Performance of Stirling Engines* (Arranging Method of Experimental Results and Performance Prediction)

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and

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Abstract

We have developed five kinds of high- and low-temperature differential Stirling engines and their engine performance was investigated experimentally. In order to determine the parameters that affect engine performance, experimental results were discussed and compared with results calculated using analytical methods. We show an arranging method for the experimental results, and consider the performance of general Stirling engines. After using the arranging method with nondimensional numbers obtained by a dimensional analysis, a prediction method, which is used at the early design stage, is formulated. One of the nondimensional numbers in this prediction method is calculated based on engine specifications, including the properties of the working gas. The prediction method can predict engine speed, output power, the effect of working gas and operating conditions.

1. Introduction

Recently, environmental pollution and energy utilization have become serious problems worldwide. As Stirling engines are a potential solution to the above problems, they have been developed actively, and the results of many studies have been reported⁽¹⁾⁻⁽⁹⁾. However, to our knowledge, no systematic investigation and research on the parameters that affect engine performance have been conducted to date.

It is important that engine performance be predicted accurately from the main specifications that are determined from the target performance during engine design. Generally, the calculation for engine performance is initiated after the main specifications are determined and the design for heat exchangers and mechanical devices is finished. But, in actual practice, it is necessary that the engine performance be determined at the same time as engine specifications, and also before finishing the design for heat exchangers and mechanical devices.

We have developed five kinds of high- and low-temperature differential Stirling engines, and measured their performance. The measured performance is compared with results calculated using analytical methods. The parameters that affect engine performance are investigated. We suggest an arranging method for the experimental results, discuss the performance of general Stirling engines and develop a simple method for predicting engine performance.

2. Previous Performance Prediction Methods

As methods for predicting output power, L_S , from engine specifications, two simple methods utilizing the Beale Number⁽⁸⁾ and the West Number⁽⁹⁾ are well known.

i) Beale Number, B_N ; This method for predicting output power was suggested by Beale⁽⁸⁾, where B_N is defined by Eq. (1).

$$B_N = \frac{L_S}{P_m V_{SE} n} \tag{1}$$

Here, P_m is mean pressure in the working space (Pa), V_{SE} is swept volume of an expansion space (m³), n

is engine speed per unit second (rps).

It is known that B_N at the engine speed where maximum output power is realized, is about 0.15 in the case of high-temperature differential Stirling engines whose heater wall temperature is about 650 °C⁽⁸⁾.

ii) West Number, W_N ; This method was suggested by West⁽⁹⁾, where W_N is defined by Eq. (2) with revision of B_N .

$$W_N = \frac{L_S}{P_m V_{SE} n \left(\frac{T_E - T_C}{T_E + T_C}\right)}$$

Here, T_E is expansion space gas temperature, and T_C is compression space gas temperature.

It was found that W_N is about 0.25 in the case of 5~150 kW class Stirling engines, and is about 0.35 in the case of smaller power engines⁽⁹⁾.

Output power can be predicted from engine specifications using the B_N or W_N . However, it is necessary to determine the engine speed at which maximum output power is realized. Then, in order to predict engine performance accurately, the relationship between output power, L_S , and engine speed, n, and that between the kind of working gas and the output power must be determined, with consideration of engine type and operating conditions.

3. Arrangement for Experimental Results

We have developed high- and low-temperature differential Stirling engines and measured their performance⁽¹⁰⁾⁻⁽¹⁴⁾. Also, engine performance has been analyzed and calculated using analytical methods⁽¹⁵⁾⁻⁽¹⁷⁾. After comparison and discussion, the parameters that affect output power significantly were clarified. As we have found that pressure loss of the working gas and mechanical loss due to friction in the mechanical devices affect output power significantly, the experimental results can be arranged as follows.

3.1 Pressure loss of working gas

The pressure of the working gas decreases when the gas flows through heat exchangers consisting of a heater, a regenerator and a cooler. The pressure drop is treated as pressure loss. Work done by the engine depends mainly on the mean pressure of the working gas, P_m, the engine size, the expansion space gas temperature, T_E , and the compression space gas temperature, T_C . Furthermore, work is affected strongly by pressure loss at heat exchangers. Pressure loss is affected by the engine type and operating conditions such as the engine speed, the density of the working gas in heat exchangers, ρ, the kinetic viscosity, v, and the shape of heat exchangers. Thus, pressure loss should be calculated precisely at the final design stage, that is, after determining the detailed sizes of heater and cooler tubes and the specifications of the regenerator matrix. However, it is considered that the size of heat exchangers is dependent on engine size, because similar types of heat exchangers have been adapted in general Stirling engines. Thus, the swept volume of the expansion space, V_{SE} , is used as the typical parameter that expresses as the size of heat exchangers, because V_{SE} is generally determined at the early design stage. Also, the gas constant, R, and the kinetic viscosity of the working gas, v, are used as the typical parameters that express the properties of pressure loss, based on the function, $\rho_m = P_m/(RT_E)$, where ρ_m is the mean density in the expansion space, T_E is the expansion space gas temperature, P_m is the mean pressure, and R is the gas constant.

3.2 Mechanical loss by friction

It can be considered that mechanical loss consists of Coulomb friction loss and viscosity friction loss. The Coulomb friction coefficient, μ_{co} , and the viscosity friction coefficient, c_{vi} , are parameters that affect output power. The Coulomb friction coefficient, μ_{co} , is the same for Stirling engines that consist of the same type of mechanical elements. Also, the viscosity friction coefficient, c_{vi} , is dependent on the swept volume of the expansion space, V_{SE} , because c_{vi} is affected strongly by the engine size. Then, the swept volume of the expansion space, V_{SE} , is determined to be a typical parameter that expresses mechanical loss.

3.3 Derivation of nondimensional numbers

From the above considerations, it is assumed that engine performance consists of the following function.

(2)

$$\boldsymbol{p} = W_s^{\boldsymbol{p}1} \cdot P_m^{\boldsymbol{p}2} \cdot V_{SE}^{\boldsymbol{p}3} \cdot R^{\boldsymbol{p}4} \cdot T_E^{\boldsymbol{p}5} \cdot \boldsymbol{n}^{\boldsymbol{p}6} \cdot n^{\boldsymbol{p}7}, \qquad (3)$$

where, W_s is output work (J), P_m is mean pressure of working gas (Pa), V_{SE} is swept volume of expansion space (m³), R is gas constant (J/kgK), T_E is expansion space gas temperature (K), ν is kinetic viscosity at temperature, T_E , and pressure, P_m , and n is engine speed (rps).

The compression space gas temperature, T_c , is assumed to be constant in the case of general Stirling engines, because it was confirmed that it does not differ markedly regardless of the engine type.

Equation (3) can be shown as Eq. (4) using fundamental units with mass, M, length, L, time, T, and temperature, θ .

$$\boldsymbol{p} = \left[ML^2 T^{-2} \right]^{p_1} \left[ML^{-1} T^{-2} \right]^{p_2} \left[L^3 \right]^{p_3} \left[L^2 T^{-2} \boldsymbol{q}^{-1} \right]^{p_4} \left[\boldsymbol{q} \right]^{p_5} \left[L^2 T^{-1} \right]^{p_6} \left[T^{-1} \right]^{p_7}$$

Then, we focus on leading nondimensional numbers that symbolize engine speed and output power. After much discussion and consideration, the following three nondimensional numbers are suggested.

$$n^* \equiv \frac{nV_{SE}}{n} \tag{5}$$

$$w_S^* \equiv \frac{W_S}{P_m V_{SE}} \tag{6}$$

$$s^* = \frac{T_E R V_{SE}^{\frac{2}{3}}}{n^2},\tag{7}$$

where, n^* is defined as nondimensional speed, and w_S^* is defined as nondimensional output work. In the case of an engine of which cylinder bore, D, is equal to piston stroke, H, Eq. (5) can be changed to Eq. (8) with mean piston speed, u (=2Hn).

$$n^* = k \frac{uD}{n},\tag{8}$$

where, k is a fixed number $[=(/4)^{2/3}/2]$. Namely, the nondimensional speed, n^{*}, is equivalent to the Reynolds number, because it is a ratio of inertia force to viscosity force of the working gas with a typical length, D. The nondimensional output work, ws^{*}, is equal to the Beale Number, B_N, defined by Eq. (1).

On the other hand, the size and specifications of seal devices such as a piston ring and a rod seal are determined from the mean pressure in the working space at maximum load, P_{lim} , at the early design stage. Also, working pressure and temperature are set as the operating conditions of the engine, and both affect mechanical loss strongly. Then, nondimensional pressure, P^* , and nondimensional temperature, T^* , are defined by Eqs. (9) and (10), respectively. They are the nondimensional numbers that express the operating conditions.

$$P^* \equiv \frac{P_m}{P_{\lim}}$$

$$T^* \equiv \frac{T_E - T_C}{T_E + T_C}$$
(9)
(10)

Here, P_m is mean pressure of working gas, P_{lim} is limited maximum mean pressure, T_E is expansion space gas temperature, and T_C is compression space gas temperature.

4. Results of Arrangement and Considerations

We have developed a 100 W class small engine and measured its performance^{(10),(16)}. The experimental results are treated using the nondimensional numbers as defined above.

Figure 1 shows the relationship between nondimensional speed, n^{*}, and nondimensional output work, ws^{*}, at nondimensional temperature, T^{*}, of 0.42 ($T_E=490$ °C, $T_C=40$ °C) using helium as the working gas. In the figure, each line represents the calculated results based on the isothermal analysis method with consideration of pressure loss and mechanical loss⁽¹⁵⁾ (called the second-order model). The solid line represents the result at nondimensional pressure, P^{*}=0.73 (P_m=0.8 MPa), the broken line represents that at P^{*}=0.55 (P_m=0.6 MPa), and the dot-dashed line represents that at P^{*}=0.36 (P_m=0.4 MPa). Symbols represent the experimental results at P^{*}=0.73, 0.64, 0.55, 0.45 or 0.36 (the mean pressure, P_m, is varied from 0.8 MPa to 0.4 MPa with 0.1 MPa steps). From the figure, the nondimensional output work, ws^{*}, decreases with increasing nondimensional speed, n^{*}. On the other hand, it was confirmed that the output power tends to decrease at higher engine speed, though the

(4)

indicated power increases at higher engine speed in this experiment⁽¹⁵⁾. The decrease in ws^{*} with increasing n^{*} is caused by the increase in pressure loss in the heat exchangers and mechanical loss caused by viscosity friction at higher n^{*}, because pressure loss and mechanical loss are affected by engine speed as described above. Also, the nondimensional output work, ws^{*}, decreases with decreasing nondimensional pressure, P^{*}. Because the specifications of seal devices such as a piston ring are determined based on P^{*} = 1 at the design stage, the rate of Coulomb friction loss of the seal devices increases with decreasing nondimensional pressure, P^{*}.

Figure 2 shows the relationship between nondimensional speed, n*, and nondimensional output work, ws*, at nondimensional pressure, P*, of 0.73 (P_m=0.8 MPa) using helium as the working gas. In the figure, the solid line represents the calculated result at $T^*=0.42$ ($T_E=490^\circ$ C, $T_C=40^\circ$ C), the broken line represents that at $T^*=0.38$ ($T_E=430^{\circ}C$, $T_C=40^{\circ}C$), and the dot-dashed line represents that at $T^*=0.35$ ($T_E=370^\circ$ C, $T_C=40^\circ$ C). Symbols represent the experimental results in the case of T* at 0.42, 0.40, 0.38, 0.37 or 0.35 (the expansion space gas temperature, $T_{\!
m E}$ is varied from 490 to 370 °C with 30 °C steps). From the figure, the nondimensional output work, ws*, decreases with increasing nondimensional speed, n^{*}, and decreasing nondimensional temperature, T^{*}, similar to the case in Fig. 1. This is due to the fact that the mechanical loss is not strongly affected by the expansion space gas temperature, T_E , and the nondimensional temperature, T^* ; furthermore the rate of mechanical loss per unit output power increases with increasing nondimensional speed, n*.

Figure 3 shows the relationship between nondimensional speed, n^{*}, and nondimensional output work, ws^{*}, at nondimensional pressure, P^{*}, of 0.73 (P_m=0.8 MPa) using nitrogen as the working gas. In the figure, the solid line represents the calculated result at T^{*}=0.40 (T_E=460°C, T_C=40°C), the broken line represents that at T^{*}=0.37 (T_E=400°C, T_C=40°C), and the dot-dashed line represents that at T^{*}=0.32 (T_E=340°C, T_C=40°C). Symbols represent the experimental results when T^{*} is set at 0.40, 0.38, 0.37, 0.35 or 0.32 (the expansion space gas







Fig. 3 Effects of nondimensional temperature (N₂)

temperature, T_E is varied from 460 to 340°C with 30°C steps). From the figure, the nondimensional output work, ws^{*}, decreases with increasing nondimensional speed, n^{*}, and decreasing nondimensional temperature, T^{*}, similar to the case in Fig. 2, though the effect becomes small at higher values of nondimensional engine speed, n^{*}. This may be due to the strong effects of pressure loss in the heat exchangers. The calculated results agree with experimental ones very well. Also, the range of nondimensional engine speed, n^{*}, is much higher, about 2000~6000, than the range of n^{*} of about 100~800 when helium is used as the working gas, as shown in Figs. 1 and 2. This is because the kinetic viscosity, v, strongly affects nondimensional speed, n^{*}, and because n^{*} is equivalent to the Reynolds number, as described above.

Figures 1, 2 and 3 show two cases of the relationship between nondimensional speed, n^* , and nondimensional output work, w_8^* , as defined by Eqs. (5) and (6), for the 100 W class Stirling engine.

One is the case of changing the mean pressure, P_m , while maintaining the same gas temperature, T_E and T_C . The other is the case of changing the gas temperature, T_E , while maintaining the same mean pressure, P_m . Furthermore, we confirmed that similar figures were obtained in the case of other operating conditions and different types of engines including low-temperature differential Stirling engines. Thus, it is considered that the experimental results can be arranged with n^* and w_S^* . Also, it is confirmed that the results of the analytical method agree well with the experimental results, and that the method has high accuracy.

5. Performance of Prototype Engines and Prediction Method

From the above considerations, the nondimensional output work, ws^* , can be arranged well according to the nondimensional speed, n^* , with nondimensional pressure, P^* , and temperature T^* , as parameters. In order to determine the relationship between engine specifications and output power, another nondimensional output work, Ws^* , and nondimensional output power, Ls^* , are defined by Eqs. (11) and (12), respectively.

$$W_{s}^{*} \equiv \frac{W_{s}}{P_{m}V_{SE}P^{*}T^{*}}$$
(11)
$$L_{s}^{*} \equiv W_{s}^{*} \cdot n^{*}$$
(12)

On the other hand, assuming that the engine is operated at the limited maximum gas temperature, T_{lim} , Eq. (13) is derived by revising Eq. (7), where the expansion space gas temperature, T_E , is changed to T_{lim} , and the kinetic viscosity, ν_{lim} is used at temperature, T_{lim} , and pressure, P_{lim} .

$$S^{*} = \frac{T_{\rm lim} R V_{SE}^{2/3}}{n_{\rm lim}^{2}}$$
(13)

Here, S is defined as a nondimensional engine specification, because it is a nondimensional number which is calculated from the engine specifications at the early design stage.

Table 1 shows the engine specifications and the operating conditions of five kinds of prototype engines. They have different engine types, operating temperatures, pressures and output power levels. In the table, Engine A is the 100 W class gamma type (see Figs. 1~3), Engine B is an alpha type of a similar power and temperature level as Engine A, and Engine C is a 2 kW class beta type. They are the high-temperature differential engines. Engine D is a 1 kW class alpha type, and Engine E is a 300 W class gamma type, both of which are low-temperature differential engines.

Figure 4 shows the relationship between nondimensional speed, \boldsymbol{n}^* and nondimensional output power, Ls* based on the experimental results of the prototype engines listed in Table 1. Each nondimensional engine specification, S, is shown in the figure. In the case of Engine A, two types of operating conditions, in which either helium or nitrogen is used as the working gas, are indicated. From the figure, each engine exhibits a maximum value of the nondimensional output power, $L_{S,max}^*$ at a given n^* ; i.e., the maximum value of L_{S,max}* is the optimal condition. The nondimensional maximum output power, L_{S,max}*, is obtained when the engine reaches the maximum output Ls, max, power, in each experiment. L_{s,max}^{*} increases and appears in the range of higher nondimensional speed, n^{*}, with

Table 1. Specifications and operating
conditions of prototype engines

Engine	A	В	С	D	E
Engine type	γ	α	ß	α	γ
Engine specifications					
Permitted pressure Plim (MPa)	0.9	0.55	4.0	1.1	0.25
Permitted temperature Tlim (°C)	650	650	700	130	130
Working gas	Helium Nitrogen	Air	Helium	Nitrogen	Air
Swept volume of exp. space, Vsɛ (cm³)	81.4	98.2	132.7	20100	40200
Operating conditions					
Operating pressure Pm (MPa)	0.4~0.9	0.1~0.3	3.5	0.3~1.0	0.1~0.2
Operating temp. of exp. space, Te(°C)	340~490	290~570	300~500	100	80~100
Engine speed n (rpm)	~1400	~1200	~1500	~300	~220



Fig. 4 Nondimensional output power as a function of nondimensional speed

increasing nondimensional engine specification, S^* . As a result, the nondimensional speed, n_{opt}^* at which the nondimensional maximum output power, $L_{S,max}^*$, is obtained according to the nondimensional engine specification, S^* .

Table 2 shows the results of nondimensional values, $L_{S,max}^*$, n_{opt}^* and S^* , based on the measured maximum output power, L_{S,max}, and engine speed, n_{opt}. Namely, the values of L_{S,max}* are the peak values (the optimal condition) for each engine shown in Fig. 4. Figure 5 shows the relationship between $n_{\text{opt}}{}^{*}$ and $L_{S,\text{max}}{}^{*}$ given in Table 2. In the figure, circles represent the results, and experimental the solid line represents the calculated result using the method of least squares. From the figure, it is considered that the relationship between maximum output power and engine speed can be arranged using the nondimensional maximum output power, $L_{S,max}^*$ and the nondimensional speed, n_{opt}^* . As the relationship between n_{pt}* and L_{8,max}* is a linear one, it is expressed by Eqs. (14) and (15).

$$L_{S,\max} = a_L \cdot n_{opt} \tag{14}$$

The coefficient, **a**, is determined to be the value given below from Fig. 5.

$$a_L = 0.24$$
 (15)

Figure 6 shows the relationship between nondimensional engine specification, S^* , and nondimensional speed, n_{opt}^* . In the figure, circles represent the experimental results, and the solid line represents the calculated result using the method of least squares. From the figure, the relationship between S^* and n_{opt}^* is expressed by Eqs. (16) and (17) in the same manner as in Fig.5.

$$n_{opt}^{*} = a_n \cdot S^{* m_n}$$
 (16)

The coefficients, a_n and m_n , are determined to be the following values from Fig. 6.

$$a_n = 6.8 \times 10^{-5}$$

 $m_n = 0.60$ (17)

Next, the relationship between nondimensional engine specification, S^* , and nondimensional maximum output power, $L_{S,max}^*$, is derived as shown in Eq. (18) from Eqs. (14)~(17).

$$L_{S,\max}^{*} = 1.6 \times 10^{-5} \cdot S^{*0.60}$$
(18)

Also from Eq. (14), Eq. (12) and the definitions of the nondimensional numbers, we can see that the coefficient $a_L=0.24$ is equivalent to the West Number, W_N , at $P^*=1$.

In order to confirm the propriety of Eqs. (14)~(18) based on the experimental results of the five kinds of engines, other experimental results that have already been published are arranged by

Table 2. Performance of prototype engines

Engine name	А	А	В	С	D	E
Engine type	γ	γ	α	β	α	γ
Pressure Pm (MPa)	0.9	0.8	0.55	3.8	1.0	0.22
Expansion space gas temp., T∉ (℃)	488	464	451	503	92	100
Working gas	He	N2	Air	He	N2	Air
Swept volume of exp. spase, Vsɛ (cm³)	81.4	81.4	98.2	132.7	20100	40200
Nondimensional engine spec., S	4.18 × 10 ¹¹	3.52 ×10 ¹²	1.49 ×10 ¹²	1.01 ×10 ¹³	1.51 ×10 ¹⁵	1.24 ×10 ¹⁴
Max. output power Ls,max (W)	102	71	59	1241	747	194
Engine speed nopt (rpm)	1103	954	560	1407	139	93
Nondimensional	9.32	6.05	4.74	1.06	1.70	2.95
output power, Ls,max*	$\times 10^{1}$	×10 ²	$\times 10^{2}$	$\times 10^{3}$	$\times 10^{4}$	$\times 10^{3}$
Nondimensional engine speed, nopt	5.17 ×10 ²	3.21 ×10 ³	1.52 ×10 ³	3.69 ×10 ³	7.84 ×10 ⁴	1.76 ×10 ⁴







Fig. 6 Nondimensional speed as a function of nondimensional engine specification

the same method, and they are compared with Figs. 5 and 6. Table 3 lists the engine specifications, the experimental results and the values of the nondimensional numbers of previously reported engines. In the table, there are several engines that do not have the measured value of the expansion space gas temperature, T_E . In the case of high-performance engines, the heater wall temperature is estimated

Name	Туре	Pm (MPa)	Te (°C)	Gas	Vse (cm ³)	S*	Ls,max (kW)	n _{opt} (rpm)	Ls,max*	Nopt*	Builder	Reference
NS-03M NS-03T	β α	6.2 6.4	698 718	He He	161 190	2.75×10^{13} 3.05×10^{13}	3.81 4.14	1401 1299	1.50 × 10 ³ 1.47 × 10 ³	4.70×10^{3} 4.86×10^{3}	Mitsubishi Toshiba	(1) (1)
NS-30A	α^{*1}	14.7 15.5	660 685	He He	148×4 145×4	3.63×10^{14} 4.17×10^{14}	30.4 45.6	1500 1805	8.43×10^{3} 1 20 × 10 ⁴	3.04×10^{4} 3.64 × 10 ⁴	Aisin Seiki Sanvo	(1)
V-160DMA	α	12.5	625	He	160	1.15×10^{14}	16.2	1500	8.12×10^3	1.15×10^4	SPS	(2)
P40/4-95 102C MP1002CA PD-46 Basic 400hp	α*1 β β β	15 1.2 1.5 10.2 11	750 800*2 700 660 694	H2 Air Air He He	95×4 67 59.4 81 17400	$\begin{array}{c} 6.58 \times 10^{14} \\ 3.85 \times 10^{12} \\ 7.03 \times 10^{12} \\ 5.21 \times 10^{13} \\ 1.74 \times 10^{15} \end{array}$	45 0.48 0.25 3 291	4000 1600 1500 3000 452	1.39×10^{4} 1.46×10^{3} 1.10×10^{3} 1.79×10^{3} 2.43×10^{4}	5.96×10^{4} 3.77×10^{3} 4.81×10^{3} 1.12×10^{4} 6.15×10^{4}	USAB Philips Philips Philips Philips	(3) (9) (3) (3) (3), (4)
MELSE II GPU-3 1-98 1-365 Mod I	α β β α*1	4.5 6.9 22 10.5 15	585 746 630 600*2 660	He H2 He H2 H2	302 120 98 365 124×4	$\begin{array}{c} 2.51\times10^{13}\\ 6.16\times10^{13}\\ 2.98\times10^{14}\\ 4.65\times10^{14}\\ 8.19\times10^{14} \end{array}$	3.1 8.95 15 30 53	1000 3600 3000 2500 4000	1.31×10^{3} 3.98×10^{3} 8.33×10^{3} 1.26×10^{4} 1.87×10^{4}	4.57×10^{3} 1.18×10^{4} 2.90×10^{4} 3.38×10^{4} 8.47×10^{4}	MEL GM Philips Philips MTI	(5) (3) (3), (4) (4), (9) (6), (9)
MT79 4-275 4-215	α*1 α*1 α*1	10 15 20	650°2 750°2 650°2	He H2 H2	189×4 275×4 215×4	2.28×10^{14} 1.37×10^{15} 2.63×10^{15}	57 118 127	2500 2600 4000	1.44×10^{4} 2.39 × 10 ⁴ 3.96 × 10 ⁴	4.14×10^{4} 8.06×10^{4} 1.67×10^{5}	Aisin Seiki USAB Philips	(7), (9) (9) (4), (9)

Table 3. Published engine performance data of high-temperature Stirling engines

*1 Four-cylinder double-acting-type Stirling engine

*2 Value estimated from heater wall temperature

generally instead of the expansion gas temperature, T_E . Thus, the value of T_E is estimated by a simple calculation of the heat transfer based on the specifications of the heat exchangers, the heater wall temperature and other measured values.

Figure 7 shows the relationship between nondimensional speed obtained from the maximum output power, n_{opt}^* , and nondimensional maximum output power, $L_{S,max}^*$. Figure 8 shows the relationship between nondimensional engine specification, S^* , and nondimensional speed, n_{opt}^* . In these figures, white circles represent the experimental results of the previously reported engines listed in Table 3, and black circles represent those of our prototype engines listed in Table 2. The broken lines were obtained using Eqs. (14) and (16), respectively. From the figures, most of the experimental results of the previously reported engines lie on the n_{opt}^* - $L_{S,max}^*$ and S^* - n_{opt}^* lines, in the same manner as that of our prototype engines listed in Table 2. As a result, it is confirmed that the performance of general Stirling engines can be estimated using Eqs. (14)~(18), because the experimental results can be arranged by the nondimensional numbers, n_{opt}^* , $L_{S,max}^*$ and S^* in engines of different types, sizes, working gas and operating conditions.









6. Flowchart of Prediction Method

As the relationship between engine specifications and output power is clarified as described above, engine performance can be predicted at the early design stage.

Figure 9 shows a flowchart of the performance prediction method for the engine First. the nondimensional design. engine specification, S^{*}, can be calculated using Eq. (13) based on the design condition and engine specifications. After the nondimensional speed, n_{opt}^{*}, and the nondimensional maximum output power, $L_{S,max}^*$, are derived from Eqs. (14)~(18), the maximum output power, L_{s,max}, and the engine speed at a particular time, n_{ppt} , can be derived. Namely, the prediction method suggested in this paper can simply predict the maximum output power and engine speed at a particular time based on engine specifications, though it was



impossible to do so using previous prediction Fig. 9 Flowchart of performance prediction method methods. Also, this prediction method can estimate the effects of the kind of working gas and the operating conditions by the derivation of the nondimensional engine specification, S^* ; thus, it is possible to predict the engine performance in detail.

7. Conclusion

In this paper, we measured the performance of high- and low-temperature differential Stirling engines, and derived the nondimensional numbers in order to estimate engine performance. Finally, the performance prediction method based on engine specifications was suggested. This study is summarized as follows.

- (1) The relationship between output work and engine speed can be arranged using the nondimensional speed, n^* , and the nondimensional output work, w_s^* , with nondimensional pressure, P^* , and temperature, T, as parameters (see Figs. 1~3). Also, because the calculated results agree well with the experimental results, the analytical method is proven to have sufficiently high accuracy.
- (2) From the relationship between nondimensional speed, n^* , and nondimensional output power, L_s^* , the optimal value of the nondimensional speed, n_{bpt}^* , is derived based on the nondimensional engine specification, S^* (see Fig. 4). Thus, there is an optimal engine speed, n_{opt} , at which maximum output power, $L_{S,max}$, is obtained.
- (3) The maximum output power, $L_{8,max}$, and the optimal engine speed, n_{opt} , are derived from Eqs. (5)~(18) when the main engine specifications are determined. Engine performance can be predicted by the flowchart shown in Fig. 9.

Additionally, as engine performance is affected by pressure loss in the heat exchangers and mechanical loss caused by friction in the mechanical devices, the effects of these should be clarified. However, engines with different types of mechanical devices and different heat exchange situations such as high- and low-temperature heat source, were investigated in this study. In particular, the heater of a high-temperature differential engine exchanges heat between hot gas and working gas, i.e., from one gas to another gas, and similar types of heat exchangers are used in such cases. In contrast, the heat exchangers and the mechanical devices of a low-temperature differential engine are differential engine described in this paper exchanges heat between hot water and working gas, i.e., from liquid to gas. In spite of the fact that various engines were estimated, engine performance agrees well, as shown in the diagrams (see Figs. $4 \sim 8$), based on the nondimensional numbers defined in this paper. The relationship between engine specification and output power is expressed by Eqs. (14)~(18). Namely, it is considered that the type of heat exchanger and the mechanical loss depend on the swept volume of the expansion space. As a result, it was found that the prediction method suggested in this study has sufficiently high accuracy for use at the design stage of the general Stirling engines.

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