}_o + [\boldsymbol{a}] + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) + 2\boldsymbol{\omega}\times[\boldsymbol{v}] + \dot{\boldsymbol{\omega}}\times\boldsymbol{r}, &(7)\end{aligned}$$

where $[\boldsymbol{v}]$ and $[\boldsymbol{a}]$ are velocity and acceleration of point P observed from the fram $o-xyz$ respectively. With these kinematic formulae of velocity and acceleration of point P, the body surface kinematic boundary condition for the acceleration field is derived in the next section.

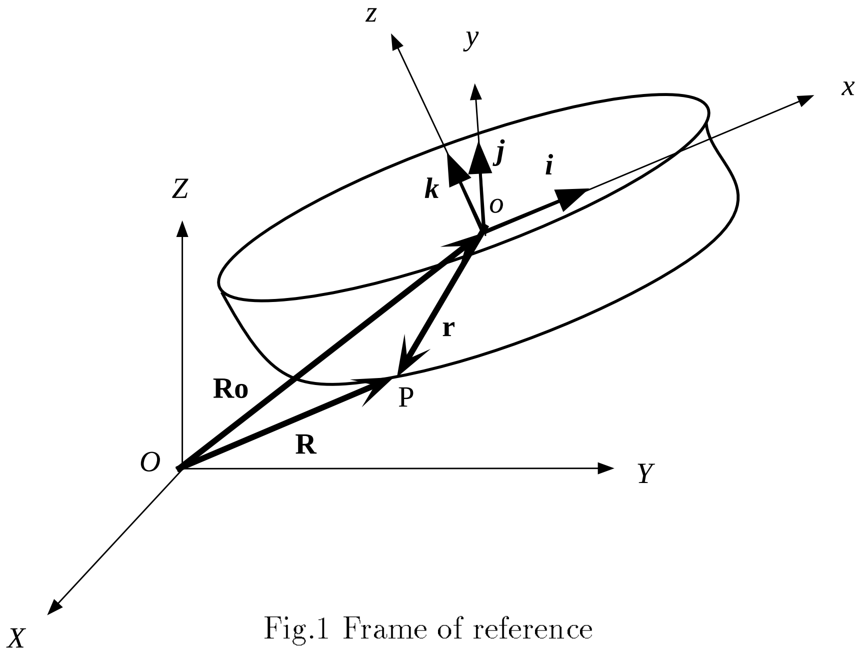


Fig.1 Frame of reference

Body surface kinematic boundary condition

Similar to the kinematic boundary condition of the velocity field, the kinematic boundary condition of the acceleration field can be expressed as scalar product of the acceleration vector of fluid particle and the unit normal vector of body surface at the fluid particle locates. That is

$$\frac{\partial\varPhi}{\partial n} = \boldsymbol{n}\cdot\nabla\varPhi = \boldsymbol{n}\cdot\boldsymbol{a}, \qquad (8)$$

where $\boldsymbol{n}$ is the unit normal vector of body surface at point P. Substituting (7) into (8) gives the following relation

$$\begin{aligned}\frac{\partial\varPhi}{\partial n} &= \boldsymbol{n}\cdot[\boldsymbol{a}] + \boldsymbol{n}\cdot(\boldsymbol{a}_o + \dot{\boldsymbol{\omega}}\times\boldsymbol{r}) \\ &\quad + \boldsymbol{n}\cdot\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) + \boldsymbol{n}\cdot 2\boldsymbol{\omega}\times[\boldsymbol{v}]. \qquad (9)\end{aligned}$$

This is the kinematic body surface boundary condition for the acceleration field. Since the fourth term of the right side includes velocity $[\boldsymbol{v}]$, this boundary condition depends on the velocity field. So, let us rewrite (9) with the velocity potential ϕ. First, considering (6), $[\boldsymbol{v}]$ can be written as

$$[\boldsymbol{v}] = \boldsymbol{v} - \boldsymbol{v}_o - \boldsymbol{\omega}\times\boldsymbol{r} = \nabla\phi - \boldsymbol{v}_o - \boldsymbol{\omega}\times\boldsymbol{r}. \qquad (10)$$

Second, normal and tangential components of $[\boldsymbol{a}]$ to the body surface can be written as

$$[\boldsymbol{a}]_n = -k_n\ [\boldsymbol{v}]^2, \qquad [\boldsymbol{a}]_s = [\dot{\boldsymbol{v}}]_s, \qquad (11)$$

where k_n is the normal curvature of the body surface along the path line of P. The value of $[\boldsymbol{a}]_s$ in (11) is unknown, but $\boldsymbol{n}\cdot[\boldsymbol{a}]_s$ is zero because $\boldsymbol{n}$ and $[\boldsymbol{a}]_s$ are orthogonal. Therefore

$$\begin{aligned}\boldsymbol{n}\cdot[\boldsymbol{a}] &= \boldsymbol{n}\cdot([\boldsymbol{a}]_n + [\boldsymbol{a}]_s) = \boldsymbol{n}\cdot[\boldsymbol{a}]_n \\ &= -k_n\ [\boldsymbol{v}]^2 = -k_n\ (\nabla\phi - \boldsymbol{v}_o - \boldsymbol{\omega}\times\boldsymbol{r})^2. \qquad (12)\end{aligned}$$

Finally with (10) and (12), the kinematic boundary condition of the acceleration field reduces to

$$\begin{aligned}\frac{\partial\varPhi}{\partial n} &= -k_n\ (\nabla\phi - \boldsymbol{v}_o - \boldsymbol{\omega}\times\boldsymbol{r})^2 \\ &\quad + \boldsymbol{n}\cdot(\boldsymbol{a}_o + \dot{\boldsymbol{\omega}}\times\boldsymbol{r}) \\ &\quad + \boldsymbol{n}\cdot\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) \qquad (13) \\ &\quad + \boldsymbol{n}\cdot 2\boldsymbol{\omega}\times(\nabla\phi - \boldsymbol{v}_o - \boldsymbol{\omega}\times\boldsymbol{r}).\end{aligned}$$

Euler's equation of 3-D solid body motions

The second term of the right side of (13) includes the body acceleration $\boldsymbol{a}_o$ and $\dot{\boldsymbol{\omega}}$. Therefore, the body surface boundary condition can not be determined explicitly when the body acceleration is unknown. In such a case, the equation of body motions can be used to eliminate the unknown body acceleration from (13). The equation of 3-D solid body motions is written as

$$\boldsymbol{\mathcal{M}}\ \boldsymbol{\alpha} + \boldsymbol{\beta} = \boldsymbol{F}, \tag{14}$$

where $\boldsymbol{\mathcal{M}}$ is generalized inertia tensor of the body, $\boldsymbol{\alpha} = (a_{ox}\boldsymbol{i} + a_{oy}\boldsymbol{j} + a_{oz}\boldsymbol{k},\ \dot{\omega}_x\boldsymbol{i} + \dot{\omega}_y\boldsymbol{j} + \dot{\omega}_z\boldsymbol{k})$ generalized acceleration vector of the body and $\boldsymbol{F} = (f_x\boldsymbol{i} + f_y\boldsymbol{j} + f_z\boldsymbol{k},\ M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k})$ generalized force acting on the body. The term $\boldsymbol{\beta}$ is so called Gyro moment. The components of(14) are written as

$$\begin{aligned}
&\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & I_{xy} & I_{xz} \\ 0 & 0 & 0 & I_{yx} & I_{yy} & I_{yz} \\ 0 & 0 & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{Bmatrix} a_{ox} \\ a_{oy} \\ a_{oz} \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{Bmatrix} \\
+ &\begin{Bmatrix} 0 \\ 0 \\ 0 \\ (I_{zz} - I_{yy})\omega_y\omega_z - I_{xy}\omega_z\omega_x + I_{zx}\omega_x\omega_y + I_{yz}(\omega_y^2 - \omega_z^2) \\ (I_{xx} - I_{zz})\omega_z\omega_x - I_{yz}\omega_x\omega_y + I_{xy}\omega_y\omega_z + I_{zx}(\omega_z^2 - \omega_x^2) \\ (I_{yy} - I_{xx})\omega_x\omega_y - I_{zx}\omega_y\omega_z + I_{yz}\omega_z\omega_x + I_{xy}(\omega_x^2 - \omega_y^2) \end{Bmatrix} \\
= &\begin{Bmatrix} f_x \\ f_y \\ f_z \\ M_x \\ M_y \\ M_z \end{Bmatrix},
\end{aligned} \tag{15}$$

Now, introducing the generalized normal vector $\boldsymbol{N} = (n_x\boldsymbol{i} + n_y\boldsymbol{j} + n_z\boldsymbol{k},\ (n_yz - n_zy)\boldsymbol{i} + (n_zx - n_xz)\boldsymbol{j} + (n_xy - n_yx)\boldsymbol{k})$, the hydraulic force is given as

$$\boldsymbol{F}_f = \int_{S_s} p\ \boldsymbol{N} ds = \int_{S_s} (-\Phi - Z)\ \boldsymbol{N} ds. \tag{16}$$

We denote here the remaining force (thrust, gravity etc.) as $\boldsymbol{F}_g$, then the total force acting on the body is written as

$$\boldsymbol{F} = \boldsymbol{F}_f + \boldsymbol{F}_g = \int_{S_s} (-\Phi - Z)\ \boldsymbol{N} ds + \boldsymbol{F}_g. \tag{17}$$

Equations (14) and (17) give the generalized Euler's equation of 3-D body motions coupled with fluid motion.

$$\boldsymbol{\mathcal{M}}\ \boldsymbol{\alpha} + \boldsymbol{\beta} = \int_{S_s} (-\Phi - Z)\ \boldsymbol{N} ds + \boldsymbol{F}_g. \tag{18}$$

Implicit body surface boundary condition

The unknown acceleration is included in the second term of the right side of (13), and other terms can be explicitly evaluated from the solution of velocity field. So, let us denote other terms as q for simplicity

$$\begin{aligned} q = &-k_n\ (\nabla\phi - \boldsymbol{v}_o - \boldsymbol{\omega} \times \boldsymbol{r})^2 + \boldsymbol{n} \cdot \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \\ &+ \boldsymbol{n} \cdot 2\boldsymbol{\omega} \times (\nabla\phi - \boldsymbol{v}_o - \boldsymbol{\omega} \times \boldsymbol{r}). \end{aligned} \tag{19}$$

With the generalized acceleration $\boldsymbol{\alpha}$ and normal vector $\boldsymbol{N}$, the second term of the right side of (13) can be written in a much simpler form

$$\boldsymbol{n} \cdot (\boldsymbol{a}_o + \dot{\boldsymbol{\omega}} \times \boldsymbol{r}) = \boldsymbol{n} \cdot \boldsymbol{a}_o + \dot{\omega} \cdot (\boldsymbol{n} \times \boldsymbol{r}) = \boldsymbol{N} \cdot \boldsymbol{\alpha}. \tag{20}$$

Then, (13) is simply written as

$$\frac{\partial\Phi}{\partial n} = \boldsymbol{N} \cdot \boldsymbol{\alpha} + q. \tag{21}$$

Eliminating the generalized acceleration from (18) and (21), an implicit body surface boundary condition

$$\begin{aligned} \frac{\partial\Phi}{\partial n} = &\ \boldsymbol{N}\ \boldsymbol{\mathcal{M}}^{-1} \int_{S_s} (-\Phi)\ \boldsymbol{N} ds \\ &+ \boldsymbol{N}\ \boldsymbol{\mathcal{M}}^{-1} \left\{ \int_{S_s} (-Z)\ \boldsymbol{N} ds + \boldsymbol{F}_g - \boldsymbol{\beta} \right\} \\ &+ q \end{aligned} \tag{22}$$

is finally derived. This condition gives the relation between the acceleration potential Φ and its flux $\partial\Phi/\partial n$ on the body surface.

Free-surface boundary condition

The free-surface boundary condition is given from (4) in the form

$$\Phi_{on\ f.s.} = -Z\ . \tag{23}$$

EXTENTION OF THE FORMULATION FOR MULTIPLE FLUID DOMAINS

The above formulation of the boundary value problem on the acceleration field can be extended to the multiple fluid domains and body interaction problem. Let us consider a problem shown in Fig.2 next. Motion of a floating body with fluid tanks inside is the target of our interest. In order to distinguish fluid domains, serial number κ is given to the fluid-domain as Ω_κ and variables of the domain are written with subscript κ as ϕ_κ, Φ_κ ,etc.

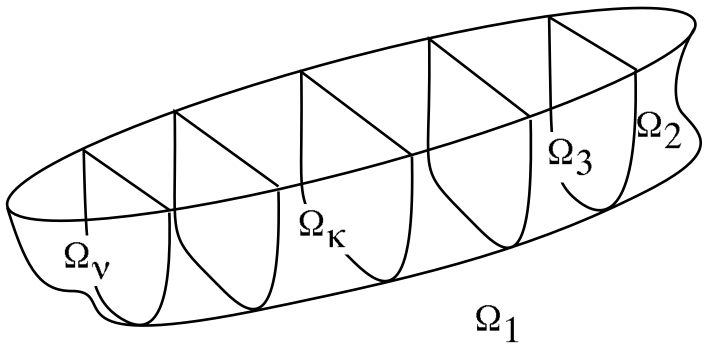


Fig.2 Fluid domains inside and outside of a ship

Similar to (21), the kinematic body surface boundary condition of the domain Ω_κ is written as

$$\frac{\partial\Phi_\kappa}{\partial n} = \boldsymbol{N}_\kappa \cdot \boldsymbol{\alpha} + q_\kappa\ , \tag{24}$$

where $\boldsymbol{N}_\kappa$ is the generalized normal unit vector of the boundary pointing to the outside of domain Ω_κ .

The hydraulic component of the right-hand side of (14) is given in this case as the sum of the hydraulic force

$$\begin{aligned} \boldsymbol{F}_f &= \sum_{\kappa=1}^{\nu} \int_{S_{\kappa s}} p\ \boldsymbol{N}_\kappa ds \\ &= \sum_{\kappa=1}^{\nu} \int_{S_{\kappa s}} \rho_\kappa(-\Phi_\kappa - Z)\ \boldsymbol{N}_\kappa ds\ , \end{aligned} \tag{25}$$

where ν is the number of fluid domains and ρ_κ is the nondimensional density of fluid domain Ω_κ . So, the generalized equation of 3-D body motions coupled with fluid motion inside and outside of the body is written as

$$\boldsymbol{\mathcal{M}}\,\boldsymbol{\alpha}+\boldsymbol{\beta}=\sum_{\kappa=1}^{\nu}\int_{S_{\kappa s}}\rho_\kappa(-\Phi_\kappa-Z)\,\boldsymbol{N}_\kappa ds+\boldsymbol{F}_g. \tag{26}$$

Eliminating the acceleration of the body from (24) and (26), an implicit body surface boundary condition is finally given as

$$\begin{aligned}\frac{\partial\Phi_\kappa}{\partial n}&=\boldsymbol{N}_\kappa\,\boldsymbol{\mathcal{M}}^{-1}\left\{\sum_{\kappa=1}^{\nu}\rho_\kappa\int_{S_{\kappa s}}-\Phi_\kappa\,\boldsymbol{N}_\kappa ds\right\}\\&+\boldsymbol{N}_\kappa\,\boldsymbol{\mathcal{M}}^{-1}\left\{\sum_{\kappa=1}^{\nu}\rho_\kappa\int_{S_{\kappa s}}-Z\,\boldsymbol{N}_\kappa ds+\boldsymbol{F}_g-\boldsymbol{\beta}\right\}\\&+q_\kappa\ .\end{aligned}\tag{27}$$

This is an extended implicit boundary condition which connects the motion of body and the motion of fluid inside and outside of the body.

ALTERNATIVE FORMULATION FOR NUMERICAL METHODS

As mentioned before, the acceleration potential Φ does not satisfy Laplace's equation. So, Φ is not adequate for numerical method like BEM. But (3) shows that the nonlinear part of Φ can be explicitly determined from the solution of velocity field. Therefore it is not necessary to solve the nonlinear part with Φ . Let us subtract this part from Φ and put linear part as

$$\phi_t=\frac{\partial\phi}{\partial t}=\Phi-\frac{1}{2}(\nabla\phi)^2\ . \tag{28}$$

Now, ϕ_t satisfies Laplace's equation. Therefore, with given boundary conditions, the boundary value problem on ϕ_t is easier to be solved than that on Φ . The boundary condition for ϕ_t is easily obtained from equations (24),(27) and (23) as follows:

- Body surface boundary condition

$$\frac{\partial\phi_{\kappa t}}{\partial n}=\boldsymbol{N}_\kappa\cdot\boldsymbol{\alpha}+q_\kappa-\frac{\partial}{\partial n}\left(\frac{1}{2}(\nabla\phi_\kappa)^2\right) \tag{29}$$

- Implicit body surface boundary condition

$$\begin{aligned}\frac{\partial\phi_{\kappa t}}{\partial n}&=\boldsymbol{N}_\kappa\,\boldsymbol{\mathcal{M}}^{-1}\left\{\sum_{\kappa=1}^{\nu}\rho_\kappa\int_{S_{\kappa s}}-\phi_{\kappa t}\,\boldsymbol{N}_\kappa ds\right\}\\&+\boldsymbol{N}_\kappa\,\boldsymbol{\mathcal{M}}^{-1}\left\{\sum_{\kappa=1}^{\nu}\rho_\kappa\int_{S_{\kappa s}}\left(-Z-\frac{1}{2}(\nabla\phi_\kappa)^2\right)\,\boldsymbol{N}_\kappa ds\right\}\\&+\boldsymbol{N}_\kappa\,\boldsymbol{\mathcal{M}}^{-1}(\boldsymbol{F}_g-\boldsymbol{\beta})+q_\kappa-\frac{\partial}{\partial n}\left(\frac{1}{2}(\nabla\phi_\kappa)^2\right)\ .\end{aligned}\tag{30}$$

- Free-surface boundary condition

$$\phi_{\kappa t}=-Z-\frac{1}{2}(\nabla\phi_\kappa)^2 \tag{31}$$

For the numerical evaluation of the term $\frac{\partial}{\partial n}(\frac{1}{2}(\nabla\phi)^2)$ in (29) and (30), the following formula is found accurate and useful.

$$\begin{aligned}&\frac{\partial}{\partial n}\left(\frac{1}{2}(\nabla\phi)^2\right)=\\&-k(\nabla\phi)^2+\frac{\partial\phi}{\partial n}\left(-\frac{\partial^2\phi}{\partial s^2}\right)+\frac{\partial\phi}{\partial s}\frac{\partial}{\partial s}\left(\frac{\partial\phi}{\partial n}\right)\ ,\end{aligned}\tag{32}$$

where k is the curvature of the body surface. Since the right side of (32) can be evaluated only from the surface values, this formula is appropriate to numerical methods like BEM.

FLOW OF THE NUMERICAL SIMULATION METHOD

Fig.3 shows the flow of the new simulation method composed of following five procedures.

(1) The boundary value problem on ϕ is solved and the velocity field is determined.

(2) The boundary condition for ϕ_t is computed using the solution of the velocity field.

(3) The boundary value problem on ϕ_t is solved and the acceleration field is determined.

(4) Using the solution of the acceleration field, the pressure distribution on the body surface, the hydraulic force and the acceleration of body are determined.

(5) Integrating the velocity and acceleration, new position and motions of the body at the next time step are estimated. For the renewal of free-surface, the mixed Eulerian and Lagrangian method is utilized.

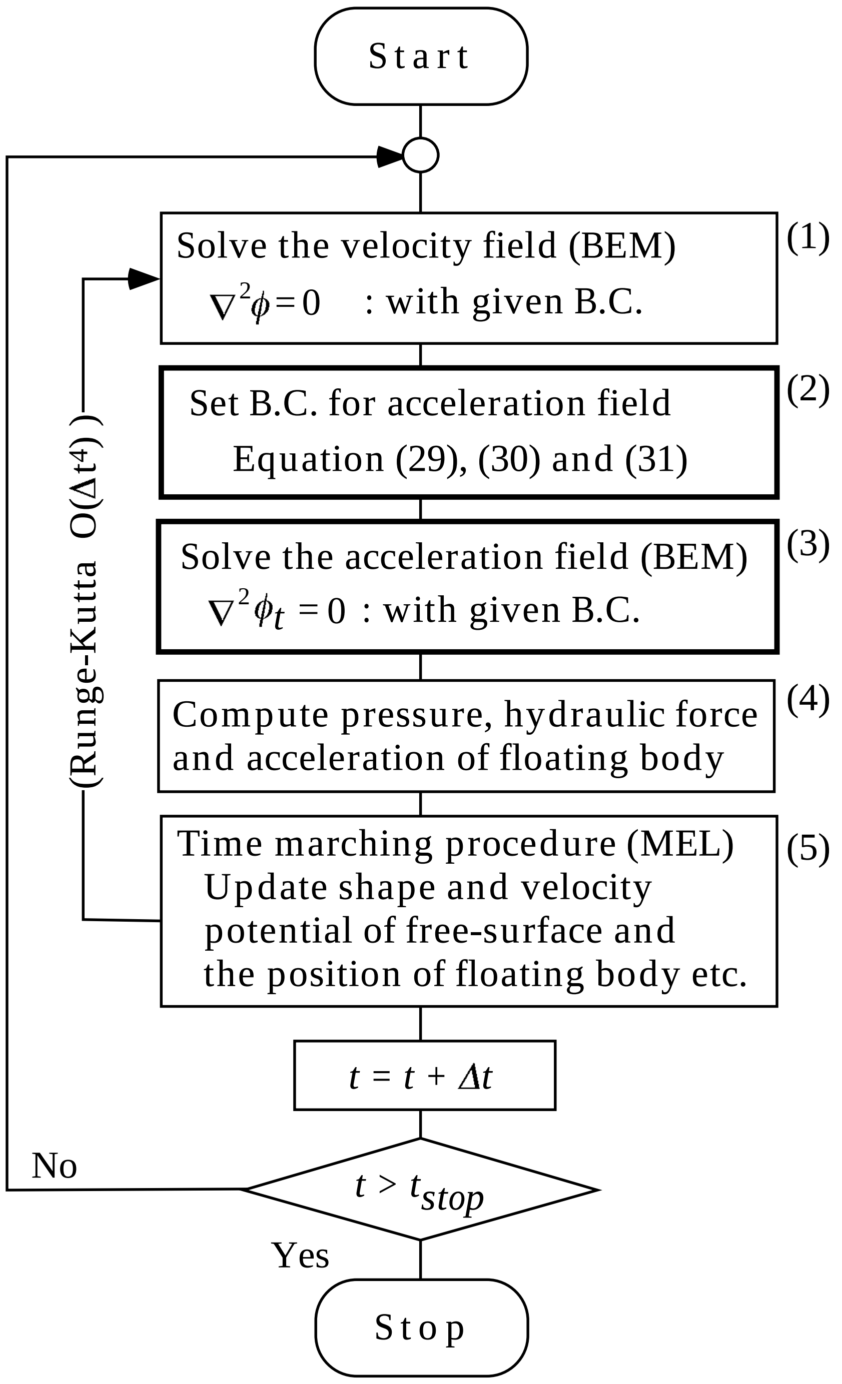
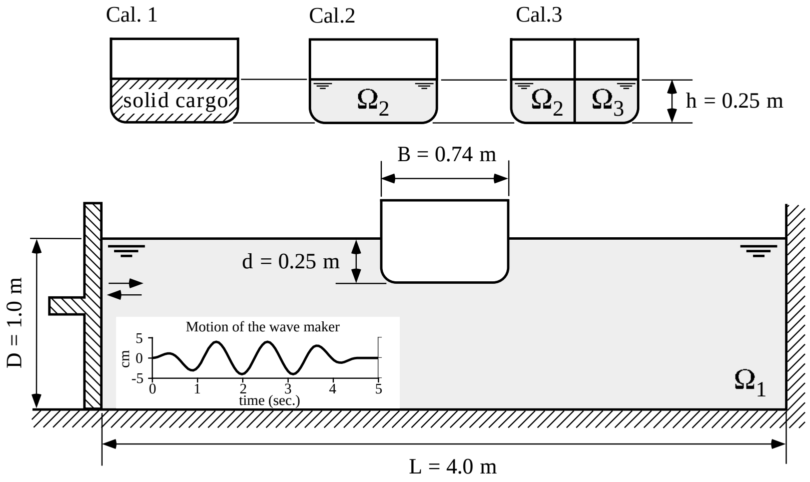


Fig.3 Computational flow formulated with the acceleration potential

This is a consistent simulation method of fluid body interaction problem. Two procedures (2) and (3) guarantee the dynamic equilibrium of forces between body and fluid. In other words, the equations of fluid and body motions can be solved simultaneously by procedures (2) and (3). Moreover, procedures (2) and (3) are effective to reduce CPU time. For the nonlinear time domain simulation, the boundary element matrix must be reconstructed every time step to trace exact moving boundary, which requires more than 50% of entire CPU time. But, since the matrix is the function of boundary shape, we can use the same matrix for both velocity and acceleration fields. Therefore, increment of CPU time by procedures (2) and (3) is not so large. On the other hand, since no backward finite difference is required to compute ϕ_t in this method, we can use larger time step Δt . As a result, CPU time can be reduced considerably.

NUMERICAL SIMULATIONS

In order to demonstrate the capability of this simulation method, three types of two dimensional fluid body interaction problems are simulated. The target of the simulation is large amplitude transient motions of midship section body with three different loading conditions, illustrated in Fig.4 as Cal.1, Cal.2 and Cal.3. In Cal.1, solid cargo is loaded. In Cal.2, fluid cargo is loaded in a single tank. And in Cal.3, the same fluid cargo is loaded in two tanks. A incident wave is generated by a piston wavemaker attached to the left side of the tank. The motion of the wave maker is plotted in the figure. These problems are nondimensionalized using the width of the floating body B , the density of the fluid outside of the body ρ_1 and the gravitational acceleration g as units.



Floating body	Fluid	Solid cargo
m = 36.0 kg	ρ_1 = 1000 kg / m^3	ρ = 800 kg / m^3
i = 2.55 kg-m^2	ρ_2 = 800 kg / m^3	Gravity
	ρ_3 = 800 kg / m^3	g = 9.8 m / s^2

Fig.4 The target of the numerical simulation

Simulated instantaneous free-surface and body motions in seven different time from $t = 9.27$ to $t = 15.45$ are shown in Fig.5-a,b,c. These three figures show that the transient body motions and fluid motion inside and outside of the body are quite large in all cases. In particular, the amplitude of the relative water level at the weather side of the body is very large. Corresponding to the loading condition, differences can be observed in the body motions.; these are plotted in Fig.6. A significant difference can be found between Cal.1 and Cal.2 particularly in roll motion. The fluid motion inside the body affects sway and roll motions strongly in this case. On the other hand, difference is small between Cal.1 and Cal.3. These results show the partition between two fluid tanks effectively reduce the strong interaction.

This method solves the boundary value problem on ϕ_t . Therefore, it is easy to compute distribution of hydrodynamic pressure ϕ_t . A contour plot of hydrodynamic pressure is shown in Fig.7. This plot shows the same instance of the last plot of Fig.5-b.

This simulation method also gives us the pressure time history of any points. Fig.8 shows the pressure time history of inside and outside of the body at the intersection of calm waterline and the weather side of the body. When the intersection point is above the water surface, the pressure is zero. The difference between the pressure inside and outside is important for ship design. Such a kind of information can be also computed by this method.

At the last of this section, the conservation of volume, momentum and energy of fluid are check by comparing the following values.

- Total volume of the fluid

$$V_f = \sum_{\kappa=1}^{\nu} \int\int_{\Omega_\kappa} dv = \sum_{\kappa=1}^{\nu} \int_{s_\kappa} Z\ n_Z\ ds \tag{33}$$

- Total momentum of the fluid

$$\boldsymbol{P}_f = \sum_{\kappa=1}^{\nu} \int\int_{\Omega_\kappa} \nabla\phi_\kappa\ dv = \sum_{\kappa=1}^{\nu} \int_{s_\kappa} \phi_\kappa\ \boldsymbol{n}\ ds \tag{34}$$

- Total energy of the fluid

$$\begin{aligned} E_f &= \sum_{\kappa=1}^{\nu} \int\int_{\Omega_\kappa} \left\{ Z + \frac{1}{2}(\nabla\phi_\kappa)^2 \right\} dv \\ &= \frac{1}{2} \sum_{\kappa=1}^{\nu} \int_{s_\kappa} \left\{ -Z^2 n_Z + \phi_\kappa\ \frac{\partial\phi_\kappa}{\partial n} \right\} ds \end{aligned} \tag{35}$$

- Impulse given from the boundary to the fluid

$$\boldsymbol{I} = \sum_{\kappa=1}^{\nu} \int_0^t \int_{s_\kappa} p\ \boldsymbol{n}\ ds\ dt \tag{36}$$

- Work done by the boundary to the fluid

$$W = \sum_{\kappa=1}^{\nu} \int_0^t \int_{s_\kappa} p\ \frac{\partial\phi_\kappa}{\partial n}\ ds\ dt \tag{37}$$

It is clear from these equations that volume V_f , momentum $\boldsymbol{P}_f$ and energy E_f are computed from the solution of velocity field and impulse $\boldsymbol{I}$ and work W are computed by integrating the solution of the acceleration field with respect to time. So, with accurate solution of both velocity and acceleration fields, computed V_f should be kept constant and $\boldsymbol{P}_f$ and $\boldsymbol{I}$ as well as E_f and W should balance.

Fig.9 shows the time history of $\boldsymbol{P}_f$ & $\boldsymbol{I}$ and E_f & W in Cal.2. These plots demonstrate that momentum and energy conservation are well satisfied. The volume conservation error is also negligibly small $(V_f(0) - V_f(t_{max}))/V_f(0) \approx 0.016\%$.

CONCLUDING REMARKS This work is aimed to develop the three dimensional full nonlinear theory on the wave body interaction problem and the numerical simulation method as its application. As the first step of this work, the mathematical formulation of the boundary value problem of the acceleration field is studied and an idea of the new time domain simulation method is presented. Following items are main results.

1. Euler's equation of ideal fluid motions is transformed to the integral equation of the acceleration potential.
2. Using the kinematic formula of the acceleration of fluid particle sliding on the body surface, the body surface boundary condition for the acceleration potential is derived.
3. Substituting the equation of three dimensional body motions into the body surface boundary condition, the implicit body surface boundary condition for the acceleration potential is derived. With this implicit boundary condition, the ideal fluid motion and the floating body motions can be solved simultaneously.
4. The free-surface boundary condition is added to complete the mathematical formulation of the boundary value problem on the acceleration potential
5. The formulation is extended to multiple fluid domains and body interaction problem.
6. For numerical method like BEM, the nonlinear term in the acceleration potential Φ is shifted from the governing equation to the boundary conditions, then alternative boundary value problem on ϕ_t is formulated.
7. The new full nonlinear time domain simulation method is presented.
8. As a demonstration of this new method, two dimensional large amplitude floating body motions are simulated and the conservation laws are checked. The results show that this method has excellent accuracy even for the large amplitude fulid-body interaction problem.

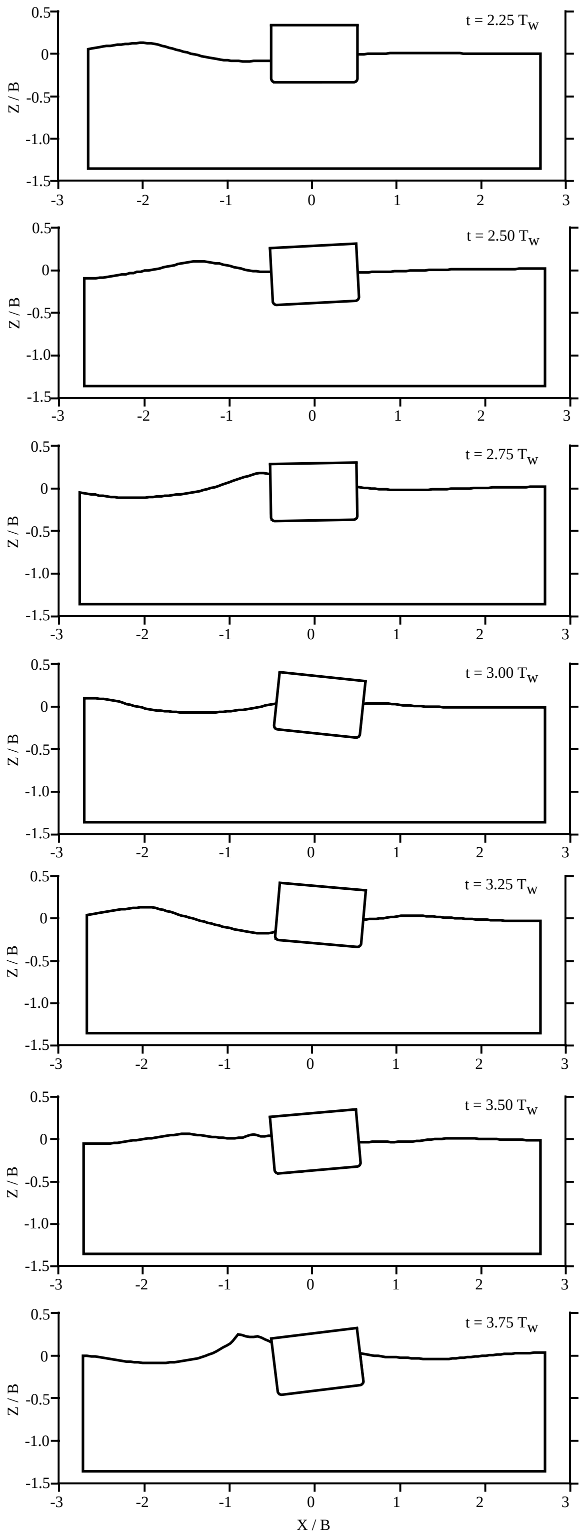


Fig.5-a Simulated fluid and body motions (Cal.1)

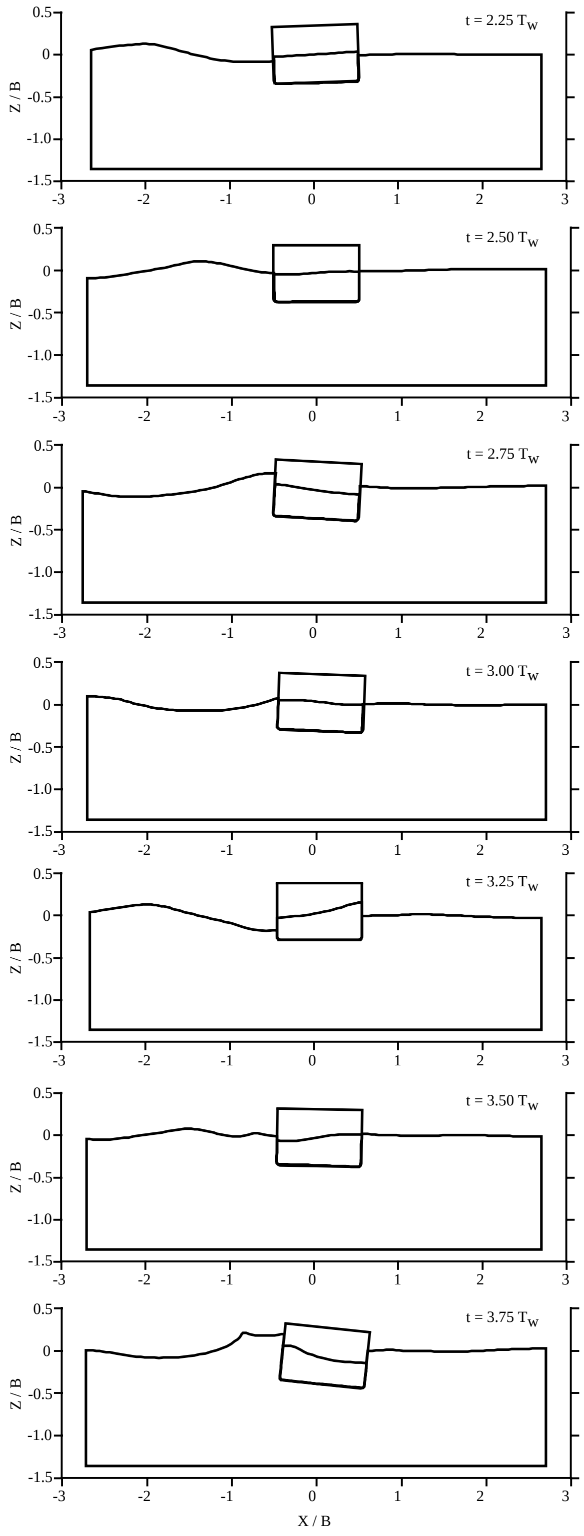


Fig.5-b Simulated fluid and body motions (Cal.2)

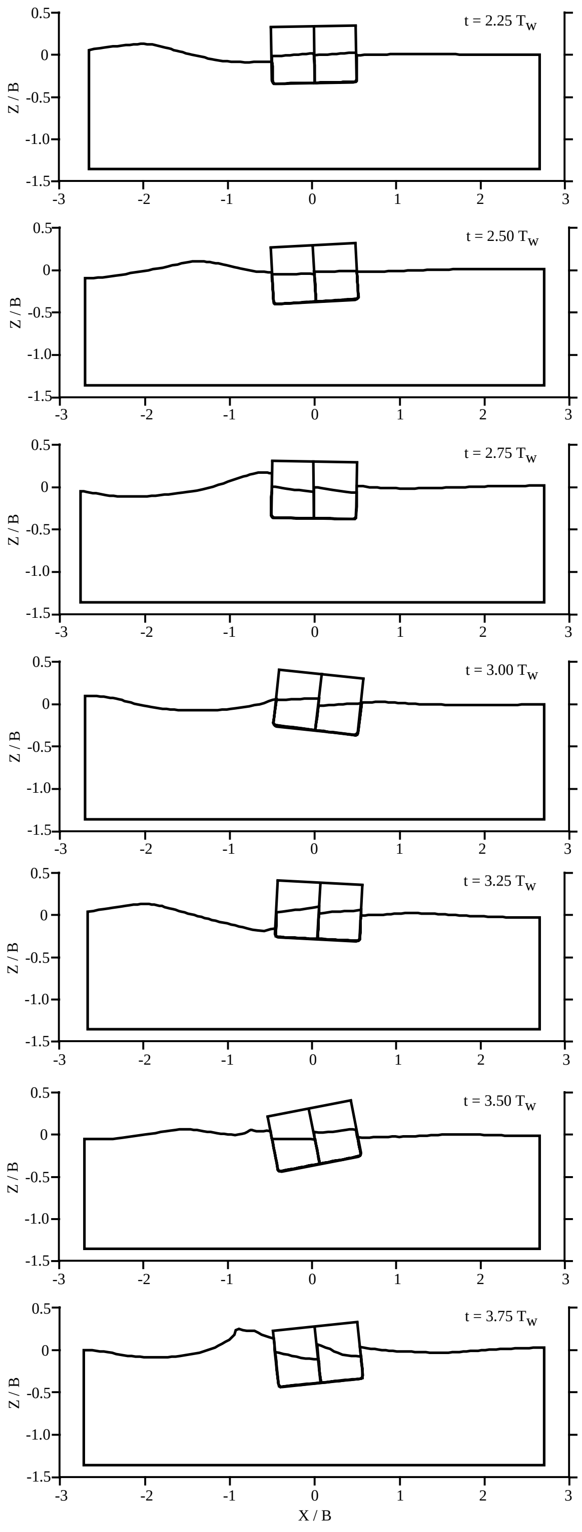


Fig.5-c Simulated fluid and body motions (Cal.3)

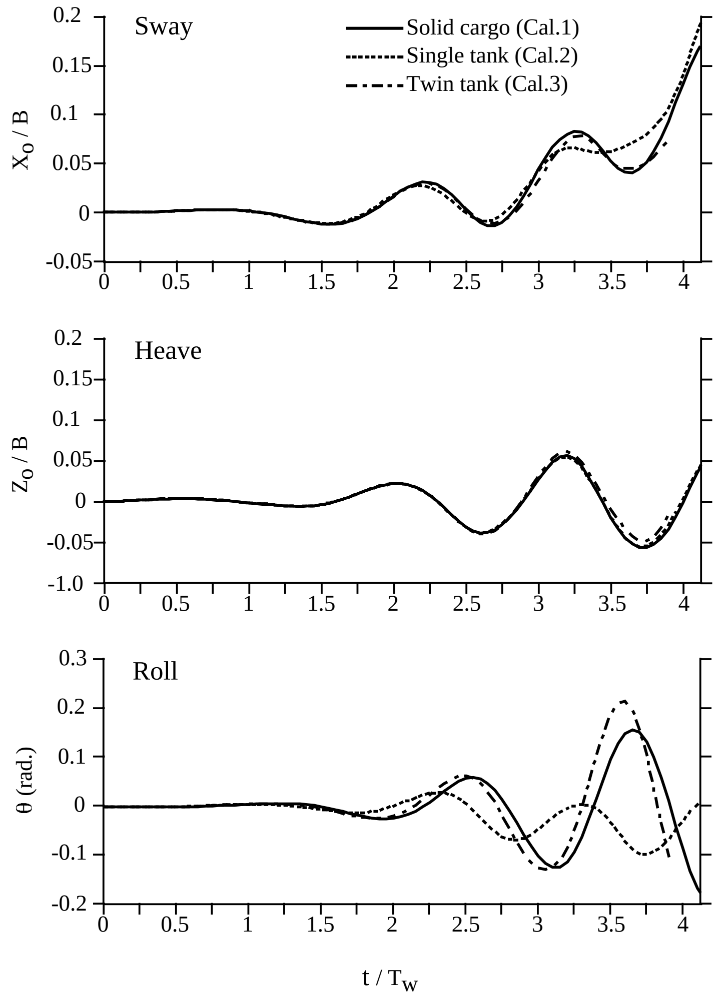


Fig.6 Floating body motions

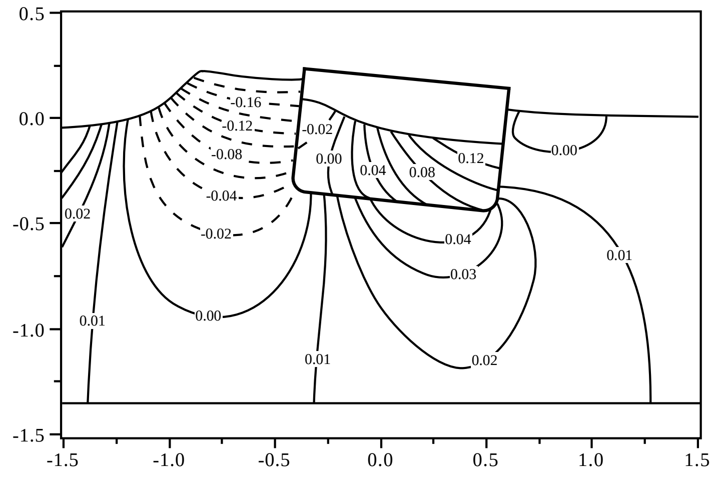


Fig.7 Contour plot of ϕ_t (Cal.2 t=15.45)

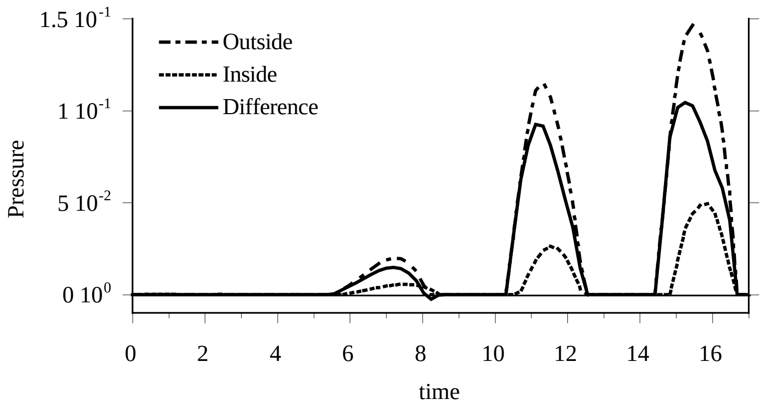


Fig.8 Pressure time history of inside and outside of the body

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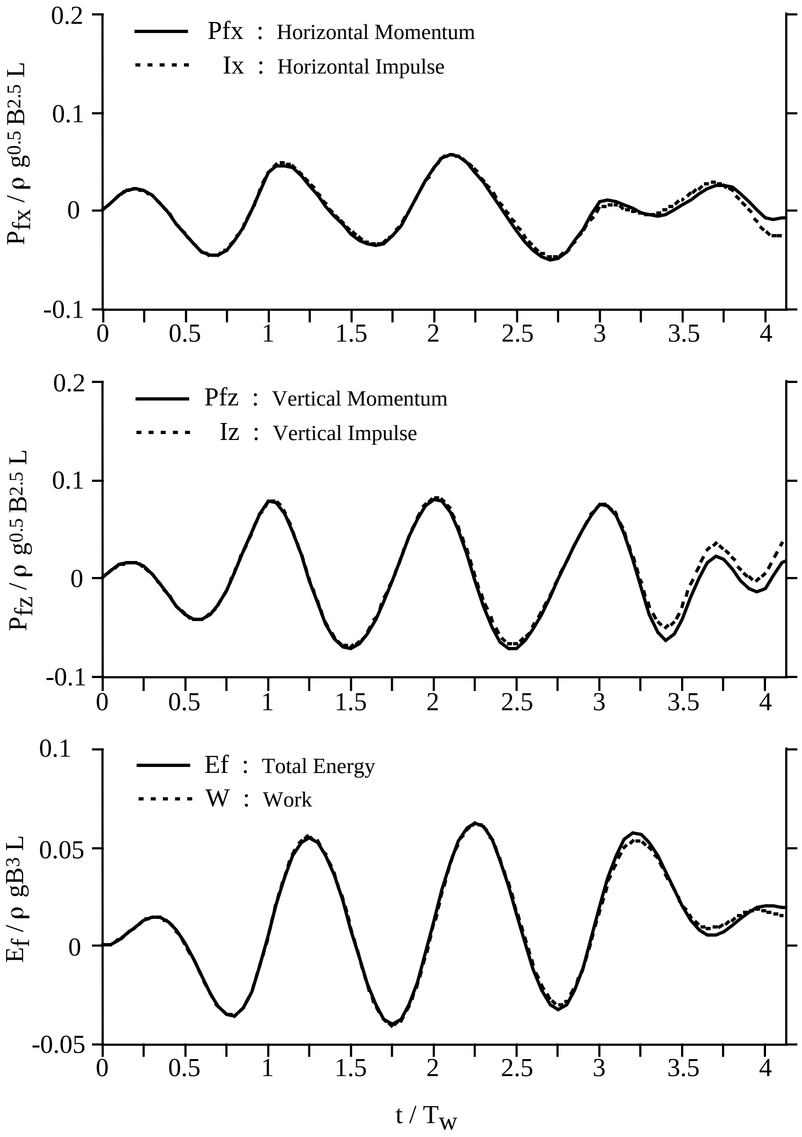

Fig.9 Conservation check of momentum and energy