Controlling A Linear Process in Nonlinear Flows*

John Kim
Department of Mechanical and Aerospace Engineering
University of California, Los Angeles
Los Angeles, CA 90095-1597
jkim@seas.ucla.edu

Abstract

Several recent studies concerning the role of linear mechanisms in turbulent flows are discussed from the perspective of how such knowledge could be utilized for turbulence control. Results from a numerical experiment designed to isolate the role of an important linear process in wall-bounded shear flows are presented to substantiate the notion that controlling a linear process in nonlinear flows can be a viable route for flow control. Other implications of the current design of linear controllers are also discussed.

1 Introduction

It has been generally accepted that nonlinearity is an essential characteristic of turbulent flows. Consequently, except for special situations in which a linear mechanism is expected to play a dominant role (e.g., rapidly straining turbulent flows to which the rapid distortion theory can be applied), the role of linear mechanisms in turbulent flows has not received much attention. Even for transitional flows, a common notion is that the most a linear theory could provide is an insight into the early stages of transition to turbulence. But several investigators have recently shown that linear mechanisms play an important role even in turbulent, and hence fully nonlinear, flows. Examples of such studies include: optimal disturbances in turbulent boundary layers (Farrell and his colleagues [2, 3, 4, 5, for example]); transient growth due to non-normality of the Navier-Stokes system (Henningson and his colleagues [6, 7, for example]); energy amplification in the linearized Navier-Stokes system [8]; essentially linear feedback controllers for drag reduction in turbulent boundary layers (Lee *et al.* [9, 10]); and successful applications of a linear control theory to transitional and turbulent boundary layers by the UCLA group [11, 12, 13, 14] and Bewley [15, 16, 17].

*This is a written version of an invited talk given at the Symposium on Smart Control of Turbulence, Tokyo, Japan, March 5-6, 2001. Portions of this paper are taken directly from Ref. [1]

The recent work by Kim and Lim [1] has shown that near-wall turbulence could not be maintained in turbulent channel flow when a linear mechanism was artificially suppressed, thus further illustrating the essential role of a linear process in the nonlinear flow.

In this paper, I shall review some of the above-mentioned work on the role of linear mechanisms in turbulent flows. This review is done from the perspective that controlling a linear processes in turbulent flows could be a viable route for flow control, especially for drag reduction in turbulent boundary layers. The reader is referred to the original references for other significances and implications of linear mechanisms in turbulent flows.

Examples in this paper, unless explicitly noted otherwise, are drawn from direct numerical simulations of a turbulent channel flow similar to those described in Kim *et al.* [18]. I shall use (x, y, z) for the streamwise, wall-normal, and spanwise coordinates, respectively, and (u, v, w) for the corresponding velocity components. Unless stated otherwise, all variables are non-dimensionalized by a characteristic velocity (either the wall-shear velocity, u_τ, or the centerline velocity, U_c) and the channel half-width, h, and Re or Re_τ denotes the corresponding Reynolds number.

2 Non-Normality of the Linearized Navier-Stokes System

The linearized Navier-Stokes (N-S) equations can be written in an operator form

$$\frac{d}{dt}\begin{bmatrix}\hat{v}\\ \hat{\omega}_y\end{bmatrix} = [A]\begin{bmatrix}\hat{v}\\ \hat{\omega}_y\end{bmatrix} = \begin{bmatrix}L_{os} & 0\\ L_c & L_{sq}\end{bmatrix}\begin{bmatrix}\hat{v}\\ \hat{\omega}_y\end{bmatrix} \tag{1}$$

where $\hat{v}$ and $\hat{\omega}_y$ represent the Fourier-transformed wall-normal velocity and vorticity, respectively, and L_{os}, L_{sq} and L_c represent the Orr-Sommerfeld, Squire, and the coupling operators, respectively (see Kim and Lim [1] for definitions).

Classical stability analysis examines the eigenvalues of the operator A and then determines the stability of the linearized system based on whether A has a positive eigenvalue. Although the classical analysis correctly predicts the asymptotic state of the linearized system, it completely ignores the initial transient period, which could play an essential role in the stability of the system. This happens because operator A is non-normal (not self-adjoint) and therefore its eigenmodes are not orthogonal to each other. When eigenmodes are not orthogonal, even if all individual eigenmodes are stable and decay asymptotically (i.e., all eigenvalues are negative), an initial condition consisting of a combination of certain modes (especially those modes almost parallel to each other) can have large transient growth [2, 6]. This transient growth is ignored by the classical analysis.

Non-normality of the N-S system is primarily due to the coupling term L_c in equation (1), which make the operator asymmetric. For two-dimensional disturbances (corresponding to $k_x \neq 0$ and $k_z = 0$, where k_x and k_z represent the streamwise and spanwise wavenumbers, respectively), the coupling term vanishes, but operator A is still non-normal because L_{os} itself is non-normal. It has been shown, however, that the transient growth associated with two-dimensional disturbances is not as large as that associated with three-dimensional disturbances [2, 4]. An optimal disturbance, defined as an initial perturbation that has the largest transient growth, corresponds to $k_x = 0$ and $k_z \neq 0$. This transient growth was attributed to a possible cause for subcritical transition in some wall-bounded shear flows. It was also argued that the transient growth is probably responsible for some bypass transition. One could raise a question, however, regarding how such an optimal (or worst, depending on the point of view) disturbance comes to exist in a real system. This issue was addressed by Bamieh and Dahleh [8], who have shown analytically that three-dimensional disturbances corresponding to $k_x = 0$ and $k_z \neq 0$ can achieve $\mathcal{O}(Re^3)$ energy amplification in response to stochastic excitation, which was introduced into the linearized N-S system as a representation of background noise. The large amplification was due to the non-normality of the linearized N-S system.

Some investigators further postulated that the same linear process is also responsible for the observed wall-layer streaky structures in turbulent boundary layers [2, 3]. The optimal disturbance looks similar to the near-wall streamwise vortices that create the streaky structures. However, this optimal disturbance occupies the entire boundary layer, in contrast to the streamwise vortices in turbulent boundary layers, which are confined to the near-wall region. In order to relate their optimal perturbation theory to those structures observed in turbulent boundary layers, a time scale corresponding to the bursting process in turbulent boundary layers, which is essentially a nonlinear process, was introduced as an additional parameter [3]. It was argued that the transient growth in turbulent boundary layers would be disrupted by turbulent motions on a time scale corresponding to the bursting process, which is smaller than the viscous time scale, and hence, the globally optimal disturbance would never have a chance to grow to its maximum possible amplitude. The notion that commonly observed wall-layer structures are related to a linear process, although it is the nonlinear process that determines the proper length scale, suggests that the same linear process may play an important role in fully nonlinear turbulent boundary layers.

3 Linear Controllers for Turbulent Flows

There is other evidence suggesting that a linear process may play an essential role in turbulent boundary layers. Lee *et al.* [9] designed a neural network to represent an inverse model of the Navier-Stokes equations, which was then used as a nonlinear adaptive controller for drag reduction in turbulent channel flow. A careful examination of the converged weight distribution and the neural network architecture led them to consider a simplified linear network. Although the dynamic range of the weights for the simplified linear network was significantly increased (this makes a hardware implementation much more difficult, but that is irrelevant for the present discussion), essentially the same performance as the nonlinear network was achieved, thus suggesting that the essential wall-layer dynamics responsible for high viscous drag in turbulent boundary layers could be approximated by the linear model.

Lee *et al.*'s [10] application of the so-called suboptimal control also neglected all nonlinear terms in the Navier-Stokes equations. Nevertheless, simple feedback controllers derived from this essentially linear procedure were shown to work quite well for drag reduction in turbulent channel flow. Incidentally, it is worth mentioning that the final feedback control resulting from this procedure is very similar to that obtained from the neural network in spite of two completely different approaches used, one essentially linear and the other nonlinear approach. Interested readers should refer to Lee *et al.* [9, 10].

Other evidence that a linear process may play an important role in turbulent boundary layers can be found in the applications of the linear systems theory to the control of transitional and turbulent flows by the UCLA group [11, 12, 13, 14, 19]. They have shown that controllers based on a systems theory approach — ranging from a simple proportional controller to controllers based on the linear-quadratic-Gaussian/loop-transfer-recovery (LQG/LTR) synthesis — performed remarkably well in suppressing target disturbances in transitional and turbulent flows. Bewley [16] also examined the applicability of a linear controller to a nonlinear flow and showed its success as well as limitations. It is mentioned in passing that Bewley [15, 17] preferred an $\mathcal{H}_\infty$ controller — another linear control theory that can be used to design controllers minimizing a cost function in the presence of a disturbance that maximizes the cost function — over the LQG/LTR controllers used by the UCLA group for its "robustness" against the worst-scenario disturbance that may present in the system. The UCLA group have used LQG/LTR controllers for their, among other things, ease of model reduction, which is essential in designing a controller for a high-order system such as the Navier-Stokes equations.

Although it is not yet understood how controllers based on a linearized model work so well for nonlinear flows and it is a subject of further investigation, these results suggest that the essential dynamics of near-wall turbulence may well be approximated by a linear model. This provides a firm basis that controller designs based on a linear systems theory can be justified, at least for certain applications where controlling near-wall dynamics alone can deliver the design objective, such as drag reduction in turbulent boundary layers.

4 Controlling a Linear Process

Examples presented in Sections 2 and 3 strongly suggest that linear processes must play an essential role in turbulent flows, particularly in the wall-bounded turbulent shear flows. Motivated by these observations, Kim and Lim [1] investigated the role of the linear coupling term, which is a source for non-normality of the linearized Navier-Stokes equations. In their attempt to isolate the role of the coupling directly, they solved the following modified Navier-Stokes equations:

$$\frac{d}{dt}\begin{bmatrix}\hat{v}\\ \hat{\omega}_y\end{bmatrix} = \begin{bmatrix}L_{os} & 0\\ 0 & L_{sq}\end{bmatrix}\begin{bmatrix}\hat{v}\\ \hat{\omega}_y\end{bmatrix} + \begin{bmatrix}\mathcal{N}_v\\ \mathcal{N}_{\omega_y}\end{bmatrix} \tag{2}$$

where $\mathcal{N}_v$ and $\mathcal{N}_{\omega_y}$ represent the nonlinear terms in the Navier-Stokes equations. This modified system can be viewed as representing a virtual turbulent flow without the

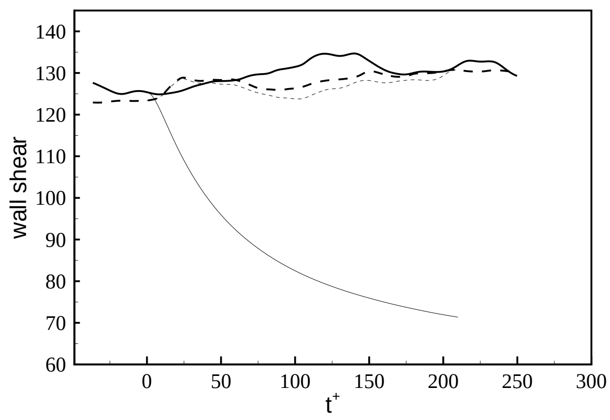

Figure 1: Time evolution of mean shear at wall: ——, upper wall; ----, lower wall. Thick lines are for a regular channel flow, while thin lines are for a channel flow with $L_c = 0$ in the upper half of the channel starting from $t^+ = 0$.

coupling term, or a turbulent flow with control by which the coupling term is suppressed completely. For instance, surface blowing and suction activated to eliminate (reduce) the spanwise variation of v (i.e., $\partial v/\partial z$) could eliminate (reduce) the effect of the coupling term. For example, the opposition control used by Choi *et al.* [20] could be considered as a control scheme trying to minimize the coupling term by suppressing the spanwise variation of v in the wall region.

Two of Kim and Lim's [1] observations are worth mentioning here. First, without the coupling term, turbulence at a low Reynolds number ($Re_\tau = 100$) could not be maintained. Starting from an initial condition obtained from a regular turbulent channel flow, near-wall streamwise vortices quickly disappeared in time and the wall-shear stress was reduced significantly (see figures 1 and 2). Note that the reduction of the wall shear in conjunction with the disappearance of the streamwise vortices is a common feature of many drag-reduced turbulent flows [9]. Turbulence intensities were also reduced drastically without the coupling term. Second, when they started from an initial condition consisting of random disturbances, i.e., without any organized structures, streamwise vortices were first formed before they eventually disappeared, suggesting that the formation of these structures is not directly related to the missing linear term.

Kim and Lim [1] also performed a numerical experiment, in which all nonlinear terms were artificially suppressed. In this experiment, streamwise vortices were formed, but in different scales from the nominal one. They concluded that both the nonlinear terms and the linear coupling term were necessary for the formation and maintaining of these structures at their proper scale. The nonlinear terms are necessary for the formation of streamwise vortices and the linear coupling term is necessary to generate the wall-layer streaks, the instability of which in turn strengthens the streamwise vortices through a nonlinear process. In the absence of either mechanism, turbulence ceases to exist. The result of this second experiment is con-

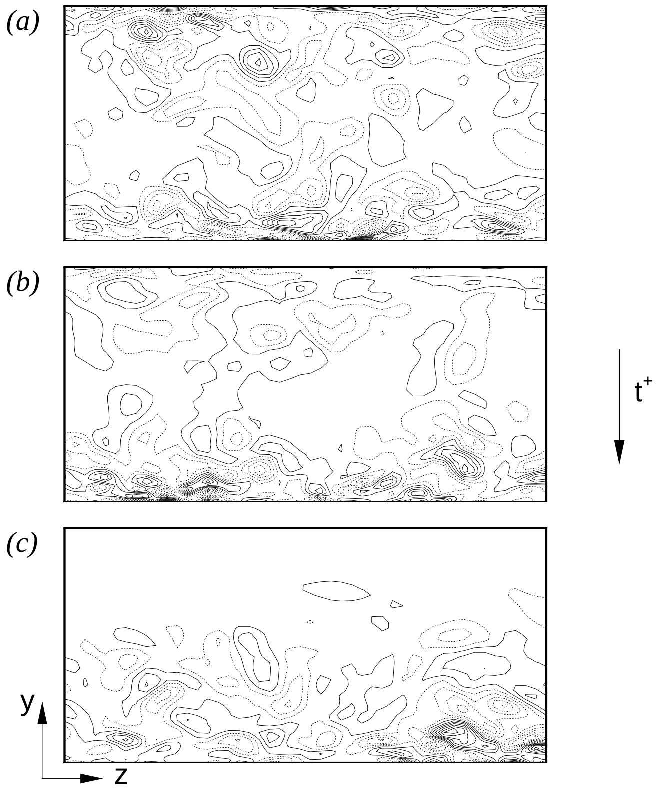

Figure 2: Contours of streamwise vorticity in $y - z$ plane: (a) $t^+ = 0$; (b) $t^+ = 20$; c) $t^+ = 200$. $-80 < \omega_x < 80$ with 18 contour levels. Note that $L_c = 0$ only in the upper-half of the channel.

sistent with Hamilton *et al.* [21] and Waleffe and Kim [22] in that the formation of the streamwise vortices is a result of a nonlinear process.

We are currently developing an LQG/LTR controller designed to reduce the role of different linear mechanisms, including that minimizes the non-normality of the linearized Navier-Stokes equations. Preliminary results obtained from this type of controllers were reported in a recent meeting [23]. Further results will be published in the near future.

5 Concluding Remarks

Several examples illustrating the role of linear mechanisms in turbulent flows have been discussed. Contrary to the common notion that linear mechanisms may not play an essential role in nonlinear flows, there are many evidences suggesting otherwise. This is particularly true for wall-bounded shear flows, for which linear terms in the Navier-Stokes equations are significant due to the large mean shear present in the wall region. It should be noted that in most of the near-wall region, the condition to which the rapid distortion theory can be applied is approximately satisfied with a large $S^* \equiv Sq^2/\epsilon$. Here, S, q^2, ϵ denote the mean shear, twice the turbulence kinetic energy and dissipation rate of the kinetic energy, respectively, and S^* denotes the ratio of the characteristic turbulence time scale over the mean time scale. Therefore, the linearized N-S system, from which the linear controllers have been designed, might be a good approximation to represent some features of the near-wall dynamics. Furthermore, it deserves to mention that the estimator, which is an essential part of LQG/LTR controllers (in control community, a controller in this context is actually referred to as a compensator, which consists of an estimator and a controller), is designed such that it continuously adjusts (through Kalman filtering) the estimated state based on continuous measurements (distributed wall shear for the present example). In this regard, it is worth investigating more carefully how well the estimator in each linear controller is doing in estimating the actual nonlinear state subject to control, and this investigation is currently underway.

Near-wall streamwise vortices are seen to be formed but cannot be sustained without the linear coupling term. The fact that the coupling term plays an essential role in maintaining the streamwise vortices, which have been found to be responsible for high skin-friction drag in turbulent boundary layers, suggests that an effective control algorithm for drag reduction should be aimed at reducing the effect of the coupling term in the wall region. As mentioned earlier, the opposition control used by Choi *et al.* [20] can be viewed as a control scheme trying to reduce the effect of the coupling term by suppressing the spanwise variation of v in the wall region. We are currently designing a control algorithm that directly accounts for the coupling term in a cost function to be minimized, and preliminary results look promising.

Controllers used in the examples discussed in the present paper have been designed in wavenumber space by taking advantage of the homogeneity present in the channel flow. This resulted in a so-called centralized approach, which makes it necessary to collect information from all sensors, transform the information into wavespace, determine control input for each wavenumber, transform the control input into physical space, and then distribute it to all actuators. There are two issues concerning the centralized approach. First, many problems, such as spatially developing boundary layers, do not have the homogeneity although the lack of homogeneity can be circumvented by an approximation. Second, the central processing may not be feasible nor desirable for real-time control when a large number of sensors and actuators are involved. Some investigators have been exploring decentralized approaches, in which a localized kernel function is used to collect/distribute sensor/actuator information [17, 24]. Many issues remain to be resolved, but this is definitely a step toward more practical control.

Acknowledgments

I thank my students and colleagues at UCLA with whom I have carried out the work described in this paper. The support by the Air Force Office of Scientific Research (F49620-97-1-0276 and F49620-00-1-0166, Dr. Marc Jacobs) is gratefully acknowledged. Computer time has been provided through DoD MSRC and NSF NPACI programs.

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