# Model-Based Active Control of Combustion, Recent Developments and Implementations

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## ABSTRACT

Model-based control of complex combustion dynamics phenomena has been successful for systems ranging from bench top experiments to full-scale power systems. The formulation of accurate models that capture the mechanism and quantitatively describe the input output relations between the heat release dynamics and the unsteady flow is a necessary component of this approach. The complexity of combustion makes it impossible to have a single model or a single modeling strategy to describe all possible systems. Similarly, it is important to use different control techniques, while ensuring that the choice of the control approach is compatible with the system mechanism.

## I. Introduction

Modeling combustion dynamics has become increasingly more sophisticated because of the introduction of heat release dynamics models that capture the impact of flow oscillations on the burning mechanism into the overall feedback loop that represents the coupling between exothermicity and acoustics. Many of these models have been intended for use in the design of active control strategies that mitigate the damaging effect of the onset of the instability without major redesign of the system and/or major degradation of its performance. These reduced order heat release dynamics models in which the response of combustion to oscillations in flow rate, velocity, pressure, etc. is modeled through a low dimensional system of ordinary differential equations fit well within the framework of optimal control design. The derivation of these equations has been pursued along one of several approaches: physically based approaches in which a model is derived using conservation equations applied to a simplified representation of the phenomena, data based approaches in which data collected from measurements or detailed numerical simulations are used to capture the essential basis for the system and a truncated version of the latter is used to develop a reduced dynamical model, or a system identification approach model in which the measurements or simulation data are fitted into a pre-assumed model structure. Progress on all these fronts has been made recently and is briefly summarized in this talk. The implementation of these models in optimal, time delay, and adaptive controllers has demonstrated an order magnitude improvement

over previous attempts, and across a range of conditions. A brief survey of these results follows.

## II. FEEDBACK LOOP MODELS

Combustion instability results from the positive coupling between heat release oscillations and acoustics, and is encountered over a wide range of applications, from high-powered rockets to relatively small gas burners. Not only do these instabilities occur as the fuel concentration and hence the power density increases, they are also frequently encountered in lean systems in which the fuel concentration is reduced to lower the burning temperature and hence NOx production. Given the range of applications and spectrum of phenomena, it is impossible to develop a single unified model to describe these processes. However, it has been established that the relationship between the heat release oscillations, q', and the pressure oscillations, p', satisfy the Rayleigh criterion:<sup>1</sup>

$$\frac{d}{dt}\left(\int_{0}^{t} e'Adx\right) = \frac{\gamma - 1}{\overline{\rho}\overline{c}^2} \int_{0}^{t} p'q'Adt - \Delta(E'A) - \Phi > 0, \qquad (1)$$

where e' is the acoustic energy, E' is the acoustic energy flux, A is the cross sectional area normal to the x direction, t is time, L is the duct length,  $\Phi$  is the dissipation of the acoustic energy. This equation has been written for an essentially one-dimensional flow, a simplification that applies over a wide range of conditions in which the acoustic wavelength is longer than most relevant flow phenomena. The Rayleigh criterion indicates that in the case of unstable combustion the phase between pressure and heat release oscillations must be less than 90°. While the Rayleigh criterion can be considered as a powerful diagnostic tool, it does not say much about why and when flow oscillations can impact the combustion zone.

The starting point in revealing these mechanisms is the linear acoustic wave equation, derived by linearizing Navier Stokes equations and neglecting dissipative effects, mean flow and mean heat addition: <sup>2</sup>

$$\frac{\partial^2 p'}{\partial t^2} - \bar{c}^2 \frac{\partial^2 p'}{\partial x^2} = (\gamma - 1) \frac{\partial q'}{\partial t}.$$
(2)

Furthermore, one can use a Galerkin expansion<sup>3</sup> of the flow variables, e.g.,

$$p' = \overline{p} \sum_{i} \Psi_i(x) \eta_i(t), \tag{3}$$

to derive a finite dimensional model for the acoustic modes of the system:

$$\frac{d^2\eta_i}{dt^2} + \omega_i^2\eta_i = \frac{\gamma - 1}{\overline{p}}E^{-1}\psi_i(x_f)\frac{dq'}{dt},\tag{4}$$

where  $E = \int_{0}^{L} \psi_{i}^{2} dx$ . To close the loop, one must then express the heat release

perturbation as a function of the flow variables, e.g.,  $q' = q'(\eta, \dot{\eta})$ , to find conditions under which the feedback would satisfy the Rayleigh criterion. Linearizing this relation we see that:  $\frac{dq}{dt} = a\eta + b\dot{\eta}$ , showing that heat release dynamics can change the characteristic frequency of the system, and may add damping. In case the latter is negative, it is a destabilizing effect. We must stress here that it is the phase relationship between heat release dynamics and the flow variables, in particular the pressure that must be examined carefully to determine the conditions for growing instabilities.<sup>4</sup> Heat release dynamics coupling with the system acoustics can be "direct", when it impacts the combustion zone, or "indirect", through the fluid dynamic field, as shown in Figure 1. We list some recent results next.

### III. HEAT RELEASE DYNAMICS MODELS

Combustion in most practical systems in which instability is encountered is turbulent. Turbulent combustion takes on different forms depending on the turbulent intensity and scales, and the combustion characteristics. Developing physically based dynamic combustion models takes advantage of the classification of turbulent combustion into different regimes in which the turbulent combustion mode is limited to one form, e.g., flame perturbed by the flow, well mixed reactants and products, flame convoluted by the flow etc. as shown schematically in Figure 2. Models have been developed to capture the impact of oscillations heat release, some of which are listed below.

### III.1. Wrinkled Flame Models

In these models turbulence intensity is relatively low, and the length scales are large so that burning occurs in the form of thin flames, possibly contorted by the large scales. In the limit, the flow is laminar and the flame is simply stabilized where the burning velocity balances the flow velocity in the direction normal to the flame. Under these conditions, the flame surface geometry is expressed by the following equation (in axisymmetric coordinates):

$$\frac{\partial \zeta}{\partial t} = u - v \frac{\partial \zeta}{\partial r} - S_{u} \sqrt{\left(\frac{\partial \zeta}{\partial r}\right)^2 + 1}$$
(5)

where *u* and *v* are the axial and radial flow velocity, *z* is the flame height and *Su* is the laminar burning velocity, and the integral heat release rate is:  $\dot{Q} = \kappa_0^R r S_u \sqrt{\left(\frac{\partial \zeta}{\partial r}\right)^2 + 1} dr$ , where  $\kappa = 2\pi\rho(\Delta h_r)$ .

Under conditions when the v << u and the flow velocity is much larger than the burning velocity, the linearized equation can be integrated and the heat release rate perturbation can be expressed as: <sup>5</sup>

$$\frac{d\hat{Q}'}{dt} + \frac{2S_u}{R}\hat{Q}' = \kappa R S_u u', \tag{6}$$

showing that the flame acts as a first order filter that attenuates higher frequencies, with a phase dependent gain on the applied oscillation frequencies. Figure 3 and Figure 4 show these trends. The model indicates that flames may possess a frequency selection mechanism that lead to the amplification of some acoustic modes, depending of the laminar burning velocity and the stabilization mechanism. This was validated against a number of experimental observations. <sup>6</sup> Note that, under the assumptions used to derive this equation, the dynamic properties do not depend on the mean flow or the average flame shape, only on the average equivalence ratio through the laminar burning velocity.

The flame surface model has been successful in describing the conditions of instability in several experiments<sup>7</sup> and has been extended to cases in which the flow perturbations are more complex<sup>8</sup> and when boundary conditions impose certain constraints on the flame response.<sup>9</sup>

The model was then extended to describe the combined effect of flame surface area oscillations and possible equivalence ratio fluctuations induced due to the presence of unchecked air or fuel flow delivery nozzles, or the utilization of oscillating fuel valves.<sup>10</sup> The extended model is:

$$\frac{d\hat{Q}'}{dt} + \omega_f \,\hat{Q}' = b \left( \frac{u'}{\overline{u}} + b_2 \,\frac{\phi'}{\overline{\phi}} + b_3 \,\frac{d\phi'/dt}{\omega_f \overline{\phi}} \right). \tag{7}$$

The relationship between the equivalence ratio fluctuations at the source and the pressure/velocity oscillations at the combustion zone is often complicated by the convective time delay between these two locations, and is often modeled using the lag expressions:<sup>11</sup>

$$\frac{\phi'}{\overline{\phi}} = -\frac{1}{\overline{u}} u'(t-\tau). \tag{8}$$

Time lag plays an important role in many combustion systems, and is known to be one of the determining factors in selecting the unstable mode. It presents a special challenge to active control strategies that will be addressed shortly.

#### III.2. Well-Stirred Reactor Model

In cases when the turbulence intensity is strong and the mixing rates of reactants and products are sufficiently high that one may assume nearly homogeneous combustion zone, the well stirred reactor combustion model has been proposed as a turbulent combustion model. Assuming single step reaction and negligible heat loss, the governing equations under adiabatic, constant pressure conditions are:<sup>12</sup>

$$\rho V c_p \frac{dT}{dt} = \dot{m}_i c_p (T_i - T) + \dot{Q}_r$$
<sup>(9)</sup>

$$\rho V \frac{dI_f}{dt} = \dot{m}(Y_{f,i} - Y_f) - Q\dot{W}_{fr}$$
<sup>(10)</sup>

Linearizing these equations around a giving steady state, we derived the flowing equation:

$$\frac{d\hat{Q}'}{dt} + \alpha \, \hat{Q}' = \beta \, m', \tag{11}$$

where

$$\alpha(\overline{\dot{m}},\phi) = \frac{1}{\tau_r} \left[ 1 + n \frac{(\overline{T} - T_i)}{\overline{T}} - \frac{(\overline{T} - T_i)}{\overline{T}^2} T_a + n \frac{(Y_{f,i} - \overline{Y}_f)}{\overline{Y}_f} \right]$$
(12)

and

$$\beta(\overline{\dot{m}},\phi) = A'_{f}\Delta h_{r}\overline{\rho}^{n-1}\overline{Y}_{f}^{n}\exp(\frac{-T_{a}}{\overline{T}})\left[n\frac{(\overline{T}-T_{i})}{\overline{T}} - \frac{(\overline{T}-T_{i})}{\overline{T}^{2}}T_{a} + n\frac{(Y_{f,i}-Y_{f})}{\overline{Y}_{f}}\right].$$
 (13)

Some facinating results can be derived from these equations. Firstly, the combustion process acts as a first order filter whose properties, contrary to the wrinkled flame model results, depends strongly on the mean flow, or residence time, and mean equivalence ratio. Contrary to the dynamic wrinkled flame model, the dependence on the mean equivalence rato in the well stirred reactor models is much stronger. Secondly, both characteristic parameters  $\alpha$  and  $\beta$  can become negative as the equivalence ratio decreases or the mass flow rate increases beyond certain values. This is shown in Figure 5 and Figure 6, both were taken form Ref. [12]. Clearly, negative values of  $\alpha$  correspond to unstable operation, or blow out. This is precisely the blow out condition predicted by the well stirred reactor theory.<sup>13</sup> Moreover,  $\beta$  reaches zero at the point of maximum average heat release rate, becoming negative for an additional change before the system blows out again. Within this small increment of either the equivalence ratio or the mean flow rate, the phase between the flow and the heat release perturbation changes by 180 degrees. The model predictions match the trends observed near blow out.<sup>14,15</sup> Many practical combustors, especially those endowed with sufficicently strong swirl exhibit very similar characteristics in which transition from stable to unstable operation occurs close to either

maximum heat release rate (maximum flow rate) or minimum equivalence ratio. The transition is often followed by blow out as the opertaing conditons change slightly. The fact that this simple model can predict these trend is very encouraging. The model has been used to predict the stability frequency in several experiments.<sup>16</sup>

# III.3. Shear Layer Stabilized Combustion

Many combustion systems, especially premixed systems, utilize a separating shear layer downstream a bluff body to anchor the flames. Under a wide range of conditions, stable operation is achieved. However, with increasing or decreasing the equivalence ratio, the noise level and pressure fluctuations increase substantially and flame blow out or flash back is observed.<sup>17,18,19</sup> In both cases, cyclic operation with flames convoluted around large scale structures are observed (similar events are also observed during stable operation, but at lower amplitudes and in many cases different frequencies). Despite many studies, the mechanisms for these phenomena are not well understood, although certain degress of control has been demonstrated.

One of the most fascinating observations in some of the reported experiments is that some spectral modes under unstable operating conditions may not correspond to some of the resident acoustic modes of the combustion system, suggesting the presence of other fluid dynamic sustained oscillation mechanisms, besides acoustics, in the system.<sup>20</sup> To explain the origin of this frequency, in a recent study, we demonstrated that under certain condition, shear layer instabilities can become absolute, i.e. sustained oscillations can be expected at the absolute mode frequency of the existing reacting shear layer. We showed using linear stability analysis of a family of local average velocity profiles that resemble those observed in separating shear flows, and for velocity profiles obtained from a numerical simulation of separating recirculating flows, that as the shear layer thickness and the backflow in the recirculation zone increase, the unstable modes become absolute and the Strouhal number of the oscillations, based on the step height, average flow velocity and oscillation frequency, are close to O(0.1), a value supported by many experimental and numerical simulation studies.<sup>21,22</sup> A sample of these results is shown We showed that increasing the equivelence ratio which imposes a in Figure 7. temperature distribution with higher tempertaure ratios, can delay the transition to absolute mode until the temperature ratio reaches higher values, as shown by the results in Figure 8.

In cases when hydrodynamic mode play the role of the resonant oscillator, we showed that the heat release dynamics can be modeled using the following second order oscillator equation:

$$\frac{d^2 \dot{Q}'}{dt^2} + 2\zeta_o \omega_o \frac{d \dot{Q}'}{dt} + \omega_o^2 \dot{Q}' = 0$$
(14)

Where  $\omega_o$  is the frequency of the absolute mode and  $\zeta_o$  is the associated damping.

Combustion instability mechanisms in which large scale structures play an important role can not be easily modeled using reduced physical models due to the complexity of the processes involved. Instead, we have proposed using data driven reduced models, e.g. POD based models to construct a useful representation of the mechanism. POD<sup>23</sup> is an approach in which the numerical results (or spatially resolved experimental data) are used to construct a space of optimal basis functions that describe the different modes in the flow and rank them according to the energy in each mode. These functions are then used as a basis in a Galerkin expansion of the flow, and the ordinary differential equations governing the amplitudes of the modes are obtained by projecting the governing equations onto that space.

# III.4. System Idenitifcation Models

As an alternative to physically based and data based models of combustion, system identification based approaches have been used in connection with experimental data or numerical simulations of combustion instability. In this case, one is interested in input-output models that captures the overall response of the system. Typically these approaches are applied after the system has achieved a stationary state such as a stable Under these conditions, the sustained oscillations are often represented limit cycle. using a lightly damped linear model in the neighborhood of the limit cucle, and system identification is used to determine the model parameters. Besides the model structure, one also needs an appropriate persistently exciting input to identify the system. Methods such as the ARMAX<sup>24</sup> and N4SID<sup>25</sup> have successfully been used to design models and model based control for combustion systems.<sup>26,27</sup> A sample of the result of the application of these technique to a swirl stabilized combustor is shown in Figure 9. Alternatively models based on representing the pressure signal using a Fourier series and captured through nonlinear observers have been used.<sup>28</sup> Note however that SI models are both system and conditions specific and unless care is taken to select the proper structure, some subtle but important dynamics may be lost. For instance, taking advantage of the insight and physically-based modelling, e.g., using Eqs. (4) and (6) or (7), or (11), averaging methods and system identification of limit cycle systems with time delays are used to identify the parameters of the model.<sup>29</sup>

# IV. ACTIVE CONTROL

Based on the basic understanding of combustion instability mechanism, phase shift controllers have been extensively applied to stabilize many combustion systems by measuring the pressure, adding the appropriate phase, generating a new presure signal that cancels the existing oscillation. Phase shift has also been implemented as pure time delay whose value is adjusted until best pressure suppression is achieved. Adaptive version, e.g. an extremum seeking controller, of the same strategy have also been attempted. <sup>30</sup> While successful, the scope of this strategy has been limited; in some cases secondary peaks were encountered following the application of the phase shift approach, and in many cases, perturbations and noise could upset the opertaion. Other

more powerful approaches that take advantage of a more detailed understanding of the phenomena and more accurate models are described next;

## IV.1. Linear Optimal Control

To reduce the pressure oscillations in the shortest possible time given an actuator with certain authority constraints, a control strategy which seeks to minimize a cost function of the form:

$$J = \int_{0}^{\infty} \left( p^{1^2} + \rho u_c^2 \right) dt$$

where  $u_c$  is the control input and  $\rho$  is chosen to represent the available control effort is used. One of the combustion models decribed above is used to determine the control input as a function of the pressure measurement and the model parameters. To minimize the effect of the modeling uncertainity, an LQG-LTR<sup>31</sup> control procedure is used so that the estimator minimizes the effect of modeling errors by representing the latter as fictitous Gaussian errors. This controller has been used successfully to<sup>32</sup> suppress existing pressure oscillationsd by 50dB, without generating secondary peaks, whereas phase shift controller only achieved 20-30dB. In<sup>33</sup>, an additional 10-15 dB reduction in pressure amplitude was realized when applying LQG-LTR to stabilize a swirl combustor over an empericial phase shift controller. Similar reduction was realized when applying the same technique to a bluffbody stabilized system in.<sup>34</sup>

Alternatively an  $H_{\infty}$  controller which ensures that desired measures of stability robustness and performance, given interms of the closed loop transfer functions, are achieved. Examples for the application of this approach are in <sup>32</sup>

### IV.2. Time Delay Control

Control strategies designed to accommodate large time delays are used for systems described by heat release dynamics accompanied by time-delays, e.g., Eqs. (7, 8). Simple PI controllers that make optimal use of actuator locations can be used to cancel out or minimize the delay effects. A more general strategy utilizes the Posicast controller which is based on the Smith controller. <sup>35</sup> The idea is to forecast the future output using the system model and use this to stabilize the system. The controller structure is given by

$$\dot{\omega}_{1} = \Lambda_{0}\omega_{1} + |u(t-\tau)$$

$$\dot{\omega}_{2} = \Lambda_{0}\omega_{2} + |y(t)$$

$$u = \theta_{1}^{T}\omega_{1} + \theta_{2}^{T}\omega_{2} + u_{1}(t)$$

$$u_{1}(t) = \int_{-t}^{0}\sum_{i=1}^{n}\alpha_{i}e^{-\beta_{i}\sigma}u(t+\sigma)d\sigma,$$
(15)

where *n* is the order of the system;  $u_1$  corresponds to the output prediction;  $A_0$  is an *n* x *n* stable matrix;  $(A_0, \mid)$  is controllable; and  $\theta_1, \theta_2, \alpha_i$ , and  $\beta_i$  are the controller parameters. The reader is referred <sup>36,37</sup> for further details regarding the stability and robustness properties and experimental and numerical results of the closed-loop performance.

Application of time delay control in<sup>38</sup> achieved a reduction of 12 dB on a reheat buzz instability using less than 3% of the fuel in the control loop.

### IV.3. Model Based Self Tuning Controller

An alternative approach to adaptive control is to exploit the structure of the dynamic model. For example, for certain actuator locations, the transfer functions corresponding to the models in Eqs. (4) and (6), or Eqs. (7, 8) and (11), can be shown to have relative degree smaller than two, with stable zeros and known high-frequency gain. For these cases, a simple adaptive phase-lead compensator can be shown to successfully suppress the pressure oscillations<sup>39</sup> and is of the form

$$u = k_{o}(t) \vec{p} + \vec{k}_{o} \vec{p}$$

$$\dot{k}_{o} = \gamma_{k} p' \vec{p}$$

$$\dot{p} = k_{c} \frac{s + z_{c}}{s + \rho_{c}} [p'], \vec{p} = \frac{1}{s + a} \vec{p} \bullet$$
(16)

#### IV.4. Adaptive Time Delay Control

As shown in the section on time-delay control, the presence of delay can be accommodated by adding a signal to the control input that attempts to anticipate the effects of the delay. The same approach can be adopted in an adaptive controller as well. The structure of the controller is of the same form as in Eq. (15), but the parameters  $\theta_1$ and  $\theta_2$  are adjusted on line  $u_1$  is chosen as

$$u_{1} = \overline{\lambda}(t)\overline{u}(t)$$

$$\dot{\theta}(t) = -y(t)\omega(t-\tau),$$
(17)

where  $\theta = [\theta_1^T, \theta_2^T, \bar{\lambda}]^T; \omega = [\omega_1^T, \omega_2^T, \bar{u}]^T; \bar{u}_i$ , the *i*th element of the vector  $\bar{u}(t)$ , is the *i*th sample of u(t), in the interval  $[t - \tau, t), i = 1, ..., p$ ; and *p* is chosen to be small enough so that the sampling error in the realization of  $u_1$  is small. A controller whose order depended on the relative degree of the plant rather than its own order was developed in.<sup>40</sup>

Results obtained during an experimental implementation of a PosiCast control algorithm to a 85 kW swirl stabilized combustor is shown in Figure 10.<sup>41</sup>



Figure 1. A schematic diagram showing the direct coupling between the acoustics and the combustion process, and their indirect coupling through the fluid dynamic field.



Figure 2. An unsteady combustion phase plane used to illustrate the impact of the turbulence intensity and its length scale on the premixed combustion process.



Figure 3. Impact of the incident wave frequency on the heat release gain as predicted by the wrinkled flame model (taken from [5]).



Figure 4. Impact of the incident wave frequency on the phase angle between the flow velocity and the heat release rate (taken from [5]).



Figure 5. Variations of the well-stirred reactor heat release dynamics model parameters with the mean mass flow rate at a fixed mean equivalence ratio (taken form [12].



Figure 6. Variation of the well stirred reactor heat release dynamics models parameters with the mean equivalence ration at a fixed mean mass flux (taken from [12].



Figure 7. Dependence of the form of the instability; absolute AU vs. convective CU, and its frequency on the fraction of backflow  $\beta$  and the thickness of the shear layer  $\delta$ . results are obtained for a family of velocity profiles obtained from a numerical simulation of a separating flow over a rearward facing step (see [21] for more detail).



Figure 8. Dependence of the shear flow instability type on the temperature ratio across the mixing zone, note how the unstable mode becomes absolutely unstable at lower equivalence ratio corresponding to lean burning conditions (see [21] for more detail).





Figure 9. System identification based modeling results for a swirl-stabilized combustion showing a comparison between the actual data and the simulation results (taken from [27]).



Figure 10. Results of the application of a PosiCast controller to a swirl stabilized 85 kW combustor using system identification based model which capture the inherent long time delays in the combustor (taken from [41]).

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