EFFECTS OF SUDDEN EXPANSION AND CONTRACTION FLOW ON PRESSURE DROPS IN THE STIRLING ENGINE REGENERATOR

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ABSTRACT

The flow losses in the regenerators greatly influence the performance of the Stirling engine. The losses mainly depend on fluid friction through the regenerator matrix, but are also generated in sudden expansion and contraction flow at the regenerator ends. The latter losses can't be neglected in the case of small area ratio (entrance area/cross-sectional area in regenerator). The pressure drops in regenerators are usually estimated assuming a uniform velocity distribution of working gas in the matrices. The estimation results, however, are generally smaller than practical data. The cross-sectional flow areas of the heater and cooler of typical Stirling engines are smaller than the cross-sectional area of the regenerator. So, it is necessary to understand the quantitative effects of the sudden change in flow area at the regenerator ends on the velocity distribution and pressure drop. In this paper, the effects of the regenerator ends are examined using stacked wire gauzes in the matrix by a steady single blow experiment and presented using the empirical equation defined by effective flow area ratio. The results show that the effective flow area ratio which is an index of the uniformity of the velocity distribution is independent of the mesh number and the Revnolds number but dependent on the entrance and exit areas and the stack thickness. Additionally, the effects of regenerator ends on the pressure drop in an actual engine are studied theoretically and experimentally.

INTRODUCTION

In Stirling engines, the regenerator is the indispensable heat exchanger. Its existence is the key factor so that the Stirling cycle may have the highest thermal efficiency. The regenerator exists between the heater and the cooler and is composed of the matrix and the housing. The matrix is made of stacks of fine size wire screens. The regenerator is usually designed according to the balance between flow loss and heat transfer characteristics. In spite of the small entrance and exit flow areas at the regenerator ends as shown in Fig. 1, however, regenerator performance is often evaluated assuming that working fluid flows through the matrix at a uniform distribution of velocity. The effects of the small flow passage on the velocity distribution of working fluid in the matrix, that is, a flow transition from tubes or channels to a regenerator matrix, can be often confirmed by the discolored matrix. Especially, the lack of a uniform distribution of velocity in the matrix causes increased flow loss and decreased thermal performance.

In this paper, using matrices made of stacks of wire screens, the effects of the entrance and exit areas and the length of the regenerator on pressure drops are examined by an unidirectional steady flow apparatus. The experimental data are arranged in an empirical equation. The lack of a uniformity of velocity distribution is visualized using smoke-wire methods. The empirical equation presented is applied to the estimation of pressure loss in an actual engine regenerator. The applicability of the equation is examined by comparison of estimated value with engine data in pressure loss.



Fig. 1 An example of heat exchanger configuration in Stirling engines

EXPERIMENTAL APPARATUS & METHODS

Fig. 2 shows the test section for measuring the pressure drop and visualizing the flow pattern in a unidirectional steady flow apparatus. The housing of the matrix is made of transparent acrylic resin. The wire screens are stacked in the housing (inner diameter: $D_0 = 38$ mm, cross-sectional area: $A_0 = \pi D_0^{2/4}$). The inner diameter of the fluid passage flowing out of the matrix is 34mm. The working fluid is atmosphere and sucked using a blower. The flow rate of air is in the region of 30 to 300L/min and is measured using a float type flow meter. The stacked wire gauzes used are made of stainless steel. The details of the wire gauzes are given in Table 1. The porosity is about 70% irrespective of mesh number. In Fig. 2, the entrance diameter $D_I(A_I = \pi D_I^{2/4})$ and the exit one $D_2(A_2 = \pi D_2^{2/4})$ can be chosen from the following sizes: 10, 19, 25, 30, or 34mm. The matrix length *L* is in the region of 3.6 to 28.6mm.



Fig. 2 Test section for visualizing of outflow and measuring pressure drop across stacked wire gauzes

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M (mesh)	60	100	150	300
Wire dia.: <i>d</i> (mm)	0.131	0.099	0.061	0.040
Pitch : $p(mm)$	0.423	0.254	0.169	0.085

Flow visualization

A smoke wire method is used for the visualization of the flow velocity distribution at the matrix end. Liquid paraffin is spread on the smoke wire in Fig. 2. The wire made of nickel is 0.1mm in diameter and stretched at a distance of 5mm from the matrix end upward. The liquid paraffin on the wire volatilizes through an electric discharge using a condenser and generates a string of smoke. The propagation patterns in which smoke is moving upward, are observed using the strobe light synchronized by an electric discharge and the video-camera having a low shutter speed (exposure time: 250ms). The mesh number of the wire screen, the stack length, and the entrance and exit diameters to be used in this experiments are as follows: Mesh number M=60, stack length L=10, 20mm, entrance diameter D_1 =10, 34mm and exit diameter D_2 =19, 38mm.

Flow loss

Pressure drop, inflow air pressure and temperature are measured using the manometer PD, P and the thermo-couple TC shown in Fig. 2. Each experiment is set up as follows:

1) Effects of entrance diameter (D_2 =34 mm) M = 60, 100, 150, 300 mesh $D_1=10, 19, 25, 30, 34 \text{ mm}$ *L* = 3.6, 8.6, 13.6, 18.6, 23.6, 28.6 mm 2) Effects of exit diameter (D_1 =34 mm) M = 150 mesh D₂=10, 19, 25, 30, 34 mm *L* = 3.6, 8.6, 13.6, 18.6, 23.6, 28.6 mm M = 60, 100, 300 mesh $D_2=10 \text{ mm}, L=8.6, 23.6 \text{ mm}$ 3) Effects of entrance and exit diameter M = 150 mesh $D_1 = D_2 = 10, 19, 25, 30, 34 \text{ mm}$ *L* = 3.6, 8.6, 13.6, 18.6, 23.6 mm M = 60, 100, 300 mesh $D_1 = D_2 = 10 \text{ mm}, L = 8.6, 23.6 \text{ mm}$

VISUALIZATION OF FLOW VELOCITY DISTRIBUTION

Fig. 3 shows the state of progression in a string of smoke

from the matrix end upward. Fig. 3(a) is the case of the large diameter at the entrance and exit, and shows the even distribution of flow velocity through the cross-sectional area. Fig. 3(b) is the case of the small entrance diameter, and shows the convex distribution of flow velocity having a high velocity at the center in the cross-sectional area. Fig. 3(c) is the same small entrance diameter as Fig. 3(b) and twice the stack length of Fig. 3(b). Fig. 3(d) is the small exit diameter and shows the concave distribution of flow velocity having a high velocity in the outskirts in the cross-sectional area.



Fig. 3 Smoke propagation observed by the smoke wire method

ARRANGEMENT FORMULAS OF PREESURE DROPS

The effects of the flow passage area at the matrix end, matrix length and each wire screen on pressure drops, are examined by a steady flow apparatus. The relation between pressure drops and flow rate is arranged using the friction factor and Reynolds number. Arrangement formulas are as follows:

Hydraulic diameter d_h is defined by

$$d_h = \varepsilon d / (1 - \varepsilon) \tag{1}$$

where, d and ε are the wire diameter and the porosity of matrix, respectively.

Flow velocity u_{h0} assuming that fluid flows uniform throughout effective opening area ϵA_0 in matrix, is defined by

$$u_{h0} = Q / (\epsilon A_0) \tag{2}$$

where, Q and A_0 are the flow rate and the cross-sectional area in matrix, respectively.

Reynolds number Re_{h0} and flow friction factor F_{h0} are defined by

$$Re_{h0} = d_h u_{h0} / v \tag{3}$$

$$F_{h0} = \frac{d_h}{L} \frac{\Delta P}{\rho u_{h0}^2 / 2} \tag{4}$$

where, ΔP , *L*, ρ and ν are pressure drop, matrix length, gas density and kinematic viscocity, respectively.

MEASURED VALUES & ARRANGEMENT OF PRESSURE DROPS

Fig. 4 shows the effects of the matrix length on pressure drop. The experimental conditions are 60mesh wire gauzes in matrix, 19mm in entrance diameter and 34mm in exit diameter. In this figure, pressure drop ΔP monotonously increases with increasing flow rate Q and matrix length L. Fig. 5 shows the effects of entrance diameter on pressure drop. The experimental conditions are 150mesh wire gauzes in the matrix, 13.6mm in the matrix length and 34mm in the exit diameter. In this figure, pressure drop ΔP increases with decreasing entrance diameter D_1 . Fig. 6 shows the relation between friction factor F_{h0} and Reynolds number Re_{h0} that Fig. 4 is arranged using eqs. (1) to (4). In this figure, a curve shows eq. (5) applying on the stacked wire gauzes with 60mesh size in a uniform flow without sudden expansion and contraction flow. Friction factor F_{h0} decreases and approaches eq.(5) with increasing matrix length L. Eq. (5) is expressed by

$$F_{h0} = \frac{129}{Re_{h0}} + 1.64 \tag{5}$$

Fig. 7 shows the relation between friction factor F_{h0} and Reynolds number Re_{h0} that Fig. 5 is arranged using eqs. (1) to (4). In this figure, a curve shows eq. (6) applying on the stacked wire gauzes with 150 mesh size without sudden expansion and contraction flow. Friction factor F_{h0} decreases and approaches eq. (6) with increasing entrance diameter D_1 . Eq.(6) is expressed by

$$F_{h0} = 110 / Re_{h0} + 1.58 \tag{6}$$

These results show that the expansion flow of fluid in the matrix decreases with decreasing entrance area and matrix length. The fluid flowing in the matrix having a small entrance area compared to cross-sectional area of the matrix, has an uneven flow distribution and causes higher pressure drop. That is, these results indicate the expansion and contraction flow pattern as shown in a dotted line in Fig. 1 and 2.



Fig. 4 Effects of stack thickness L on pressure drop







Fig. 6 Effects of stack thickness L on friction factor Fh0



Fig. 7 Effects of entrance diameter D_1 on friction factor F_{h0}



Fig. 8 Effects of entrance diameter D_1 and Reynolds number Reh0 on effective flow area ratio A_E / A_0

So, the effective flow area ratio A_E/A_0 is introduced as an index indicating an expansion extent or a uniformity in velocity distribution of the fluid flow in a matrix. The ratio A_E/A_0 is defined by the ratio of the imaginary cross-sectional area A_E to the cross-sectional area A_0 in a matrix. $A_E(=\pi D_E^{-2}/4)$ is defined using the diameter of an imaginary cylindrical flow passage as shown in a chain line in Fig. 2. D_E is estimated assuming a uniform flow velocity distribution and the same pressure drop to measured data. So, the ratio is defined by $A_E/A_0 = (D_E/D_0)^2$. The velocity in an imaginary cylindrical passage u_{hE} is defined by

$$u_{hE} = A_0 u_{h0} / A_E = Q / (\epsilon A_E)$$
 (7)

 A_E is calculated by solving the simultaneous equation replacing u_{h0} , Re_{h0} and F_{h0} in eq.(2), (3) and (4) with u_{hE} , Re_{hE} and F_{hE} .

The effective flow area ratio A_E/A_0 calculated using Fig. 5 and 7 is shown in Fig. 8. Fig. 8 shows that A_E/A_0 doesn't depend on Reynolds number Re_{h0} and flow rate Q. A_E/A_0 calculated on the basis of effects of entrance flow area and length in the matrix on pressure drops is shown in Fig. 9. A_E/A_0 value on L = 0 is A_I/A_0 . Fig. 9 shows that A_E/A_0 doesn't depend on the mesh size of wire gauzes. Namely, the



Fig. 9 Effects of entrance diameter D_1 and stack thickness L on effective flow area ratio A_E/A_0 for four kinds of wire gauzes



Fig. 10 Effects of entrance diameter D_1 , exit diameter D_2 and stack thickness L on effective flow area ratio A_E/A_0

ratio A_E/A_0 depends on entrance diameter D_1 and length L in the matrix. Moreover, effects of exit diameter D_2 on A_E/A_0 are the same to ones of entrance diameter D_1 . For example, in Fig. 10, the symbol \bullet to indicate the effects of exit diameter D_2 on A_E/A_0 in stacks of 150 mesh wire screens, is nearly the same as the symbol \circ , ones of entrance diameter. Fig. 10 also shows the effects of both of entrance and exit diameter. The symbol Δ shows that the entrance and exit diameters equal each other.

EMPIRICAL EQUATION FOR EFFECTIVE FLOW AREA RATIO

An empirical equation for the effective flow area ratio A_E / A_0 shown in Fig. 9 and 10 is suggested in order to easily apply to the regenerator design. To formulate this equation, the diameters of the entrance and exit flow passage D_1 , D_2 are compared with each other, so the diameter having a larger passage is expressed as D_L and that of smaller passage is done as D_S . And, considering that A_E / A_0 corresponds to A_S / A_0 $(=(D_S / D_0)^2)$ at L = 0 and unity at $L = \infty$ or $D_S = D_L = D_0$, the empirical equation is formulated by

$$\frac{A_E}{A_0} = \left\{ 1 - \left(1 - \frac{D_S}{D_0} \right) \frac{1 - e^{-\phi}}{\phi} \right\}^2$$
(8)
where, $\phi = \frac{1.4}{1 - D_S / D_0} \left(\frac{L_E}{D_0} \right)^{0.7}$
 $L_E = L \left\{ 0.4 + 0.6 \left(\frac{D_L - D_S}{D_0 - D_S} \right)^{0.6} \right\}$

In Fig. 9 and 10, the solid lines calculated from eq.(8) correspond to the experimental values. So, it is realized that eq.(8) can greatly estimate the actual expansion and contraction flows in the matrix.

APPLICATION OF THE EMPIRICAL EQUATION TO THE STIRLING ENGINE REGENERATOR

Experimental engine

The experimental engine is a small 100W displacer type engine¹⁾. A simplified cross-section is shown in Fig. 11 and a brief specification is given in Table 2. The regenerator is located in the displacer piston with inner heating and cooling tubes. The regenerator housing of an annular configuration is 27mm in inner diameter, 66mm in outer diameter and 35mm in length. The matrix supplied with the engine is Spring Mesh²⁾. Details of this matrix are given in Table 3. The electric heating method is used to heat the heater tubes. The outer wall temperature of the heater tubes is maintained at 973K. Maximum heat input is 1kW. The cooler is cooled with water at a controlled flow rate of 3L/min of constant temperature of 12? This engine is operated at a constant revolutionary speed using the DC motor with an arm for measuring the load. The data measured are: heat input, output, heat absorption in cooler,



Fig. 11 Experimental engine

time averaged working gas temperatures, and instantaneous pressures in expansion, compression and power piston space.

 Table 2 Specifications of experimental engine

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Engine configuration	γ-type
Working gas	Helium
Rotational speed	700 to 1400 rpm
Maximum output	100 W (1 MPa, 1000 rpm)
Heating method	Electric heating
Cooling method	Water cooling
Displacer & Power pisto	n
$Bore \times Stroke$	φ 72× 20 mm
Phase angle	90 deg.
Heater & Cooler	-
Heater tube	10-\$ 7.0(\$ 9.5)\$× \$ 12.4(\$ 13.8)\$× 54mm
Cooler tube	24-\$4.0(\$5.0)\$×\$\$\$ 7.4 (\$8.0)\$×\$56mm
Regenerator	
Housing	¢ 27(¢ 66)× 35 mm
Matrix	Spring Mesh

Table 3	Details	of regenerator	matrix
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Matrix	Wire dia.	Porosity	Weight	Surface area
Spring Mesh	0.100 mm	0.740	232 g	$1.08 \mathrm{m}^2$

Estimation of expansion and contraction flow in regenerator

The expansion and contraction flow at both ends of the experimental engine regenerator is estimated using the empirical equation (8). The calculated values are shown in Table 4. Table 4 shows how to estimate the effective flow ratio in the regenerator of experimental engine. The diameter of entrance passage area at cooler ends of the regenerator, D_S is defined by

$$D_{\rm S} = D_C (n_C / n_H)^{0.5} \tag{9}$$

The diameter of the regenerator cross-section per a heater tube is the value that the cross-sectional area is divided by the number of heater tubes. The flow area ratios at two entrance areas are obtained by $A_L/A_0 = (D_L/D_0)^2$ and $A_S/A_0 = (D_S/D_0)^2$.

Table 4 Effective flow area ratio in experimental engine regenerator

	Heater end	Cooler end	
Number of heater & cooler tubes	n _H =10	<i>n</i> _c =24	
Dia. of heater & cooler tubes	$D_{H}=7 \mathrm{mm}$	$D_c=4 \text{ mm}$	
Dia. of entrance area	$D_L=7 \mathrm{mm}$	<i>D</i> _S =6.2 mm	
Dia. of regenerator cross- section per a heater tube	<i>D</i> ₀ =19.04 mm		
Flow area ratio at entrance	AL/A0=0.135	$A_{S}A_{0}=0.106$	
Regenerator length	<i>L</i> =35 mm		
Effective flow area ratio	$A_E / A_0 = 0.501$		

So, the effective flow area ratio 0.501 is estimated. This value shows the average value of the expansion and contraction flow in regenerator (a chain line) shown in Fig. 1 2. In Fig. 12, a solid line shows the flow pattern of working gas in the regenerator.



Fig. 12 Expansion and contraction flow pattern in regenerator

Estimation and experiment of pressure drop in regenerator

Pressure drop is estimated using the empirical equation to express the relation between friction factor and Reynolds number for a Spring Mesh matrix that is investigated under an uniform flow using a steady flow apparatus. The empirical equation for friction factor F_{h0} is defined by²⁾

$$F_{h0} = \frac{26.5}{(1-\epsilon)Re_{h0}} + \frac{0.246}{\epsilon(1-\epsilon)}$$
(10)

where friction factor F_{h0} , Reynolds number Re_{h0} and hydraulic diameter d_h are expressed by eq.(3), eq.(4) and eq.(1). And ε is the porosity of the matrix. However, flow velocity u_{h0} uses the eq.(2) divided by the effective flow area ratio $A_E/A_0=0.501$. The gas flow rate required for calculating the flow velocity is obtained by a simple isothermal analysis.



Fig. 13 Comparison of experimental results with calculated results for pressure loss L_P

The operating conditions of this engine are set up in the following way to properly make an analysis: working gas pressure 0.8 MPa, and the gas temperature in the expansion space and the compression space 703 K, 313 K, respectively.

The pressure loss L_P is defined by

$$L_p = N / 60 \oint (P_c - P_e) dV_e \tag{11}$$

where, N is the engine speed, Pc and Pe are the pressure in the compression and expansion space, and Ve is the stroke volume in the expansion space.

As to the flow loss, the calculated pressure loss L_P and the pressure drop ΔP (=*Pc-Pe*) are compared with the experimental data as shown in Fig. 13 and 14. In Fig. 13, the symbol • is the experimental value, the solid line is the calculated result considering expansion flow, and the dotted line is the calculated result without considering expansion flow. In Fig. 14, the solid line is the experimental result, the chain line is the calculated results considering expansion flow and the dotted line is the calculated result without considering expansion flow. Fig. 13 and 14 show that the estimation of pressure drop in regenerator considering the expansion and contraction flow corresponds to the experimental results. So, the proof of the empirical equation eq. (8) proposed in this paper is confirmed.



Fig.14 Comparison of experimental result with calculated results for pressure drop $\triangle P$

CONCLUSIONS

The sudden expansion and contraction flow in the regenerator has been experimentally investigated and the empirical equation formulated, and the effects of this equation on the estimation of regenerator flow loss examined theoretically and experimentally. The results are as follows; 1. The expansion and contraction flows in the regenerator matrix made of stacked wire screens don't depend on the mesh number of screens and the flow rate.

2. The estimation of the flow loss in the regenerator using the empirical equation proposed corresponds to the engine data.

REFERENCES

- 1. K. Hirata et al., "Test results of applicable 100 W Stirling engine", Proc. 31st IECEC, Vol.2, p.1259(1996).
- K. Hamaguchi et al., "Basic Characteristics and applied tests of Spring Mesh' as a new regenerator matrix for Stirling engine", Proc. 8th ISEC, p.51(1997).