(Abstract for the 5th Symposium on Nonlinear and Free-Surface Flow, October 1997)

A Nonlinear Simulation Method of Hydro-Elastic Problem

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1 Introduction

Ships and ocean structures have elastic body in the real world, but they are usually treated as solid in popular seakeeping theories for simplicity. When the scale of these structures are small, this simplification is considered to be valid. But, as the scale of the structure enlarged, relative rigidity of the body decrease and so elasticity becomes important. For wave radiation and diffraction problem of large scale floating structure, elastic response sometimes plays more important roll than solid mode body motions. Elastic response of floating airport is a good example. Even if the scale is small, elastic response can not be neglected when wave load is large. Wave impact and slamming is the typical problems we should consider the elastic response. Besides of these problems, there is a variety of problems we should consider elastic effects, if possible. For example, even though distortion of VLCC is not so small in heavy seas, elastic effects are neglected in the estimation of hydrodynamic load on the hull. If we have fully nonlinear simulation method applicable to fluid and elastic body interaction problem, we should make full use of it for the safety design of ships and ocean structures. The aim of this study is to pave the way for developing such a powerful tool in near future. At the first stage of this work, a nonlinear fluid and elastic body interaction problem is formulated. In the formulation, interaction between fluid and elastic vibration superposed on large amplitude solid mode body motions 5) is taken into consideration. A two dimensional simulation program is developed to validate the formulation and vibration of a beam in unbounded fluid is simulated, as the most basic trial. Results of this simulation are presented in comparison with analytical results.

2 General formulation of fluid and elastic body interaction problem

2.1 Velocity and acceleration of fluid on a elastic body surface

As illustrated in Fig.1, $O-XYZ$ is the space fixed reference frame and $o-xyz$ is the body fixed reference frame which origin o is fixed at the center of gravity. $\boldsymbol{R}$, $\boldsymbol{R}_o$ and $\boldsymbol{r}$ are positioning vectors. Motion of elastic body can be expressed by superposition of elastic vibration on the solid mode motions (simply written as vibration and motions in the following formulation). P is a point fixed to the fluid sliding on the body surface. Using the positioning vectors, the position, velocity and acceleration of point P can be written as

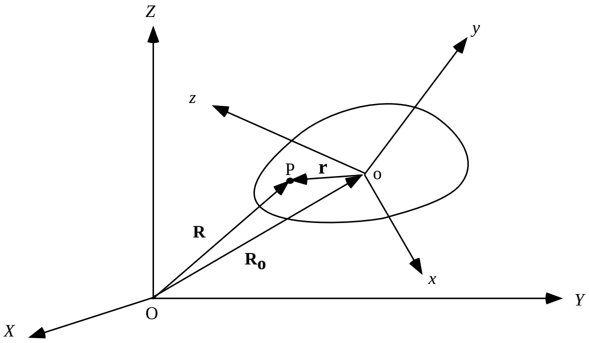

Fig.1 Frame of reference

$$\boldsymbol{R} = \boldsymbol{R}_o + \boldsymbol{r} \tag{1}$$

$$\dot{\boldsymbol{R}} = \dot{\boldsymbol{R}}_o + \dot{\boldsymbol{r}} = \dot{\boldsymbol{R}}_o + \boldsymbol{\omega} \times \boldsymbol{r} + [\dot{\boldsymbol{r}}] \tag{2}$$

$$\ddot{\boldsymbol{R}} = \ddot{\boldsymbol{R}}_o + \ddot{\boldsymbol{r}} = \ddot{\boldsymbol{R}}_o + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + [\ddot{\boldsymbol{r}}] + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2\boldsymbol{\omega} \times [\dot{\boldsymbol{r}}] \ , \tag{3}$$

where $\dot{\boldsymbol{R}}_o$ and $\boldsymbol{\omega}$ are translating and angular velocity of the body and $[\dot{\boldsymbol{r}}]$ and $[\ddot{\boldsymbol{r}}]$ are velocity and acceleration of point P observed from $o-xyz$ frame respectively.

2.2 Boundary value problems of fluid domain

Ideal fluid is assumed and the velocity potential ϕ and the nonlinear acceleration potential $\varPhi$ 3) are introduced to describe the fluid motion. Kinematic boundary condition of the velocity potential and the

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acceleration potential can be written as

$$\phi_n = \boldsymbol{n}\cdot\dot{\boldsymbol{R}} \tag{4}$$

$$\varPhi_n = \boldsymbol{n}\cdot\ddot{\boldsymbol{R}}\ , \tag{5}$$

where $\phi_n = \partial\phi/\partial n$, $\varPhi_n = \partial\varPhi/\partial n$ and $\boldsymbol{n}$ is the normal vector of body surface.

Substituting (2) into (4) and (3) into (5), we have following boundary conditions.

$$\phi_n = v_n + \boldsymbol{n}\cdot[\dot{\boldsymbol{r}}] \tag{6}$$

$$\varPhi_n = a_n + \boldsymbol{n}\cdot[\ddot{\boldsymbol{r}}] + \boldsymbol{n}\cdot\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) + \boldsymbol{n}\cdot 2\boldsymbol{\omega}\times[\dot{\boldsymbol{r}}] \tag{7}$$

where v_n and a_n are normal component of velocity and acceleration of body surface due to body motions respectively. v_n and a_n are given as

$$v_n = \boldsymbol{n}\cdot(\dot{\boldsymbol{R}}_o + \boldsymbol{\omega}\times\boldsymbol{r}) \tag{8}$$

$$a_n = \boldsymbol{n}\cdot(\ddot{\boldsymbol{R}}_o + \dot{\omega}\times\boldsymbol{r})\ . \tag{9}$$

In case of solid body, the term $\boldsymbol{n}\cdot[\dot{\boldsymbol{r}}]$ in (6) disappears because of orthogonality between $\boldsymbol{n}$ and $[\dot{\boldsymbol{r}}]$. But in case of elastic body, $[\dot{\boldsymbol{r}}]$ has normal component caused by vibration. Here, the normal distortion of the body u is introduced. Using u , we have

$$\boldsymbol{n}\cdot[\dot{\boldsymbol{r}}] = u_t \tag{10}$$

$$\boldsymbol{n}\cdot[\ddot{\boldsymbol{r}}] = u_{tt} - k_n[\dot{\boldsymbol{r}}]^2\ , \tag{11}$$

where $u_t = \partial u/\partial t$, $u_{tt} = \partial^2 u/\partial t^2$ and k_n is normal curvature of the body.

Taking these relations into account, the explicit kinematic boundary conditions on the elastic body surface

$$\{\phi_n\} = \{v_n\} + \{u_t\} \tag{12}$$

$$\{\varPhi_n\} = \{a_n\} + \{u_{tt}\} + \{q\} \tag{13}$$

are derived, where

$$q = \boldsymbol{n}\cdot\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) + \boldsymbol{n}\cdot 2\boldsymbol{\omega}\times[\dot{\boldsymbol{r}}] - k_n[\dot{\boldsymbol{r}}]^2 \tag{14}$$

is normal component of acceleration due to fluid flow and it can be evaluated from the solution of velocity field. In (12) and (13), variables are put into $\{array\ formula\}$ to explicitly show that these boundary conditions are applied every collocational points on wetted body surface.

2.3 Acceleration of body surface due to motions

The generalized equation of body motion is given as

$$\boldsymbol{\mathcal{M}}\cdot\boldsymbol{\alpha} + \boldsymbol{\beta} = \boldsymbol{F}, \tag{15}$$

where $\boldsymbol{\mathcal{M}}$, $\boldsymbol{\alpha}$ and $\boldsymbol{F}$ are generalized inertia tensor, acceleration and external forces respectively and $\boldsymbol{\beta}$ is a term so called Gyro-moment. Introducing the generalized normal vector $\boldsymbol{N}$, the generalized hydrodynamic force is written as

$$\boldsymbol{F}_f = \int_s p\ \boldsymbol{N}ds = \int_s (-\varPhi - Z)\ \boldsymbol{N}ds, \tag{16}$$

where integral is taken on the wet surface s . Denoting the other forces (thrust, gravity etc.) as $\boldsymbol{F}_g$, total force acts on the body is written as

$$\boldsymbol{F} = \boldsymbol{F}_f + \boldsymbol{F}_g = \int_s (-\varPhi - Z)\ \boldsymbol{N}ds + \boldsymbol{F}_g. \tag{17}$$

Using these equations, the normal component of body surface acceleration due to body motions is obtained.

$$a_n = \boldsymbol{N}\cdot\boldsymbol{\alpha} = \boldsymbol{N}\boldsymbol{\mathcal{M}}^{-1}\left\{\int_s (-\varPhi - Z)\boldsymbol{N}ds + \boldsymbol{F}_g + \boldsymbol{\beta}\right\} \tag{18}$$

Using boundary elements, (18) can be discretized and written in matrix form 5) as

$$\{a_n\} = [\boldsymbol{A}]\{\varPhi\} + \{B\}\ . \tag{19}$$

2.4 Acceleration of body surface due to vibration

It is not easy to formulate general equation of vibration for arbitrary elastic body. But, if we consider the discretized equation, it is possible to write the general equation of vibration in matrix form as

$$[\boldsymbol{M}]\{u_{tt}\} + [\boldsymbol{K}]\{u\} = [\boldsymbol{F}]\{p\} \ , \tag{20}$$

where $[\boldsymbol{M}]$ is a mass matrix, $[\boldsymbol{K}]$ is a rigidity matrix, $\{p\}$ is pressure on body surface and $[\boldsymbol{F}]$ is transform matrix of pressure to force on nodal points. Generally, this equation in matrix form can be derived from the principle of virtual work and popularly used in dynamic structure analysis by FEM. Solving the acceleration of vibration, we have

$$\{u_{tt}\} = -[\boldsymbol{M}]^{-1}[\boldsymbol{K}]\{u\} + [\boldsymbol{M}]^{-1}[\boldsymbol{F}]\{p\} \ . \tag{21}$$

Substituting $p = -\Phi - Z$ into (21), linear relational expression between acceleration of vibration and the acceleration potential

$$\{u_{tt}\} = [\boldsymbol{C}]\{\Phi\} + \{D\} \tag{22}$$

is obtained, where

$$[\boldsymbol{C}] = -[\boldsymbol{M}]^{-1}[\boldsymbol{F}] \tag{23}$$

$$\{D\} = -[\boldsymbol{M}]^{-1}[\boldsymbol{K}]\{u\} + [\boldsymbol{C}]\{Z\} \ . \tag{24}$$

2.5 Implicit body surface boundary condition of Φ

Normal acceleration a_n and u_{tt} in the boundary condition (13) are unknown, but these values can be determined by solving the simultaneous equation of fluid and elastic body motions using the implicit boundary condition. Substituting (19) and (22) into (13), we can eliminate unknown a_n and u_{tt} and the implicit boundary condition is derived.

$$\{\Phi_n\} = [\boldsymbol{A} + \boldsymbol{C}]\{\Phi\} + \{B + D + q\} \tag{25}$$

This implicit condition prescribes the relation between the acceleration potential Φ and its flux Φ_n . Since this condition is derived from both kinematic boundary condition of Φ and equations of body motions and elastic vibration, this is kinematic and at the same time dynamic boundary condition which connects the fluid and elastic body motions. Using this implicit boundary condition, we can solve fluid and elastic body interaction problem without modal decomposition.

3 Validation of the formulation by simple simulation

To validate the formulation, a simple beam vibration in unbounded fluid is simulated. In this simulation, the beam is divided into 20 segments, both ends of the beam are hinged and solid mode body motions are omitted. Pure hydro-elastic vibration is simulated from given initial distortion

$$u(x)_{at\ t=0} = u_o \sin \pi x, \ [0 \le x \le 1] \ , \tag{26}$$

where x axis is taken along with neutral axis of the beam and amplitude of vibration u_o is fixed to 0.1% of the beam length. For the simulation, the problem is non-dimensionalized using beam length L , bending rigidity EI , and linear density of the beam $h\rho_b$ as units, where h and ρ_b are thickness and density of the beam respectively.

Fig.2 shows simulated beam distortion at the center of the beam. From these simulation in different fluid density ρ_w , dependency of natural frequency to the density ratio between beam and fluid is estimated as shown in Fig.3. The horizontal axis is the density ratio and the vertical axis is the natural frequency normalized by that of in vacuum. The simulated results well agree with theoretical line by Kito 1).

$$\frac{\omega}{\omega_o} = \sqrt{\frac{1}{1+\varepsilon}}, \quad \varepsilon = \frac{2\rho_w}{\pi h \rho_b} \ , \tag{27}$$

4 Conclusion

In this study, a nonlinear fluid and elastic body interaction problem is formulated and following results are obtained.

1. The kinematic boundary conditions on elastic body surface are derived for the velocity potential ϕ and the acceleration potential $\varPhi$.
2. Relational expression between $\varPhi$ and body surface normal acceleration due to elastic vibration u_{tt} are derived.
3. The implicit boundary condition on elastic body surface is derived.
4. A simulation program based on this formulation is developed and a hinged beam vibration in unbounded fluid is simulated.
5. Natural frequency obtained by the simulation well agrees with theoretical value.

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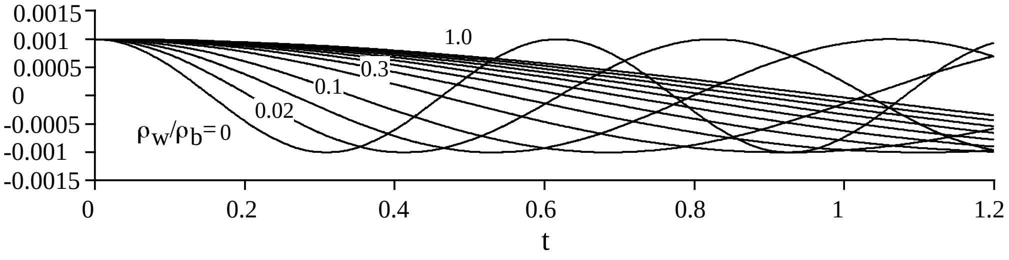

Fig.2: Simulated hydro-elastic vibration of a hinged beam : u at the center of beam

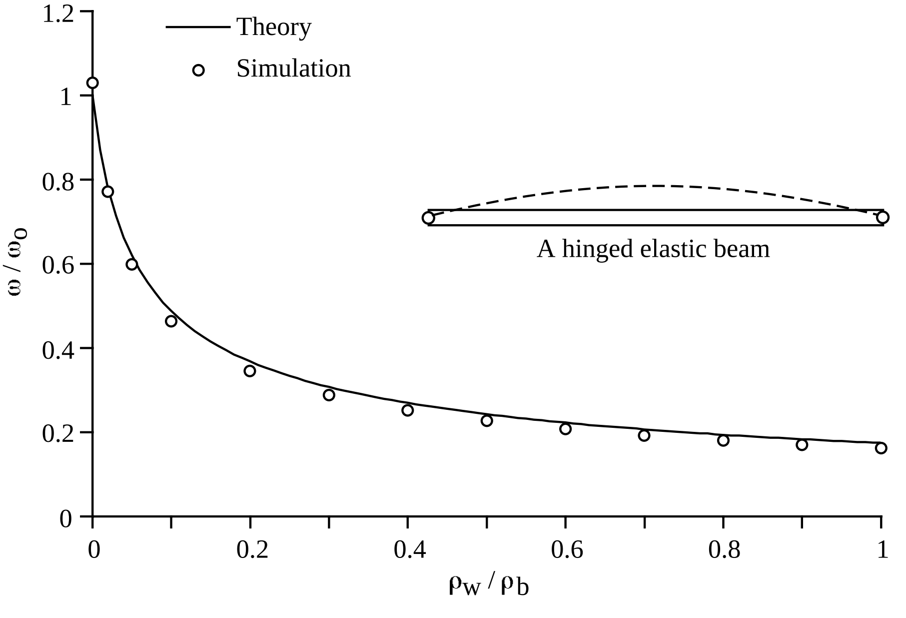

Fig.3: Natural frequency of hydro-elastic vibration of a hinged beam