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A Nonlinear Simulation Method of 3-D Body Motions in Waves Extended Formulation for Multiple Fluid Domains

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1 Introduction

The numerical treatment for full nonlinear wave simulation was firstly given by Longuet-Higgins in 1976 and well known as Mixed Eulerian and Lagrangian method (MEL). Detonated by this break through, many time domain simulation methods for nonlinear waves and fluid-body interaction problems were developed in past two decades. The first consistent simulation method for two dimensional fluid-body interaction problem was developed by Vinje & Brevig [2)] in 1981. They showed the idea to solve the simultaneous equations of fluid and body motions by decomposing of the acceleration field into four modes corresponding to the unit acceleration of the three body motions (heave sway and roll) and the other accelerations like the centripetal acceleration due to the fluid velocity. The authors developed further rational method to solve the simultaneous equations of fluid and body motions in the acceleration field in 1990 [5)]. The authors introduced the implicit body surface boundary condition derived from the kinematic body surface boundary condition and the equation of body motions, and showed the simultaneous equations of ideal fluid motion and floating body motions could be solved without decomposition. Van Daalen also came up with the same idea independently in 1993 [6)]. In 1995, the author introduced Prandtl's nonlinear acceleration potential, formulated the boundary value problem on the acceleration potential and clarify the physical meaning to solve the acceleration field [9, 10)]. As an application, the numerical simulation method was also given. Following these works, the multiple fluid domains and body interaction problem is formulated in this paper. Numerical simulations based on this formulation are also presented.

2 The formulation of the boundary value problem on the acceleration field

At the last workshop, the author introduced Prandtl's nonlinear acceleration potential Φ defined as

$$\Phi = \frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla \phi)^2 \tag{1}$$

and showed the acceleration of fluid $\boldsymbol{a}$ is given by the gradient of the acceleration potential as $\boldsymbol{a} = \nabla \Phi$. The boundary conditions for this acceleration potential were also given as follows.

Kinematic body surface boundary condition of Φ is given as

$$\frac{\partial \Phi}{\partial n} = \boldsymbol{N} \cdot \boldsymbol{\alpha} + q \ , \tag{2}$$

where $\boldsymbol{N}$ is the generalized normal vector of the body surface, $\boldsymbol{\alpha}$ is the generalized acceleration of the body and q is the term due to the fluid velocity. Using the velocity potential ϕ , q is written as

$$\begin{aligned} q = & -k_n \left(\nabla \phi - \boldsymbol{v}_o - \boldsymbol{\omega} \times \boldsymbol{r}\right)^2 + \boldsymbol{n} \cdot \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \\ & + \boldsymbol{n} \cdot 2\boldsymbol{\omega} \times \left(\nabla \phi - \boldsymbol{v}_o - \boldsymbol{\omega} \times \boldsymbol{r}\right) \ , \end{aligned} \tag{3}$$

where k_n is the normal curvature, $\boldsymbol{v}_o$ and $\boldsymbol{\omega}$ are translational and angular velocity, $\boldsymbol{n}$ is the unit normal vector of the body.

The acceleration of the body $\boldsymbol{\alpha}$ in equation (2) can be eliminated by Euler's equation of 3-D solid body motions coupled with fluid motion

$$\boldsymbol{\mathcal{M}} \ \boldsymbol{\alpha} + \boldsymbol{\beta} = \int_{S_s} (-\Phi - Z) \ \boldsymbol{N} ds + \boldsymbol{F}_g \ , \tag{4}$$

where $\boldsymbol{\mathcal{M}}$ is the generalized inertia tensor, $\boldsymbol{\beta}$ is so called Gyro moment, $\int_{S_s}(-\Phi - Z) \ \boldsymbol{N} ds$ is the generalized fluid force and $\boldsymbol{F}_g$ is other forces (thrust,gravity,etc.). Substituting equation (4) into (2), the implicit boundary condition on body surface is derived as

$$\begin{aligned} \frac{\partial \Phi}{\partial n} = & \boldsymbol{N}\boldsymbol{\mathcal{M}}^{-1} \int_{S_s} -\Phi \boldsymbol{N} ds \\ & + \boldsymbol{N}\boldsymbol{\mathcal{M}}^{-1} \left\{ \int_{S_s} -Z\boldsymbol{N} ds + \boldsymbol{F}_g - \boldsymbol{\beta} \right\} + q \ . \end{aligned} \tag{5}$$

This condition gives the relation between the acceleration potential Φ and its flux $\partial \Phi / \partial n$ on the body surface.

Since the nonlinear acceleration potential is the hydrodynamic pressure, the free-surface boundary condition is simply written as

$$\Phi_{on \ f.s.} = -Z \tag{6}$$

Detail of this formulation is presented in the reference paper 10).

3 The extended formulation for multiple fluid domains

The above formulation of the boundary value problem on the acceleration field can be extended to multiple fluid domains and body interaction problems. Let us denote a fluid domain as Ω_κ and variables of the domain as ϕ_κ, Φ_κ ,etc.

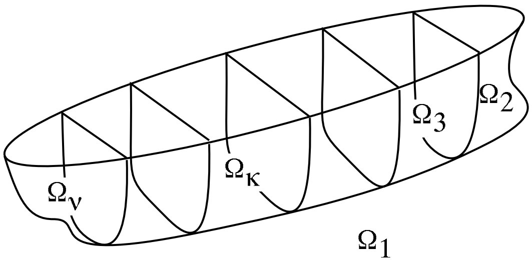


Fig.1 Fluid domains inside and outside of a ship

Similar to equation (2), the kinematic body surface boundary condition of the domain Ω_κ is written as

$$\frac{\partial \Phi_\kappa}{\partial n} = \boldsymbol{N}_\kappa \cdot \boldsymbol{\alpha} + q_\kappa \ , \tag{7}$$

where $\boldsymbol{N}_\kappa$ is the generalized normal unit vector of the boundary which points to the outside of domain Ω_κ .

The total hydraulic force acts to the body $\boldsymbol{F}_f$ is given as the sum of the hydraulic force of each fluid domain

$$\boldsymbol{F}_f = \sum_{\kappa=1}^{\nu} \int_{S_{\kappa s}} \rho_\kappa(-\Phi_\kappa - Z) \ \boldsymbol{N}_\kappa ds \ , \tag{8}$$

where ν is the number of fluid domains and ρ_κ is the nondimensional density of fluid domain Ω_κ . So, the generalized equation of 3-D body motions coupled with fluid motion is written as

$$\boldsymbol{\mathcal{M}} \ \boldsymbol{\alpha} + \boldsymbol{\beta} = \sum_{\kappa=1}^{\nu} \int_{S_{\kappa s}} \rho_\kappa(-\Phi_\kappa - Z) \ \boldsymbol{N}_\kappa ds + \boldsymbol{F}_g . \tag{9}$$

Eliminating the acceleration of the body from equation (7) and (9), the implicit body surface boundary condition is given as

$$\begin{aligned} \frac{\partial \Phi_\kappa}{\partial n} = & \ \boldsymbol{N}_\kappa \ \boldsymbol{\mathcal{M}}^{-1} \left\{ \sum_{\kappa=1}^{\nu} \rho_\kappa \int_{S_{\kappa s}} -\Phi_\kappa \ \boldsymbol{N}_\kappa ds \right\} \\ & + \boldsymbol{N}_\kappa \ \boldsymbol{\mathcal{M}}^{-1} \left\{ \sum_{\kappa=1}^{\nu} \rho_\kappa \int_{S_{\kappa s}} -Z \ \boldsymbol{N}_\kappa ds + \boldsymbol{F}_g - \boldsymbol{\beta} \right\} \\ & + q_\kappa \ . \end{aligned} \tag{10}$$

This is the extended implicit boundary condition which connects the motion of body and motion of fluid inside and outside of the body.

4 The alternative formulation for numerical methods

Because of the nonlinear term in equation (1), the acceleration potential Φ does not satisfy Laplace's equation. So, Φ is not adequate for numerical method like BEM. But this nonlinear term of Φ can be explicitly determined from the solution of velocity field. Therefore it is not necessary to solve the nonlinear part with Φ . Let us subtract this part from Φ and put linear part as

$$\phi_t = \frac{\partial \phi}{\partial t} = \Phi - \frac{1}{2}(\nabla \phi)^2 \ . \tag{11}$$

Now, ϕ_t satisfies Laplace's equation. So, with given boundary conditions, boundary value problem on ϕ_t is easier to be solved than that on Φ . The boundary condition for ϕ_t is easily obtained from equation (7),(10) and (6) as follows.

- Body surface boundary condition

$$\frac{\partial \phi_{\kappa t}}{\partial n} = \boldsymbol{N}_\kappa \cdot \boldsymbol{\alpha} + q_\kappa - \frac{\partial}{\partial n}\left(\frac{1}{2}(\nabla \phi_\kappa)^2\right) \tag{12}$$

- Implicit body surface boundary condition

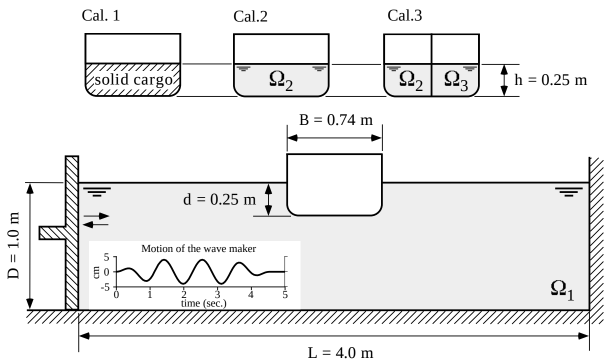
$$\begin{aligned} \frac{\partial \phi_{\kappa t}}{\partial n} = & \ \boldsymbol{N}_\kappa \boldsymbol{\mathcal{M}}^{-1} \left\{ \sum_{\kappa=1}^{\nu} \rho_\kappa \int_{S_{\kappa s}} -\phi_{\kappa t} \boldsymbol{N}_\kappa ds \right\} \\ & + \boldsymbol{N}_\kappa \boldsymbol{\mathcal{M}}^{-1} \left\{ \sum_{\kappa=1}^{\nu} \rho_\kappa \int_{S_{\kappa s}} \left(-Z - \frac{1}{2}(\nabla \phi_\kappa)^2\right) \boldsymbol{N}_\kappa ds + \boldsymbol{F}_g - \boldsymbol{\beta} \right\} \\ & + q_\kappa - \frac{\partial}{\partial n}\left(\frac{1}{2}(\nabla \phi_\kappa)^2\right) \end{aligned} \tag{13}$$

- Free-surface boundary condition

$$\phi_{\kappa t} = -Z - \frac{1}{2}(\nabla \phi_\kappa)^2 \tag{14}$$

5 Numerical Simulations

In order to demonstrate the capability of the simulation method based on the above formulation, three types of two dimensional fluid body interaction problems are simulated. The target of the simulation is large amplitude transient motions of midship section body with three different loading conditions illustrated in Fig.2 as Cal.1, Cal.2 and Cal.3.



Floating body	Fluid	Solid cargo
m = 36.0 kg	ρ_1 = 1000 kg / m^3	ρ = 800 kg / m^3
i = 2.55 kg-m^2	ρ_2 = 800 kg / m^3	Gravity
	ρ_3 = 800 kg / m^3	g = 9.8 m / s^2

Fig.2 The target of the numerical simulation

In Cal.1, solid cargo is loaded. In Cal.2, fluid cargo is loaded in a single tank. And in Cal.3, the same fluid cargo is loaded in two tanks. The incident wave is generated by a piston wavemaker attached to the left side of the tank. The motion of the wave maker is plotted in the figure. These problems are nondimensionalized using the width of the floating body B, the density of the fluid outside of the body ρ_1 and the gravitational acceleration g as units.

First of all, comparison of the simulated body motions among these three loading conditions are presented in Fig.3. Corresponding to the loading condition, the difference can be observed in the body motions. In particular, significant difference can be found between Cal.1 and Cal.2 in roll motion. The fluid motion inside the body affects sway and roll motions strongly in this case. On the other hand, no significant difference can be found between Cal.1 and Cal.3. This result shows the partition between two fluid tanks effectively reduce the strong interaction.

Next, the simulated instantaneous free-surface and body motions in seven different time from $t = 9.27$ to $t = 15.45$ are shown in Fig.4a,4b and 4c. These three figures show that the transient body motions and fluid motion inside and outside of the body are quite large in all cases. Particularly, the amplitude of the relative water level at the weather side of the body are large.

This method solves the boundary value problem on ϕ_t. Therefore, it is easy to compute distribution of ϕ_t inside of the fluid domains. A contour plot of ϕ_t, it is identical to the contour plot of hydrodynamic pressure, is shown in Fig.5. This plot shows the same instance of the last plot of Fig.4b.

This simulation method also gives us the pressure time history of any points. Fig.6 shows the pressure time history of inside and outside of the body at the intersection of calm waterline and the weather side of the body. When the intersection point is above the water surface, the pressure is zero. The difference between the pressure inside and outside is important for ship design. Such a kind of information can be also computed by this method.

Last, the conservation laws of momentum and energy are checked in Fig.7. The upper two plots show the balance between total fluid momentum and total impulse given from the boundary to the fluid in the simulation of Cal.2. And the lower plot shows the balance between total energy of the fluid and total work in the same simulation. These plots demonstrate that momentum and energy conservation are well satisfied. The volume conservation error is also very small, less than 0.016% in Cal.2.

6 Concluding remarks

The formulation is extended to multiple fluid domains and body interaction problem. As a demonstration of this new method, two dimensional large amplitude floating body motions are simulated and the conservation laws are checked. The results show that this method has excellent accuracy even for the large amplitude multiple fluid domains and body interaction problem.

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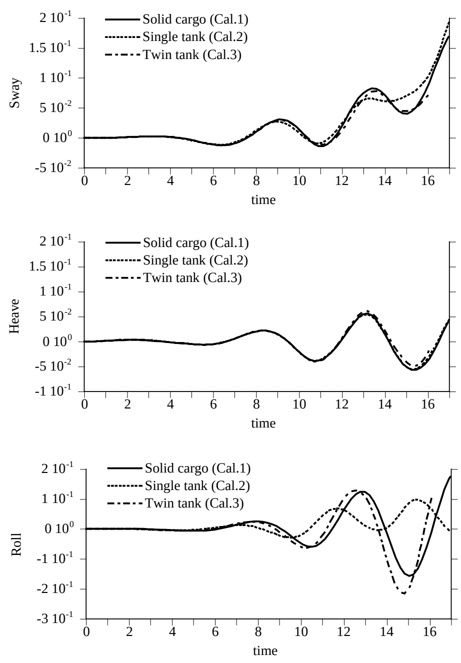

Fig.3 Floating body motions

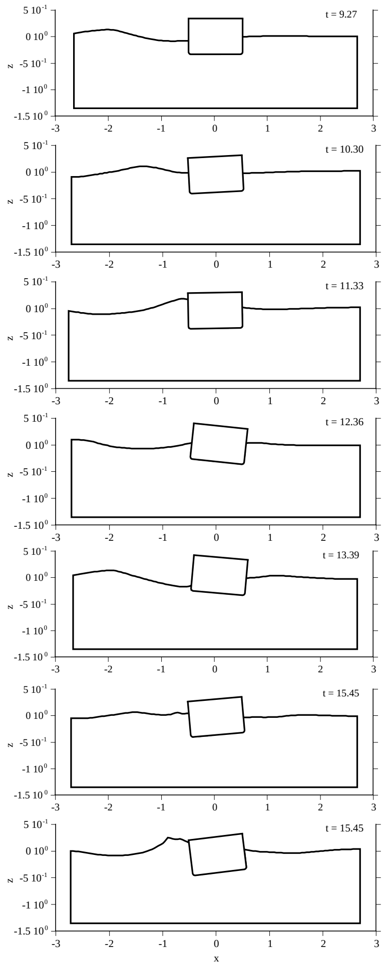

Fig.4-a Simulated fluid and body motions (Cal.1)

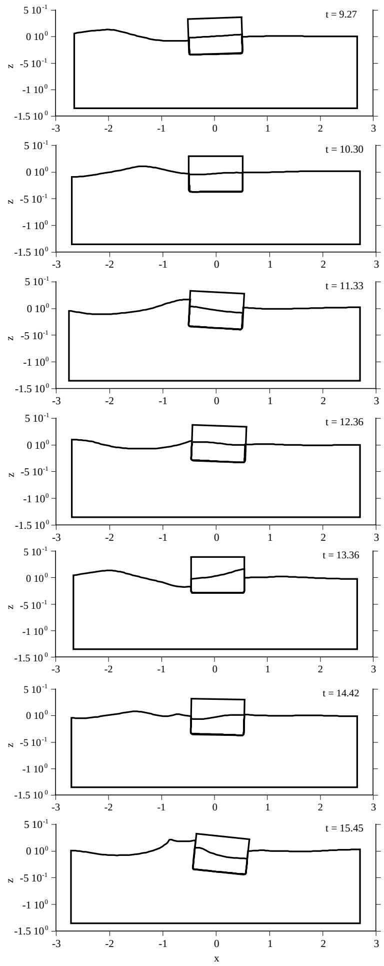

Fig.4-b Simulated fluid and body motions (Cal.2)

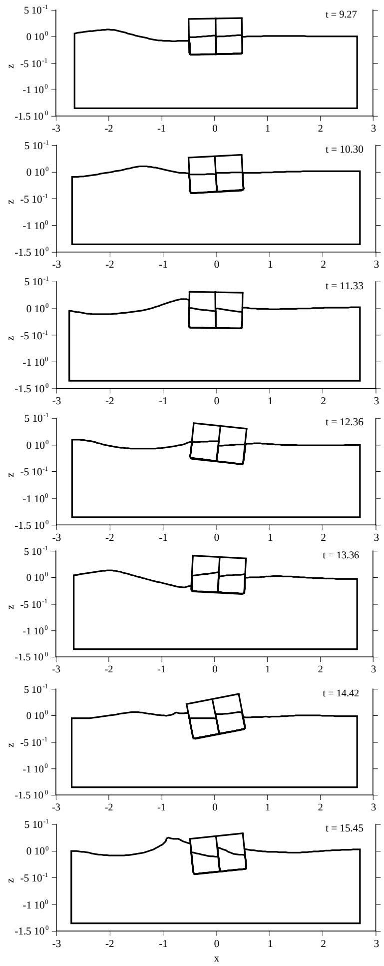

Fig.4-c Simulated fluid and body motions (Cal.3)

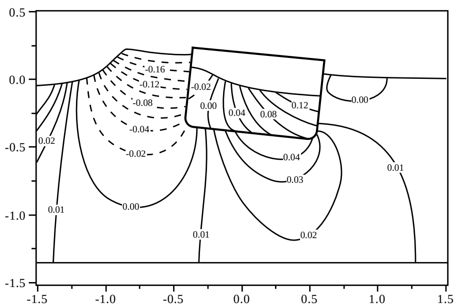

Fig.5 Contour plot of ϕ_t (Cal.2 t=15.45)

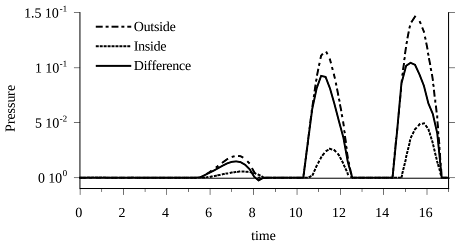

Fig.6 Pressure time history of inside and outside of the body (Cal.2)

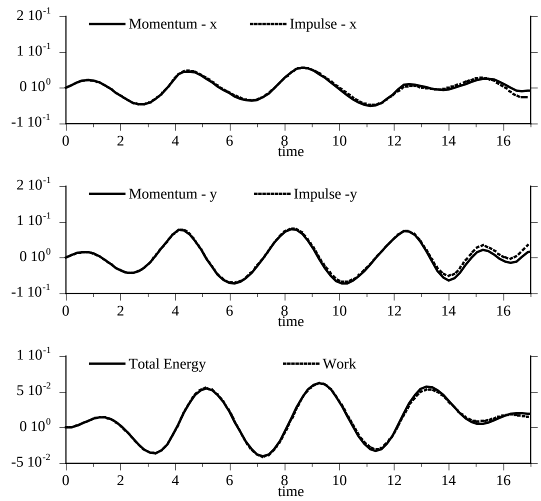

Fig.7 Conservation check of total momentum and total energy of fluid domains (Cal.2)