

A Nonlinear Simulation Method of 3-D Body Motions in Waves

Formulation of the Method with Acceleration Potential (1st report)

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Summary

A full nonlinear method to simulate three dimensional body motions in waves is presented. This is a time domain method to simulate Euler's equation of ideal fluid motion coupled with the equation of solid body motions.

Introducing Prandtl's nonlinear acceleration potential, whose gradient gives acceleration of the fluid, Euler's differential equation of the ideal fluid motion is converted to the integral equation of the acceleration potential. The boundary condition of the acceleration potential on the body surface is systematically derived from the kinematic relation between the acceleration of the solid body and the acceleration of the fluid on the body surface. Since this kinematic boundary condition is a function of the body acceleration, the boundary values on the floating body can not be evaluated explicitly. To overcome this point, the unknown acceleration of the free floating body is eliminated by substituting the equation of body motion into kinematic condition, then implicit body surface boundary condition is derived. This is the kinematic and dynamic condition which guarantees dynamic equilibrium of forces between ideal fluid and the solid body at any instance. With the free-surface boundary condition of the acceleration potential, the formulation of the boundary value problem for the acceleration field is completed.

Although this formulation of the acceleration field is mathematically correct, this is not appropriate to numerical computation, because Prandtl's nonlinear acceleration potential does not satisfy Laplace's equation. Therefore, the nonlinear part is shifted from the governing equation to the boundary condition, then the alternative formulation for the numerical computation is derived. The computational flow of the nonlinear simulation method based on this alternative formulation is also given. In order to show the accuracy of this new method, two dimensional numerical results are presented. They show that the conservation of mass, momentum and energy are satisfied excellently.

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1 Introduction

Time domain full nonlinear simulation is the most direct approach to compute large amplitude motions of arbitrary shaped bodies in waves. But this approach was neither appropriate to analytical methods nor numerical methods when high speed CPU was expensive and hard to available. Therefore, linear theories like Slender body theory, Strip theory, Unified theory etc. have been investigated and used as practical tools to study the seakeeping performance of ships. ^{1) 2) 3)} These linear theories are based on four basic assumptions, (1) Ideal fluid, (2) Small displacement of free-surface, (3) Small motions of floating body and (4) Slender body shape. Nowadays, the assumption (4) is going to be removed by the development of three dimensional higher order panel methods. ^{4) 5) 6)} Anyway, the linear theories describe arbitrary but small amplitude body motions by superposing frequency response functions of the body to regular waves, then solve the problems in frequency domain. So, the time dependent variables are assumed to be sinusoidal and can be separated as $e^{i\omega t}$ form, where ω is encounter frequency of body and wave component. Therefore, the acceleration of fluid and body can be described as $i\omega e^{i\omega t}$ form, then the simultaneous equation of fluid and body motions can be solved in frequency domain to determine added mass and damping coefficient. The linear theories are very useful as far as motions of body and free-surface are small enough to hold the assumptions (1),(2) and (3), but can not be applied to estimate large body motions. Of course, perturbation method can be used to extend linear theories to weak nonlinear problem, but not to the full nonlinear problem like the bottom emerging ship motions in heavy seas or capsizing of small vessels in a plunging wave.

However, recent design office of ship yards are craving for powerful numerical tools for the advanced ship design by nonlinear analysis and so development of the time domain simulation method is an issue of burning concern. Fortunately, recent development of high speed CPU and parallel processing technology will open the possibility to apply three dimensional time domain simulation methods as a design tool in near future.

The numerical treatment for full nonlinear wave simulation was firstly given by Longuet-Higgins in 1976 and his method is well known as Mixed Eulerian and Lagrangian method (MEL). Detonated by this break through, many time domain simulation methods for nonlinear wave body interaction problem were developed in past two decades, but many of them are not consistent from hydrodynamical point of view because the hydrodynamic pressure is computed by the backward finite difference of the velocity potential and so hydrodynamic equilibrium of forces between water and floating bodies are not guaranteed consequently. For the consistent time domain simulation, it is indispensable to solve the simultaneous equations of ideal fluid motion and floating body motions. The first consistent simulation method for two dimensional problem was developed by Vinje & Brevig ⁸⁾ in 1981. They decomposed the acceleration field into four modes corresponding to the unit acceleration of the three body motions (heave sway and roll) and the other accelerations like the centripetal acceleration comes from the velocity field, then solved the boundary value problem corresponding to each modes in their simulation method. The solutions of the each modes were used with the equation of floating body motions to determine heave, sway and roll acceleration of the body. Since Vinje's method solves the acceleration field four times for two dimensional problem

and seven times if applied to three dimensional case, it is CPU time consuming. So, the authors¹²⁾ developed further rational method to solve the simultaneous equations in the acceleration field in 1990. The authors introduced the implicit body surface boundary condition derived from the kinematic body surface boundary condition and the equation of body motions, and showed the simultaneous equations of ideal fluid motion and floating body motions could be solved without decomposition. Van Daalen¹³⁾ also came up to the same idea independently in 1993.

But, still remained question is the exact kinematic body surface boundary conditions for the acceleration field. Since the relation between the fluid acceleration and the boundary condition is not clearly shown in these works, physical meaning to solve the acceleration field is obscure even now. Therefore, Prandtl's nonlinear acceleration potential is introduced to formulate the boundary value problem of the acceleration field in this paper. Since Prandtl's nonlinear acceleration potential is hydrodynamic pressure itself and its gradient gives acceleration of the fluid, the physical meaning to solve the acceleration field can be clearly shown. Thus §2 and §3 of this paper are spent for the formulation of the boundary value problem on Prandtl's nonlinear acceleration potential. The construction of the numerical simulation method is written in §4 and §5 and some examples of numerical simulations and their accuracy checks are presented in §6.

2 Euler's equation of ideal fluid motion and the acceleration potential

First of all, let us introduce the nonlinear acceleration potential from Euler's equation of the ideal fluid motion. Nondimensional Euler's equation of the ideal fluid motion is written as

$$\mathbf{a} = \frac{D\mathbf{v}}{Dt} = \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p - \nabla Z, \quad (1)$$

where the density of the fluid and the gravitational acceleration are unities, D/Dt is the material derivative, \mathbf{v} and \mathbf{a} are velocity and acceleration of the fluid respectively. Vectors are written in bold type face in this article. Similar to the velocity potential ϕ whose gradient gives the velocity of the fluid as $\mathbf{v} = \nabla\phi$, a scalar function which gradient gives the acceleration of the fluid can be derived from equation (1). Substituting the relation $\mathbf{v} = \nabla\phi$, equation (1) is written as

$$\mathbf{a} = \frac{D\nabla\phi}{Dt} = \frac{\partial\nabla\phi}{\partial t} + (\nabla\phi \cdot \nabla)\nabla\phi = \nabla\frac{\partial\phi}{\partial t} + \nabla\left(\frac{1}{2}(\nabla\phi)^2\right) = \nabla\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right), \quad (2)$$

then gradient of $\partial\phi/\partial t + 1/2(\nabla\phi)^2$ gives the acceleration of the fluid. Here, let us define the nonlinear acceleration potential Φ as

$$\Phi = \frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2. \quad (3)$$

From the definition, fluid acceleration is expressed as $\mathbf{a} = \nabla\Phi$. This is the general form of Prandtl's acceleration potential. Since the second term of the right side of equation (3) is nonlinear, this acceleration potential dose not satisfy Laplace's equation $\nabla^2\Phi \neq 0$, but the acceleration field described by this acceleration potential is irrotational. When Prandtl applied the acceleration potential to his wing theory, he assumed the velocity disturbance is small compared to the wing

forward speed V and used the linearized form $\Phi = \partial\phi/\partial t + V\nabla\phi$. Therefore the linearized acceleration potential is much well known. But, in this article, nonlinear acceleration potential defined by equation (3) is used to solve fluid and body motions. From equations (1),(2) and (3), the acceleration potential is written in another form as

$$\Phi = -p - Z + \text{const.} \quad (\text{Integral constant can be set to zero.}), \quad (4)$$

so that physical meaning of the acceleration potential is very clear. Despite of this clearness, the acceleration potential is rarely used to solve the hydrodynamic problems. The reason seems to be that the acceleration field is not necessarily solved in the framework of linear theory. But in addition, there exist two unsolved problems. These are (1) the body surface boundary condition of the acceleration potential is not clearly obtained and (2) the acceleration potential is nonlinear and dose not satisfy Laplace's equation. These problems are overcome in the following sections.

3 Boundary conditions of the acceleration field

3.1 Acceleration of fluid particle sliding on the body surface

In order to get the kinematic body surface boundary condition, the acceleration of fluid particle sliding on the body surface is studied first. As illustrated in Fig.1, the space fixed reference frame $O - XYZ$ and the body fixed reference frame $o - xyz$ are employed. The origin o is situated at the center of gravity of the body and the frame $o - xyz$ moves with translating velocity \mathbf{v}_o and angular velocity $\boldsymbol{\omega}$. The relation between these two frames is described by the fundamental vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$. The fundamental vector means polar vector for the translating motion and axial vector for the angular motion. So, both \mathbf{v}_o and $\boldsymbol{\omega}$ can be written as $\mathbf{v}_o = v_{ox}\mathbf{i} + v_{oy}\mathbf{j} + v_{oz}\mathbf{k}$ and $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$ respectively. In Fig.1, P is a point fixed to the fluid particle sliding on the body surface. Using the positioning vectors \mathbf{R}_o and $\mathbf{r} = x_P\mathbf{i} + y_P\mathbf{j} + z_P\mathbf{k}$, the positioning vector \mathbf{R} of point P is expressed as

$$\mathbf{R} = \mathbf{R}_o + \mathbf{r}. \quad (5)$$

Here, we know the relation between time differential operators d/dt in $O - XYZ$ frame and $[d/dt]$ in $o - xyz$ frame as $d/dt \equiv [d/dt] + \boldsymbol{\omega} \times$. Using the relation, the derivative of equation (5) with respect to time gives the velocity of point P

$$\mathbf{v} = \dot{\mathbf{R}}_o + \dot{\mathbf{r}} = \mathbf{v}_o + [\dot{\mathbf{r}}] + \boldsymbol{\omega} \times \mathbf{r} = \mathbf{v}_o + [\mathbf{v}] + \boldsymbol{\omega} \times \mathbf{r}, \quad (6)$$

where $[\mathbf{v}] = [\dot{\mathbf{r}}] = \dot{x}_P\mathbf{i} + \dot{y}_P\mathbf{j} + \dot{z}_P\mathbf{k}$. Similarly, the derivative of equation (6) with respect to time gives the acceleration of point P

$$\mathbf{a} = \ddot{\mathbf{R}}_o + \ddot{\mathbf{r}} = \mathbf{a}_o + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + [\mathbf{a}] + 2\boldsymbol{\omega} \times [\mathbf{v}], \quad (7)$$

where $\mathbf{a}_o + \dot{\boldsymbol{\omega}} \times \mathbf{r}$ is the acceleration due to the translational and angular acceleration of the body, $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ is the centripetal acceleration due to the angular velocity of the body, $[\mathbf{a}] = [\ddot{\mathbf{r}}] = \ddot{x}_P\mathbf{i} + \ddot{y}_P\mathbf{j} + \ddot{z}_P\mathbf{k}$ is the acceleration of point P observed from $o - xyz$ frame and $-2\boldsymbol{\omega} \times [\mathbf{v}]$ is Coriolis acceleration orthogonal to $\boldsymbol{\omega}$ and $[\mathbf{v}]$.

3.2 Kinematic boundary condition on the body surface

Similar to the kinematic boundary condition for the velocity potential, the kinematic boundary condition for the acceleration potential can be expressed as a scalar product of the acceleration vector of the fluid particle and the unit normal vector of the body surface at the fluid particle considered. The kinematic boundary conditions for velocity and acceleration potentials are

$$\frac{\partial \phi}{\partial n} = \mathbf{n} \cdot \nabla \phi = \mathbf{n} \cdot \mathbf{v} \quad (8)$$

$$\frac{\partial \Phi}{\partial n} = \mathbf{n} \cdot \nabla \Phi = \mathbf{n} \cdot \mathbf{a} , \quad (9)$$

where $\mathbf{n} = n_x \mathbf{i} + n_y \mathbf{j} + n_z \mathbf{k}$ is the unit normal vector of the body surface. Using equation (6), we have

$$\mathbf{n} \cdot \mathbf{v} = \mathbf{n} \cdot (\mathbf{v}_o + \boldsymbol{\omega} \times \mathbf{r}) + \mathbf{n} \cdot [\mathbf{v}] .$$

Since \mathbf{n} and $[\mathbf{v}]$ are orthogonal, the second term disappears and following well known body surface boundary condition for the velocity potential is obtained.

$$\frac{\partial \phi}{\partial n} = \mathbf{n} \cdot \mathbf{v} = \mathbf{n} \cdot (\mathbf{v}_o + \boldsymbol{\omega} \times \mathbf{r}) \quad (10)$$

Similarly, equation (7) is used to have

$$\mathbf{n} \cdot \mathbf{a} = \mathbf{n} \cdot (\mathbf{a}_o + \dot{\boldsymbol{\omega}} \times \mathbf{r}) + \mathbf{n} \cdot \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \mathbf{n} \cdot [\mathbf{a}] + \mathbf{n} \cdot 2\boldsymbol{\omega} \times [\mathbf{v}] . \quad (11)$$

Here the velocity potential ϕ can be used to express $[\mathbf{v}]$ and $[\mathbf{a}]$. From equation (6), $[\mathbf{v}]$ is written as

$$[\mathbf{v}] = \mathbf{v} - \mathbf{v}_o - \boldsymbol{\omega} \times \mathbf{r} = \nabla \phi - \mathbf{v}_o - \boldsymbol{\omega} \times \mathbf{r} . \quad (12)$$

Since $[\mathbf{v}]$ is tangential to the body surface as far as no separation occurs, normal and tangential components of $[\mathbf{a}]$ are given by

$$\begin{aligned} [\mathbf{a}]_n &= -k_n ([\mathbf{v}])^2 \\ [\mathbf{a}]_s &= [\dot{\mathbf{v}}]_s , \end{aligned} \quad (13)$$

where k_n is normal curvature of the body surface along the path line of fluid. As shown in Fig.2, if the piece of body surface around point P is smooth enough and be expressed by a pair of parameters u, v as $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, and if the path line of point P is expressed by projection of line $u(s), v(s)$ on the $u - v$ plane as $\mathbf{P}(s) = \mathbf{P}(u(s), v(s))$, the normal curvature k_n is given by the second fundamental form of differential geometry

$$k_n(s) = \text{II} \left(\frac{d\mathbf{P}(s)}{ds}, \frac{d\mathbf{P}(s)}{ds} \right) . \quad (14)$$

Term $[\mathbf{a}]_s$ in equation (13) is unknown, but $\mathbf{n} \cdot [\mathbf{a}]_s$ disappears because of orthogonality. Then, $\mathbf{n} \cdot [\mathbf{a}]$ becomes

$$\begin{aligned} \mathbf{n} \cdot [\mathbf{a}] &= \mathbf{n} \cdot ([\mathbf{a}]_n + [\mathbf{a}]_s) = \mathbf{n} \cdot [\mathbf{a}]_n \\ &= -k_n ([\mathbf{v}])^2 = -k_n (\nabla \phi - \mathbf{v}_o - \boldsymbol{\omega} \times \mathbf{r})^2 . \end{aligned} \quad (15)$$

Finally the kinematic boundary condition of the acceleration field is reduced to be

$$\begin{aligned} \frac{\partial \Phi}{\partial n} &= \mathbf{n} \cdot (\mathbf{a}_o + \dot{\boldsymbol{\omega}} \times \mathbf{r}) + \mathbf{n} \cdot \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \\ &\quad - k_n (\nabla \phi - \mathbf{v}_o - \boldsymbol{\omega} \times \mathbf{r})^2 + \mathbf{n} \cdot 2\boldsymbol{\omega} \times (\nabla \phi - \mathbf{v}_o - \boldsymbol{\omega} \times \mathbf{r}) . \end{aligned} \quad (16)$$

3.3 Euler's equation of solid body motions coupled with fluid motion

Since the body surface boundary condition given by equation (16) includes the body acceleration \mathbf{a}_o and $\dot{\boldsymbol{\omega}}$, this boundary condition can not be applied to the free floating body surface because the body acceleration is unknown before we solve the acceleration field. In such a case, the equation of body motion can be used to eliminate unknown body acceleration from equation (16).

The equation of translational body motion is given by

$$m\mathbf{a}_o = m\dot{\mathbf{v}}_o = m[\dot{\mathbf{v}}_o] + \boldsymbol{\omega} \times m\mathbf{v}_o = \mathbf{f} , \quad (17)$$

where m is the body mass, $\mathbf{a}_o = a_{ox}\mathbf{i} + a_{oy}\mathbf{j} + a_{oz}\mathbf{k}$ is the acceleration of the center of gravity and $\mathbf{f} = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$ is the force acts to the body. Taking notice of that the relation between $O - XYZ$ frame and $o - xyz$ frame is described by the fundamental vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, the components of above equation are given by

$$\left. \begin{aligned} ma_{ox} &= m(\dot{v}_{ox} + v_{oz}\omega_y - v_{oy}\omega_z) = f_x \\ ma_{oy} &= m(\dot{v}_{oy} + v_{ox}\omega_z - v_{oz}\omega_x) = f_y \\ ma_{oz} &= m(\dot{v}_{oz} + v_{oy}\omega_x - v_{ox}\omega_y) = f_z \end{aligned} \right\} , \quad (18)$$

Similarly, the equation of angular body motion and its components are given by

$$\dot{\mathbf{H}} = [\dot{\mathbf{H}}] + \boldsymbol{\omega} \times \mathbf{H} = \mathbf{M} , \quad (19)$$

$$\left. \begin{aligned} \dot{H}_x + H_z\omega_y - H_y\omega_z &= M_x \\ \dot{H}_y + H_x\omega_z - H_z\omega_x &= M_y \\ \dot{H}_z + H_y\omega_x - H_x\omega_y &= M_z \end{aligned} \right\} \quad (20)$$

where $\mathbf{H} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$ is angular momentum of the body and $\mathbf{M} = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$ is moment acts to the body. With the angular velocity $\boldsymbol{\omega}$ and the inertia tensor

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} , \quad (21)$$

the components of angular momentum $\mathbf{H} = \mathbf{I} \boldsymbol{\omega}$ are given by

$$\left\{ \begin{array}{l} H_x \\ H_y \\ H_z \end{array} \right\} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \left\{ \begin{array}{l} \omega_x \\ \omega_y \\ \omega_z \end{array} \right\} . \quad (22)$$

Substituting this equation into equation (20) gives following equation.

$$\left. \begin{aligned} I_{xx}\dot{\omega}_x &+ (I_{zz} - I_{yy})\omega_y\omega_z &+ I_{xy}(\dot{\omega}_y - \omega_z\omega_x) \\ &+ I_{zx}(\dot{\omega}_z + \omega_x\omega_y) &+ I_{yz}(\omega_y^2 - \omega_z^2) &= M_x \\ I_{yy}\dot{\omega}_y &+ (I_{xx} - I_{zz})\omega_z\omega_x &+ I_{yz}(\dot{\omega}_z - \omega_x\omega_y) \\ &+ I_{xy}(\dot{\omega}_x + \omega_y\omega_z) &+ I_{zx}(\omega_z^2 - \omega_x^2) &= M_y \\ I_{zz}\dot{\omega}_z &+ (I_{yy} - I_{xx})\omega_x\omega_y &+ I_{zx}(\dot{\omega}_x - \omega_y\omega_z) \\ &+ I_{yz}(\dot{\omega}_y + \omega_z\omega_x) &+ I_{xy}(\omega_x^2 - \omega_y^2) &= M_z \end{aligned} \right\} \quad (23)$$

This equation is called Euler's equation of three dimensional angular body motions.

Next, let us introduce the generalized values. The generalized inertia tensor of the body is defined as

$$\mathcal{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & I_{xy} & I_{xz} \\ 0 & 0 & 0 & I_{yx} & I_{yy} & I_{yz} \\ 0 & 0 & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix}. \quad (24)$$

The generalized body acceleration $\boldsymbol{\alpha}$ is defined as a pair of translational acceleration $\mathbf{a}_o = a_{ox}\mathbf{i} + a_{oy}\mathbf{j} + a_{oz}\mathbf{k}$ and angular acceleration $\dot{\boldsymbol{\omega}} = [\dot{\boldsymbol{\omega}}] + \boldsymbol{\omega} \times \boldsymbol{\omega} = [\dot{\boldsymbol{\omega}}] = \dot{\omega}_x\mathbf{i} + \dot{\omega}_y\mathbf{j} + \dot{\omega}_z\mathbf{k}$

$$\boldsymbol{\alpha} = (a_{ox}\mathbf{i} + a_{oy}\mathbf{j} + a_{oz}\mathbf{k}, \dot{\omega}_x\mathbf{i} + \dot{\omega}_y\mathbf{j} + \dot{\omega}_z\mathbf{k}). \quad (25)$$

In the same way, the generalized force acts to the body is also defined as a pair of force and moment

$$\mathbf{F} = (f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}, M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}). \quad (26)$$

Using these generalized values, equation (17) can be combined with (19) to have single equation

$$\mathcal{M} \boldsymbol{\alpha} + \boldsymbol{\beta} = \mathbf{F}. \quad (27)$$

The components of this equation are

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & I_{xy} & I_{xz} \\ 0 & 0 & 0 & I_{yx} & I_{yy} & I_{yz} \\ 0 & 0 & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{Bmatrix} a_{ox} \\ a_{oy} \\ a_{oz} \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ (I_{zz} - I_{yy})\omega_y\omega_z - I_{xy}\omega_z\omega_x + I_{zx}\omega_x\omega_y + I_{yz}(\omega_y^2 - \omega_z^2) \\ (I_{xx} - I_{zz})\omega_z\omega_x - I_{yz}\omega_x\omega_y + I_{xy}\omega_y\omega_z + I_{zx}(\omega_z^2 - \omega_x^2) \\ (I_{yy} - I_{xx})\omega_x\omega_y - I_{zx}\omega_y\omega_z + I_{yz}\omega_z\omega_x + I_{xy}(\omega_x^2 - \omega_y^2) \end{Bmatrix} = \begin{Bmatrix} f_x \\ f_y \\ f_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}, \quad (28)$$

where the second term of the left side is the components of $\boldsymbol{\beta}$ which appears because $o - xyz$ frame has angular velocity.

On the other hand, hydraulic force \mathbf{f}_f and moment \mathbf{M}_f act to the body are expressed by the pressure integral on the body surface

$$\mathbf{f}_f = \int_{S_s} p \mathbf{n} ds \quad (29)$$

$$\mathbf{M}_f = \int_{S_s} p \mathbf{n} \times \mathbf{r} ds. \quad (30)$$

Using the hydraulic force $\mathbf{f}_f = f_{fx}\mathbf{i} + f_{fy}\mathbf{j} + f_{fz}\mathbf{k}$ and moment $\mathbf{M}_f = M_{fx}\mathbf{i} + M_{fy}\mathbf{j} + M_{fz}\mathbf{k}$, the generalized hydraulic forth can be defined as

$$\mathbf{F}_f = (f_{fx}\mathbf{i} + f_{fy}\mathbf{j} + f_{fz}\mathbf{k}, M_{fx}\mathbf{i} + M_{fy}\mathbf{j} + M_{fz}\mathbf{k}). \quad (31)$$

Moreover, normal vector can be also generalized with \mathbf{n} and $\mathbf{n} \times \mathbf{r}$ as

$$\begin{aligned} \mathbf{N} &= (\mathbf{n}, \mathbf{n} \times \mathbf{r}) \\ &= (n_x \mathbf{i} + n_y \mathbf{j} + n_z \mathbf{k}, (n_y z - n_z y) \mathbf{i} + (n_z x - n_x z) \mathbf{j} + (n_x y - n_y x) \mathbf{k}) . \end{aligned} \quad (32)$$

Using these generalized values, equation (29) and (30) can be brought into single form

$$\mathbf{F}_f = \int_{S_s} p \mathbf{N} ds . \quad (33)$$

Substituting equation (4) into equation (33), the generalized hydraulic forth is written as a function of the acceleration potential

$$\mathbf{F}_f = \int_{S_s} (-\Phi - Z) \mathbf{N} ds . \quad (34)$$

Then, denoting other forces like gravity, thrust etc. as a generalized force \mathbf{F}_g , we have the generalized total force acts on the body

$$\mathbf{F} = \mathbf{F}_f + \mathbf{F}_g = \int_{S_s} (-\Phi - Z) \mathbf{N} ds + \mathbf{F}_g . \quad (35)$$

Equation (27) and (35) gives the generalized Euler's equation of three dimensional body motions coupled with fluid motion.

$$\mathcal{M} \boldsymbol{\alpha} + \boldsymbol{\beta} = \int_{S_s} (-\Phi - Z) \mathbf{N} ds + \mathbf{F}_g . \quad (36)$$

3.4 Implicit body surface boundary condition

Using the generalized acceleration $\boldsymbol{\alpha}$ and the generalized normal vector \mathbf{N} , the first term of the right side of equation (16) is simply written as

$$\mathbf{n} \cdot (\mathbf{a}_o + \dot{\boldsymbol{\omega}} \times \mathbf{r}) = \mathbf{n} \cdot \mathbf{a}_o + \dot{\boldsymbol{\omega}} \cdot (\mathbf{n} \times \mathbf{r}) = \mathbf{N} \cdot \boldsymbol{\alpha} . \quad (37)$$

The other terms of the right side of equation (16) can be evaluated from the solution of the velocity field and denoted here as q for simplicity.

$$\begin{aligned} q &= \mathbf{n} \cdot \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \\ &\quad - k_n (\nabla \phi - \mathbf{v}_o - \boldsymbol{\omega} \times \mathbf{r})^2 + \mathbf{n} \cdot 2\boldsymbol{\omega} \times (\nabla \phi - \mathbf{v}_o - \boldsymbol{\omega} \times \mathbf{r}) \end{aligned} \quad (38)$$

Then, equation (16) is simply written in the form

$$\frac{\partial \Phi}{\partial n} = \mathbf{N} \cdot \boldsymbol{\alpha} + q . \quad (39)$$

On the other hand, equation (36) gives

$$\boldsymbol{\alpha} = \mathcal{M}^{-1} \left\{ \int_{S_s} (-\Phi - Z) \mathbf{N} ds + \mathbf{F}_g - \boldsymbol{\beta} \right\} . \quad (40)$$

Substituting equation (40) into equation (39), the implicit body surface boundary condition

$$\begin{aligned} \frac{\partial \Phi}{\partial n} &= \mathbf{N} \mathcal{M}^{-1} \int_{S_s} -\Phi \mathbf{N} ds \\ &\quad + \mathbf{N} \mathcal{M}^{-1} \left\{ \int_{S_s} -Z \mathbf{N} ds + \mathbf{F}_g - \boldsymbol{\beta} \right\} + q \end{aligned} \quad (41)$$

is given. This implicit boundary condition gives the relation between the acceleration potential and its flux on the body surface. This is the kinematic and dynamic condition which connects Euler's equation of fluid motion and Euler's equation of body motion.

3.5 Free-surface boundary condition

Equating the dynamic free-surface boundary condition $p = 0$ and equation (4), we have simple free-surface boundary condition for the acceleration potential

$$\Phi = -Z . \quad (42)$$

4 The formulation for numerical method

As mentioned in §2, the acceleration potential Φ dose not satisfy Laplace's equation because of the nonlinear term $\frac{1}{2}(\nabla\phi)^2$ in equation (3). So, Φ is not appropriate to numerical methods like BEM. But this nonlinear term can be explicitly evaluated from the solution of velocity field. Therefore it is not necessary to solve the nonlinear part with Φ . Let us subtract the nonlinear term from Φ and put linear part as

$$\phi_t = \frac{\partial\phi}{\partial t} = \Phi - \frac{1}{2}(\nabla\phi)^2 . \quad (43)$$

Since ϕ_t satisfies Laplace's equation, boundary value problem on ϕ_t is easier to be solved than that is on Φ . But, we have to remind that gradient of ϕ_t does not give the fluid acceleration.

The boundary conditions for ϕ_t is easily obtained by substituting equation (43) into (39),(41) and (42). First, the body surface kinematic boundary condition for ϕ_t is

$$\frac{\partial\phi_t}{\partial n} = \mathbf{N} \cdot \boldsymbol{\alpha} + q - \frac{\partial}{\partial n} \left(\frac{1}{2}(\nabla\phi)^2 \right) \quad (44)$$

Next, the implicit body surface boundary condition for ϕ_t is

$$\begin{aligned} \frac{\partial\phi_t}{\partial n} = & \mathbf{N} \mathcal{M}^{-1} \int_{S_s} -\phi_t \mathbf{N} ds \\ & + \mathbf{N} \mathcal{M}^{-1} \left\{ \int_{S_s} \left(-Z - \frac{1}{2}(\nabla\phi)^2 \right) \mathbf{N} ds + \mathbf{F}_g - \boldsymbol{\beta} \right\} + q - \frac{\partial}{\partial n} \left(\frac{1}{2}(\nabla\phi)^2 \right) \end{aligned} \quad (45)$$

Binding the terms which can be evaluated from the solution of velocity field as Q

$$Q = \mathbf{N} \mathcal{M}^{-1} \left\{ \int_{S_s} \left(-Z - \frac{1}{2}(\nabla\phi)^2 \right) \mathbf{N} ds + \mathbf{F}_g - \boldsymbol{\beta} \right\} + q - \frac{\partial}{\partial n} \left(\frac{1}{2}(\nabla\phi)^2 \right) , \quad (46)$$

we have final form of the implicit body surface boundary condition

$$\frac{\partial\phi_t}{\partial n} = \mathbf{N} \mathcal{M}^{-1} \int_{S_s} -\phi_t \mathbf{N} ds + Q . \quad (47)$$

The implicit boundary condition should be discretized by boundary elements when used for BEM. Lastly, free-surface boundary condition for ϕ_t is

$$\phi_t = -Z - \frac{1}{2}(\nabla\phi)^2 . \quad (48)$$

The kinematic body surface boundary condition for the nonlinear acceleration potential Φ is derived from the physical correspondence of $\nabla\Phi$ to the acceleration of fluid \mathbf{a} . On the other hand, ϕ_t does not have such a direct physical correspondence. So, boundary conditions for ϕ_t is derived indirectly but systematically from those for Φ .

5 Application of the acceleration potential to the numerical simulation

Let us apply the boundary value problem on ϕ_t to the nonlinear time domain simulation. Fig.3 shows the flow of the new simulation method which traces the body and free-surface motions step by step from the give initial condition. This method is composed of following five procedures.

- (1) The boundary value problem on ϕ is solved and the velocity field is determined.
- (2) The boundary condition for ϕ_t is computed using the solution of the velocity field.
- (3) The boundary value problem on ϕ_t is solved and the acceleration field is determined.
- (4) Using the solution of the acceleration field, the pressure distribution on the body surface, hydraulic force and the accelerations of body are determined.
- (5) Integrating the velocity and the acceleration, new position and motions of the body at the next time step are estimated. For the renewal of free-surface, the mixed Eulerian and Lagrangian method is utilized.

It can be said that this new simulation method is much consistent compared with methods those do not have above procedures (2) and (3) (referred to as the former method hereinafter). Fig.4 shows the typical flow of the former method. Since the former method does not solve the acceleration field, ϕ_t is computed by the backward finite difference. In brief, denoting the velocity potential of the collocational point i on the body surface at time t as $\phi_i(t)$ and the velocity of the collocational point i as \mathbf{v}_i , $d\phi_i(t)/dt$ is written as

$$\frac{d\phi_i(t)}{dt} = \frac{\partial\phi_i(t)}{\partial t} + \mathbf{v}_i \cdot \nabla\phi_i(t) . \quad (49)$$

Here $d\phi_i(t)/dt$ is approximated by backward finite difference to have

$$\frac{\partial\phi_i(t)}{\partial t} = \frac{\phi_i(t) - \phi_i(t - \Delta t)}{\Delta t} - \mathbf{v}_i \cdot \nabla\phi_i(t) . \quad (50)$$

Thus, the former method computes the hydraulic force by integrating the pressure distribution on the body given by this backward finite difference, computes the acceleration of the body using this hydraulic force and integrates this acceleration to estimate the motion and position of the body for next time step. But again, the backward finite difference is applied to compute pressure in next time step. This sequence of computation is not consistent from the hydrodynamical point of view because the sequence is no more than the repetition of backward finite difference and forward time integral. So, there is no reason to have the converged result by this sequence. As a matter of fact, the numerical example shown in the next section shows that the former method gives no converged result even if very small time step Δt is used. This inconsistency is overcome by introducing the procedure (2) and (3). These two procedures guarantee the dynamic equilibrium forces between body and fluid. In other words, the equations of fluid motion and body motions can be solved simultaneously by procedure (2) and (3).

In addition, procedure (2) and (3) are effective to reduce CPU time. For the nonlinear time domain simulation, the boundary element matrix must be reconstructed every time step to trace exact moving boundary and this reconstruction requires more than 50% of entire CPU time. But, since the matrix is the function of boundary shape, we can use same matrix for both velocity field and acceleration field. Therefore, increment of CPU time by procedure (2) and (3) is not so large. On the other hand, since no backward finite difference is required to compute ϕ_t in the new method, we can use larger time step Δt as shown in the next section. As a result, we can reduce CPU time considerably by the new method.

6 Accuracy check of the new simulation method.

As mentioned in the previous section, both the velocity field and the acceleration field are solved in the new simulation method. The physical meaning to solve the velocity field is to get the solution which conserve fluid mass and physical meaning to solve the acceleration field is to get the solution which conserves momentum and energy because we assume ideal fluid without energy loss. Accordingly, the simulated results must satisfy the conservation laws of mass, momentum and energy. Thus, the accuracy of the new simulation method can be verified by checking these conservation laws.

In this report, simple two dimensional simulations are chosen to check the conservation laws. Fig.5 is an illustrative drawing of the simulation. The length of wave tank L , gravitational acceleration g and density of the water ρ are chosen as units and other values are nondimensionalized with these units in the simulations. The number of boundary elements to discretize the boundaries are 36 for floating body, 18 for piston wave-maker and right wall, 20 for bottom of the tank and 184 for free-surface.

Check of the conservation laws are performed by comparing the following values.

- Volume of the fluid

$$V_f = \int \int_v dv = \int_s Y n_Y ds = \int_s Z n_Z ds \quad (51)$$

- Momentum of the fluid

$$\mathbf{P}_f = \int \int_v \nabla \phi dv = \int_s \phi \mathbf{n} ds \quad (52)$$

- Total energy of the fluid

$$E_f = \int \int_v Z + \frac{1}{2}(\nabla \phi)^2 dv = \frac{1}{2} \int_s -Z^2 n_Z + \phi \frac{\partial \phi}{\partial n} ds \quad (53)$$

- Impulse given from the boundary to the fluid

$$\mathbf{I} = \int_0^t \int_s p \mathbf{n} ds dt \quad (54)$$

- Work done by the boundary to the fluid

$$W = \int_0^t \int_s p \frac{\partial \phi}{\partial n} ds dt \quad (55)$$

It is clear from these equations that volume V_f , momentum \mathbf{P}_f and energy E_f are determined by the velocity field and impulse \mathbf{I} and work W are determined by the time integral of the

acceleration field. So, with accurate solution of both velocity and acceleration field, computed V_f should keep constant and \mathbf{P}_f and \mathbf{I} as well as E_f and W should balance.

As the first trial, transient heave motion of the body floating at the center of the wave tank is simulated. The heave motion is initiated by the unbalance between the weight and the displacement. The initial displacement is set to the half of the weight and the body is released calmly at time $t = 0$. Five different time steps $\Delta t = 0.007, 0.014, 0.028, 0.056$ and 0.112 are tried for this simulation. Since the natural heave period T_h of this body is about 2.24, number of time steps per single heave motion correspond to these Δt are 320, 160, 80, 40 and 20 respectively.

The simulated heave motions are presented in Fig.6. We can see the new method gives almost same results for all Δt . These results demonstrate the new method gives converged result even for the largest $\Delta t \approx T_h/20$. On the other hand, the former method does not give the converged result for $\Delta t > 0.014 \approx T_h/160$.

The difference between these two methods can be much clearly shown by conservation check. Next, the volume change while the simulation is plotted in Fig.7. This figure shows that both methods have quite high accuracy on the volume conservation and the maximum error is only $(V_f(0) - V_f(t_{max}))/V_f(0) \approx 0.007\%$. This results well demonstrate that velocity field is solved very accurately by both method. In contrast, momentum conservation plot in Fig.8 shows the big difference between two methods. In this figure, the vertical momentum of the fluid P_{fZ} is plotted by solid line and vertical impulse I_Z given from boundary to the fluid is plotted by broken line. This figure shows that P_{fZ} and I_Z by the new method agree with each other for all Δt , but P_{fZ} and I_Z by the former method disagree even for the smallest $\Delta t = 0.007 \approx T_h/320$. We can see the same result on the energy conservation plot in Fig.9. The new method gives better results for all Δt than the former method. Since the potential energy is much larger than the kinetic energy, the difference between two method is not as clear as Fig.8. So, R.M.S. error of momentum conservation and energy conservation is plotted in Fig.10. The horizontal axis is number of time steps per natural heave period T_h . This figure quantitatively shows that the new method is very stable for the size of time step and gives converged result even for the largest Δt . On the other hand, the converged result is hard to be obtained by the former method. In particular, on the conservation of momentum, the convergent speed of the former method is so slow that the converged result is hopeless to have.

Next, as the second trial, the floating body motions in a wave generated by the piston wave-maker are simulated. The amplitude and period of the wave-maker motion is $a = 0.01$ and $T_w = 1.587$ respectively. For this simulation, the former method can not be applied because it break down halfway. So, only the results by the new method are presented here. The simulated body motions are shown in Fig.11. Time step $\Delta t = 0.04 \approx T_w/40$ is used for this simulation. This figure shows, the simulated double amplitude of heave and sway are about 80% and 50% of initial draft respectively and the double amplitude of roll transient motion is about 12 degree. So, the transient body motions are considerably large. The balance between momentum \mathbf{P}_f and impulse \mathbf{I} and the balance between total energy E_f and work W are plotted in Fig.12(1),(2),(3). Even for the large body motions, the momentum and energy conservation are well satisfied. The loss of fluid volume is also negligibly small, less than 0.003%, in the simulation.

At the end of this section, a note is added for accurate numerical evaluation of the term $\frac{\partial}{\partial n}(\frac{1}{2}(\nabla\phi)^2)$ in the equation (44) and (46). This is the nonlinear term shifted from the governing equation of Φ to the boundary condition for ϕ_t . Therefore, accurate numerical evaluation of this term is indispensable to get the accurate solution of the acceleration field. At first, as the most direct approach to evaluate this term, the values of ϕ at some internal points of fluid domain are computed to take finite difference among values on the body surface and the internal points. But, this method was found to be too sensitive to the location of the internal points and converged results could not be obtained. So, the following formula is used to evaluate this term in this paper.

$$\frac{\partial}{\partial n} \left(\frac{1}{2}(\nabla\phi)^2 \right) = -k(\nabla\phi)^2 + \frac{\partial\phi}{\partial n} \left(-\frac{\partial^2\phi}{\partial s^2} \right) + \frac{\partial\phi}{\partial s} \frac{\partial}{\partial s} \left(\frac{\partial\phi}{\partial n} \right) \quad (56)$$

where k is the curvature of the body surface. Since the right side of equation (56) can be evaluated only from the surface values, this formula is appropriate to numerical methods like BEM. Moreover, equation (56) gives stable result to the panel size. The derivation of this equation is given in Appendix.A.

7 Concluding remarks

This work is aimed to develop the three dimensional full nonlinear theory on the wave body interaction problem and the numerical simulation method as its application. As the first step of this work, the mathematical formulation of the boundary value problem of the acceleration field is studied and an idea of the new time domain simulation method is presented. Following items are main results of this report.

1. Euler's equation of ideal fluid motions is transformed to the integral equation of the acceleration potential Φ .
2. Using the kinematic formula of the acceleration of fluid particle sliding on the body surface, the body surface boundary condition for the acceleration potential Φ is derived.
3. Substituting the equation of three dimensional body motions into the body surface boundary condition, the implicit body surface boundary condition for Φ is derived. With this implicit boundary condition, the ideal fluid motion and the floating body motions can be solved simultaneously.
4. The free-surface boundary condition for Φ is added to complete the mathematical formulation of the boundary value problem on the acceleration potential Φ .
5. Physical meaning to solve the acceleration field is clearly understood from above arguments.
6. For numerical method like BEM, the nonlinear term in Φ is shifted from the governing equation to the boundary conditions, then alternative boundary value problem on ϕ_t is formulated.
7. As an application of ϕ_t , the new nonlinear time domain simulation method is proposed.

8. To demonstrate the accuracy of this new method, two dimensional full nonlinear simulations of floating body motions are presented. The results show that the simulated results are accurate and satisfy the conservation law of mass, momentum and energy excellently.

The three dimensional simulation code based on this new method is under development. Hopefully, some results of three dimensional full nonlinear simulations will be presented in the next report.

8 Acknowledgement

On the body surface kinematic boundary condition for the acceleration potential Φ , Dr.Tomita, Dr.Hinatsu, and Dr.Murashige of Ship Research Institute provided many discussions, opinions and hints to the author. They are quite helpful to derive the boundary condition and hit the point to understand the physical meaning. The author records here the warmest acknowledgement to them for their help and favor.

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Appendix.A Formula for numerical computation of $\frac{\partial}{\partial n} \left(\frac{1}{2} (\nabla \phi)^2 \right)$

Using the local polar coordinate system (r, θ) which origin located at the local center of the curvature as shown in Fig.A, the r-directional gradient of square of the fluid velocity is written as

$$\frac{\partial}{\partial r} \left(\frac{1}{2} (\nabla \phi)^2 \right) = \frac{\partial \phi}{\partial r} \frac{\partial^2 \phi}{\partial r^2} + \frac{\partial \phi}{r \partial \theta} \left\{ -\frac{1}{r} \frac{\partial \phi}{r \partial \theta} + \frac{1}{r} \frac{\partial^2 \phi}{\partial r \partial \theta} \right\} \quad (A-1)$$

Substituting the Laplace's equation

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0 \quad (A-2)$$

into equation (A-1), following relation can be obtained.

$$\frac{\partial}{\partial r} \left(\frac{1}{2} (\nabla \phi)^2 \right) = -\frac{1}{r} \left\{ \left(\frac{\partial \phi}{\partial r} \right)^2 + \left(\frac{\partial \phi}{r \partial \theta} \right)^2 \right\} + \frac{\partial \phi}{\partial r} \left(-\frac{\partial^2 \phi}{r^2 \partial \theta^2} \right) + \frac{\partial \phi}{r \partial \theta} \frac{\partial}{r \partial \theta} \left(\frac{\partial \phi}{\partial r} \right) \quad (A-3)$$

Furthermore, relations

$$\frac{\partial}{\partial r} \equiv \frac{\partial}{\partial n}, \quad \frac{\partial}{r \partial \theta} \equiv \frac{\partial}{\partial s}, \quad \frac{1}{r} \equiv k \quad (A-4)$$

can be taken to rewrite equation (A-3) to get the final form

$$\frac{\partial}{\partial n} \left(\frac{1}{2} (\nabla \phi)^2 \right) = -k (\nabla \phi)^2 + \frac{\partial \phi}{\partial n} \left(-\frac{\partial^2 \phi}{\partial s^2} \right) + \frac{\partial \phi}{\partial s} \frac{\partial}{\partial s} \left(\frac{\partial \phi}{\partial n} \right). \quad (A-5)$$

Appendix.B Relation between Eulerian angles and fundamental vectors.

In this article, the fundamental vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are used to relate the space fixed reference frame and the body fixed reference frame. But in many cases, the angular position of ship is presented by Eulerian angles. Therefore, the relation between Eulerian angles and fundamental vectors is given here. As Fig.B shows, let us use pitch angle θ , roll angle φ and yaw angle ψ as Eulerian angles, then apply sequence of rotations ψ , θ and φ to $O - XYZ$ frame so that it coincide to $o - xyz$ frame. Keeping the sequence of rotations, the relation between (X, Y, Z) and (x, y, z) can be written as

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \cos \theta \cos \psi & \sin \varphi \sin \theta \cos \psi - \cos \varphi \sin \psi \\ \cos \theta \sin \psi & \sin \varphi \sin \theta \sin \psi + \cos \varphi \cos \psi \\ -\sin \theta & \sin \varphi \cos \theta \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}. \quad (B-1)$$

Since (X, Y, Z) is identical to $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the relation between Eulerian angles and fundamental vectors is clearly understood from this equation.

Moreover, the relation between angular velocity $\boldsymbol{\omega} = \omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k}$ and Eulerian angular velocity $\dot{\theta}, \dot{\varphi}, \dot{\psi}$ can be written as

$$\begin{Bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & \sin \varphi \tan \theta & \cos \varphi \tan \theta \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi \sec \theta & \cos \varphi \sec \theta \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}. \quad (B-2)$$

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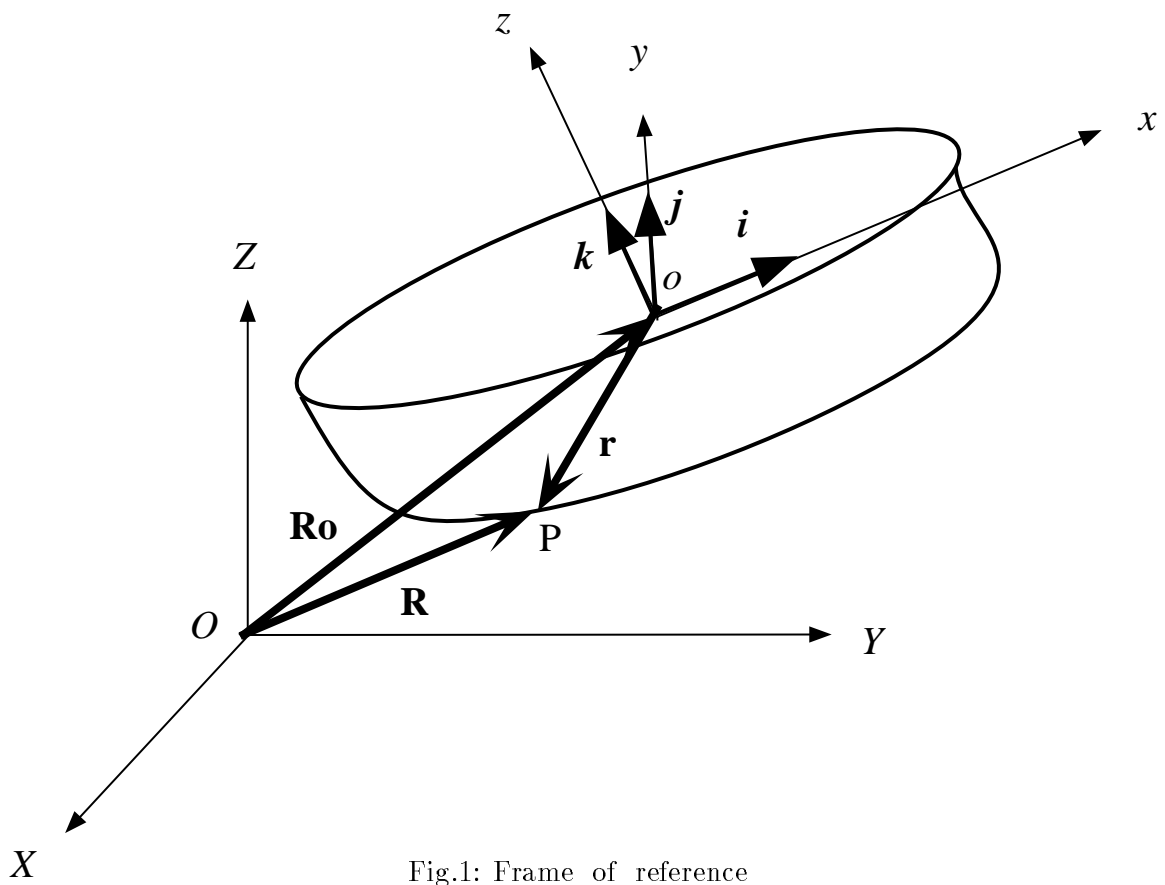


Fig.1: Frame of reference

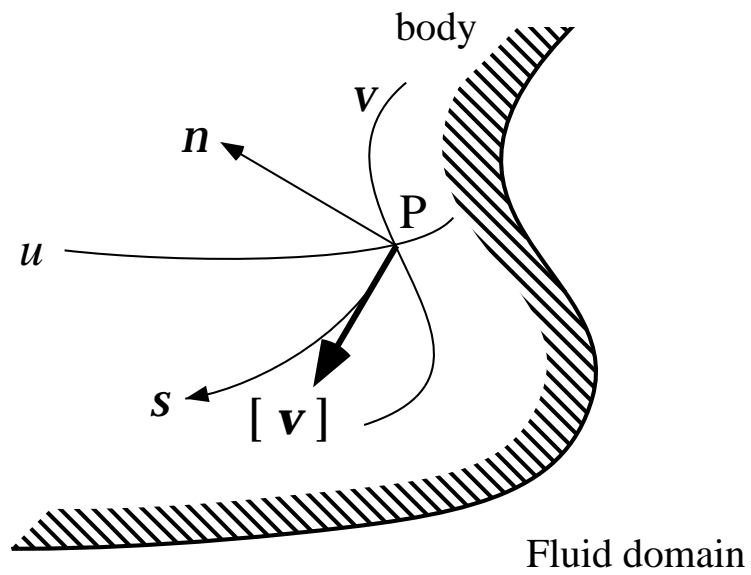


Fig.2: Local coordinate system

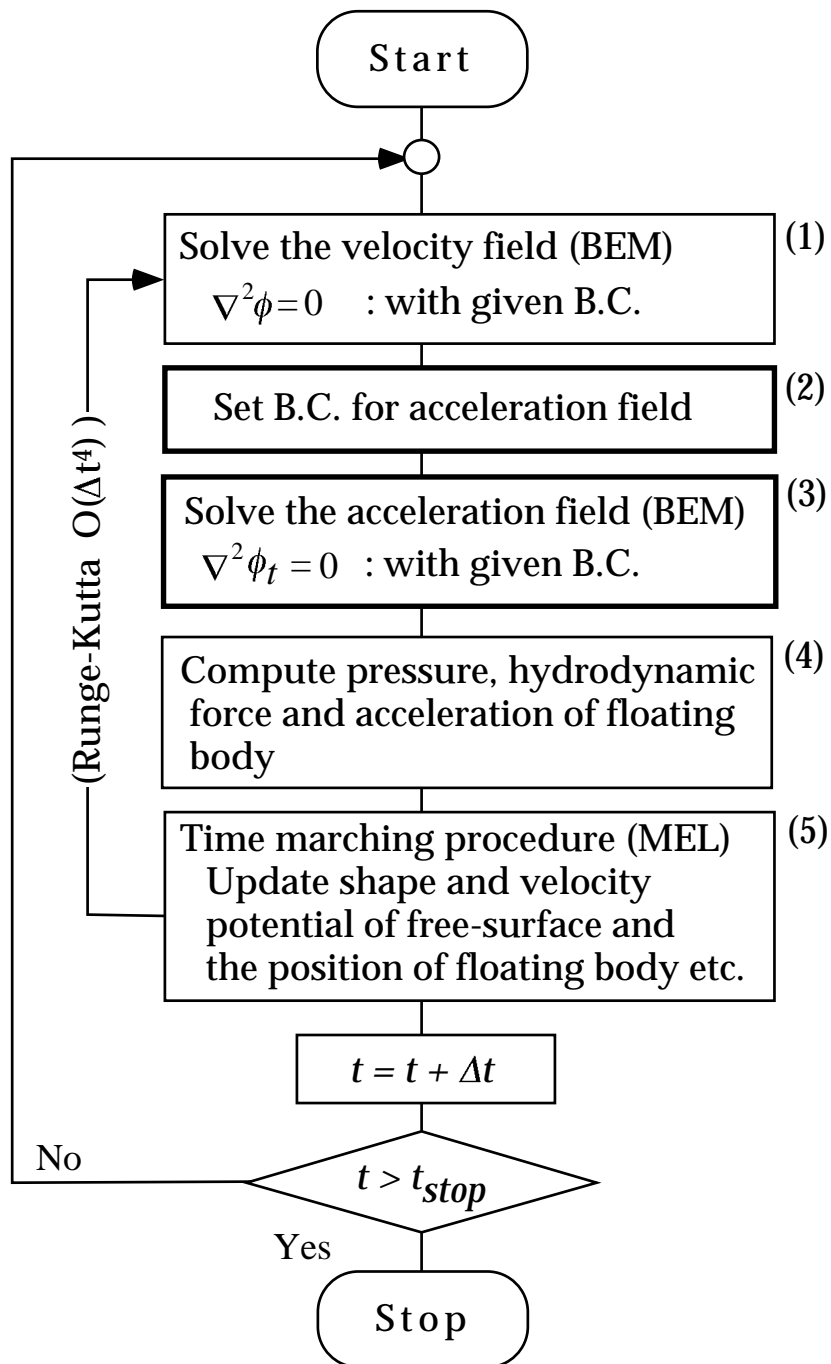


Fig.3: Computational flow formulated with the acceleration potential : Method.2

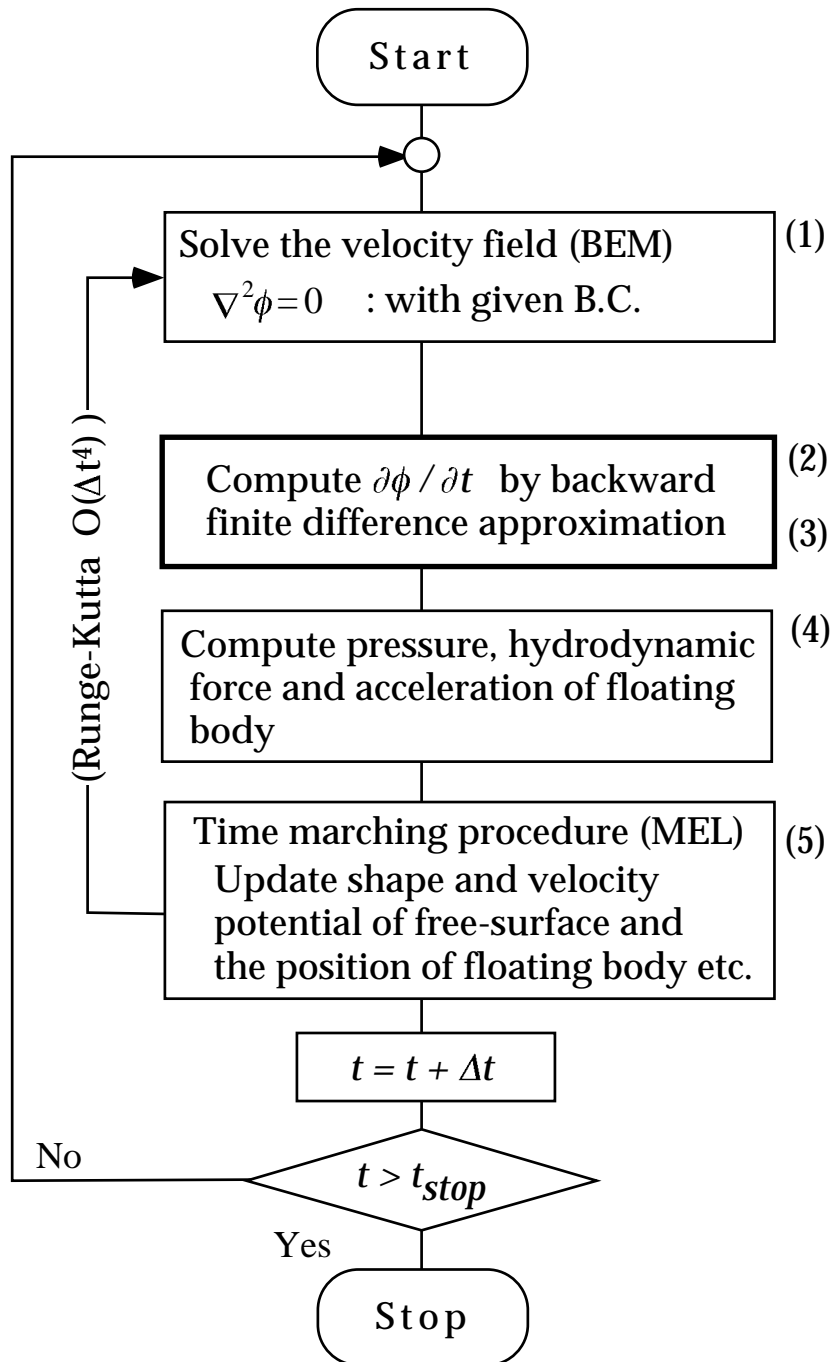


Fig.4: Computational flow formulated without the acceleration potential : Method.1

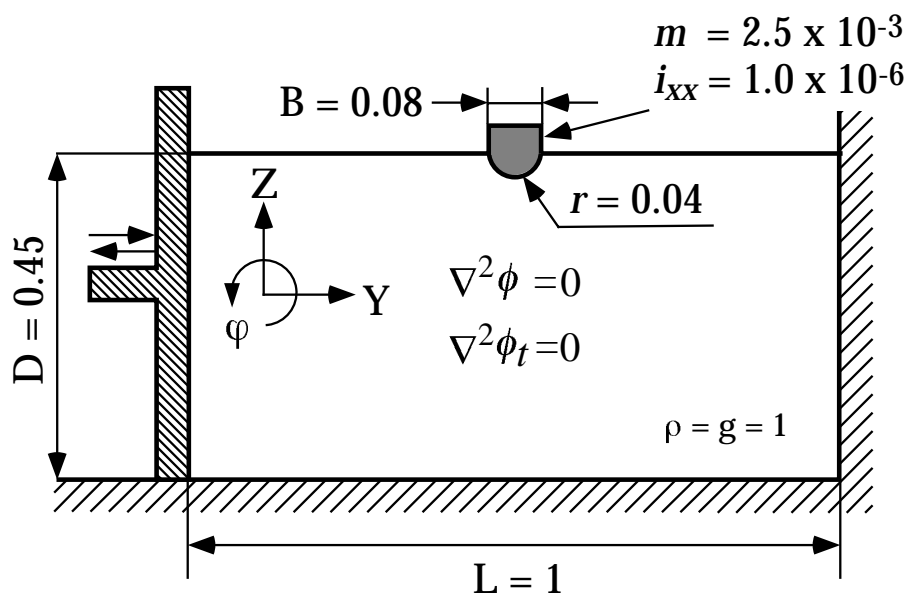


Fig.5: Target of the computation : *Case.1* Wave field generated by free heave motion of the floating body, *Case.2* Wave field generated by the piston wave maker and the the free floating body

Fig.6: Simulated free heave motion of the body : *Case.1*

Fig.7: Conservation check of fluid volume : *Case.1*

Fig.8: Conservation check of vertical momentum : *Case.1*

Fig.9: Conservation check of fluid energy : *Case.1*

Fig.10: Convergence plot of momentum and energy conservation error to the number of time steps per one heave cycle : *Case.1*

Fig.11: Simulated body motions in a wave generated by the piston wave maker : *Case.2*

Fig.12: Conservation check of momentum and energy: *Case.2*

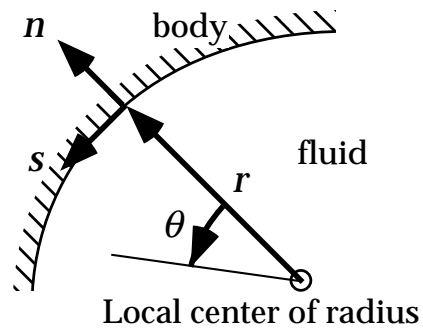


Fig.A Local polar coordinate

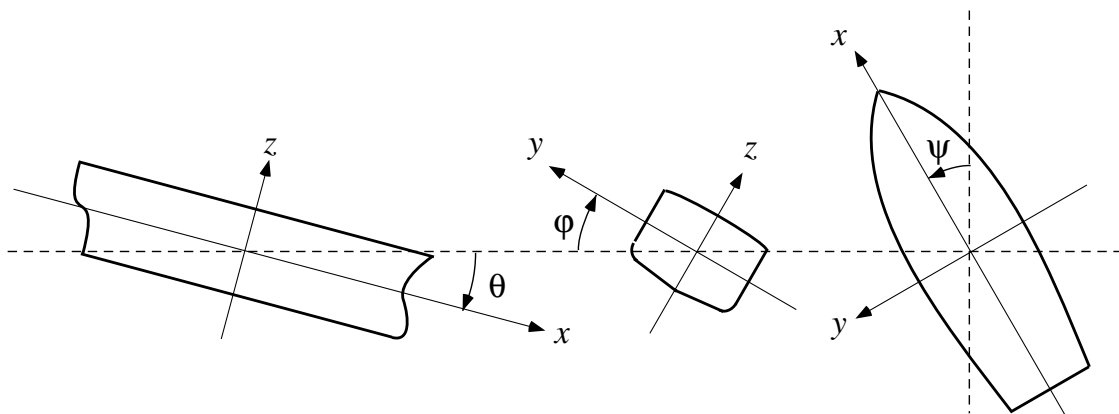


Fig.B Euler angles