Dynamical Processes in Active Control of Combustion Instabilities

S. Candel*, S. Ducruix, T. Schuller and D. Durox and F. Lacas

Laboratoire EM2C, CNRS UPR 288, Ecole Centrale Paris
92295 Châtenay-Malabry, France
*and Institut Universitaire de France

Summary

Strong coupling of combustion with pressure waves gives rise to oscillations and instabilities. Such dynamical phenomena raise difficult practical issues and their understanding and control constitute challenging scientific and technological problems. These topics have received considerable attention during the last period in relation with incidents encountered in gas turbines operating in the premixed mode. Combustion dynamics is also to be considered in the development of high performance devices like rocket motors and jet engines but the problem also appears in less demanding systems like boilers, furnaces and drying equipement. Much research has concerned fundamental aspects and progress has been achieved in the understanding, modeling and simulation of combustion dynamics. Dynamical processes have been investigated in elementary configurations and much has been learnt on flame/vortex interactions, flame shortening by mutual annihilation, collisions with boundaries. The coupling between flames and acoustics waves has been analyzed in well controlled configurations yielding a reasonable understanding of the mechanisms driving instabilities and the coupling between combustion and pressure waves. Numerical simulations of flame dynamics have been completed for both laminar and turbulent flames. This has allowed advances in prediction methods for combustion instabilities. The capacity to model and predict combustion dynamics has allowed progress in the related area of combustion control. Experiments on laboratory scale combustors have indicated that the amplitude of combustion instabilities could be reduced by applying control principles. Much effort has been carried out on the theoretical level to develop control methodologies and deal with the related technical issues (actuators, sensors, integration). Full scale terrestrial gas turbine systems have been equipped with active control indicating that the concepts were applicable. The scale-up has also served to show the limitations of the methods. This article gives a review of of the state of the art in this general area. It focuses on (1) Elementary processes in combustion/acoustics coupling, (2) Simulation of combustion dynamics, (3) Active control issues and multi-dimensional simulation. The first part of this paper concerns experiments on various types of premixed flames. These are used to illustrate some fundamental interactions like those arising when a flame impinges on a solid boundary or those occuring when two flame sheets collide and result in the rapid destruction of flame area. It is shown that this can yield an intense source of sound radiation and that such mechanisms could drive instabilities. It is next shown that simulation of unsteady flames has reached a state which could allow developments of predictive tools for instabilities. These tools will be useful to engineering development and they could also allow developments of control methods. The last part of this paper discusses the many issues raised by active controle and emphasizes the new possibilities of multi-dimensional simulation.

1 Introduction

Smart control of turbulence is a difficult subject but also one which could have a large impact on many applications. It is the subject of this symposium and of previous editions and it has many facets which will be covered during this meeting. The present article focuses on the special problem of control when the flow is reactive. One may think that by adding combustion to an already complicated subject this will make the problem even less tractable. It is true that combustion is in itself a complex phenomenon and it will bring further complication to the subject but the scope of control is also narrowed to the problem of reducing or suppressing instabilities. There is also an advantage in having an exothermic chemical reaction in the flow. This constitutes a source of energy which could be tapped to provide the power required for actuation. It is then clear that much effort should be expanded

to understand the mechanisms which give rise to combustion instabilities and then to implement active control principles which could be integrated in practical devices.
There are many types of flame instabilities but those which are most prevalent in combustion systems are induced by a coupling with pressure waves (see Figure 1 (a) and (b) for a schematic representations of the coupling and active control). The resonant behavior leading to instability is particularly effective when the flame is confined as is the case in propulsion systems (aircraft engine combustors, ramjets, rocket motors), gas turbines, power-plants, industrial process devices. Confinement is favorable to resonant interactions which in turn induce powerful dynamical phenomena and have serious consequences in the form of "combustion instabilities". One also finds self-induced instabilities in unconfined geometries as shown for example in Ref.

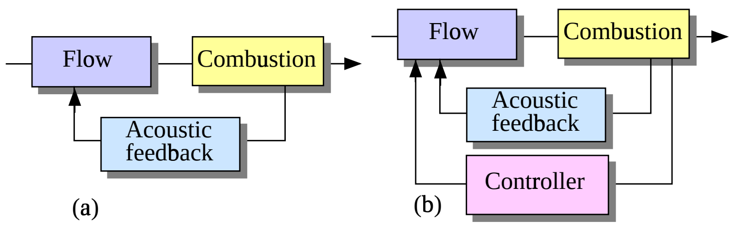

Figure 1: (a) Block diagram describing combustion instability mechanism. (b) Block diagram describing active control of combustion instability.

Coupling of combustion and pressure waves has been extensively investigated over the past fifty years and it has evolved from phenomenology to a situation where numerical simulations can be carried out (see Refs. [1-4] for recent reviews). Active control of instabilities has also evolved during this period from some early concepts of feedback stabilization of combustion to more recent demonstrations in laboratory experiments of active control and full scale application in gas turbine combustors. The design objective is to keep the combustion process under control and avoid possible flashback, blow-out and resonant instability. Control is a difficult matter because the combustion process is nonlinear, involves a variety of coupled mechanisms and is not easily described mathematically or simulated numerically. On the practical side, there are difficulties associated with the harsh environment prevailing in combustors. It is not easy to integrate actuators and sensors in practical systems. There are also many problems related to the maximum power output achievable by the actuators, the bandwidth and time lag induced by the actuator location with respect to the reactive region and the time lags inherent to the combustion process itself.
At this point it is worth examining some of the causes of combustion sensitivity to pressure waves.

- In many combustion systems the power density is quite sizable. A minor fraction of this power suffices to drive a large amplitude oscillation. In a modern cryogenic rocket engine for example, the mean pressure is typically $\overline{p} = 10$ MPa, the mean power density reaches $E_c = 50$ GW m^{-3}. If an instability develops the rms pressure perturbation may typically reach 1 to 10 % of the mean pressure. For a 10% fluctuation, the power density associated with the unsteady motion is $E_a = 0.1$ MW m^{-3}. which is just a fraction of the power density in the combustion chamber : $E_a/E_c \simeq 10^{-6}$. This shows that the oscillation may be driven by a small fraction of the heat release.

- Combustion features time lags. Reactants introduced in the combustor at one instant are converted into burnt gases at a later time. It is known that systems with delays are inherently unstable. This is exemplified by examining a second order model featuring a linear damping and a delayed restauring force :

$$\frac{d^2y}{dt^2} + 2\zeta\omega_0\frac{dy}{dt} + \omega_0^2 y(t-\tau) = 0 \tag{1}$$

If the delay τ is small, the last term may be expanded in a Taylor series. This yields to first order

$$\frac{d^2y}{dt^2} + \omega_0(2\zeta - \omega_0\tau)\frac{dy}{dt} + \omega_0^2 y(t) = 0 \tag{2}$$

This yields a negative damping coefficient if $\omega_0\tau > 2\zeta$. If the delay is sufficient, one expects an exponential growth of an initially small perturbation. Time lags in a system are destabilizing. This observation forms the basis of the early work on rocket instability. The further observation that the lag is not constant but depends on physical parameters like pressure or temperature in the chamber has lead to the sensitive time lag (STL) theory developed during the fifties and sixties (see for example Crocco and Cheng [5]). Consider the simple case where one reactant is injected as a spray, an increase in pressure augments the ambiant density thus decreasing the droplet size. Vaporization proceeds faster and conversion of reactants into burnt gases is accelerated. Thus, an increase in pressure produces a decrease of the time lag τ between injection and combustion. If the reactant is injected at a constant rate and if one assumes that only pressure influences the time lag one finds that the relative change in the mass rate of burnt gases $\mu_b(t) = (\dot{m}_b - \overline{\dot{m}}_b)/\overline{\dot{m}}_b$ is a function of the relative change in pressure $\varphi = (p - \overline{p})/\overline{p}$ and that these two quantities are related by $\mu_b(t) = n\,[\varphi(t) - \varphi(t-\tau)]$. In this expression the interaction index n describes the sensitivity of the combustion process to pressure. The relative change of burnt gas mass flow rate is proportional to the difference in pressures at times t and $t - \tau$.

- There are many resonant modes in the confined geometries used in practical combustors. Acoustic modes often dominate the coupling process. Resonance is sharp because the losses are weak. For a low frequency resonance, the wavelength exceeds the typical transverse dimension, wave propagation is longitudinal and generally involves the complete installation giving rise to “system instabilities”. High frequency resonances ($f > 1$ kHz), feature wavelength which are of the order or less than the tranverse dimension and the coupling usually involves transverse modes of the chamber. These “chamber instabilities” are found in systems where the combustor is well decoupled from upstream and downstream systems (reactant supply lines, turbomachinery, vessels …). High frequency transverse instabilities are particularly damaging in rocket engines.

- Energy may be fed into the the perturbed motion by a range of processes. It is known from an early work of Rayleigh that the gain is positive if the heat release fluctuation is in phase with pressure or more exactly if $\int_T p'q'\,dt > 0$. Flow perturbations which may initiate the process involve vortex roll-up, shear flow instabilities, flame acceleration, collective interactions of reactant jets, collisions with boundaries, flame shortening resulting from mutual annihilation of flame surface area. If the feed lines and the chamber are not well decoupled, for example by a sizable injection head loss, instabilities can also be driven by the differential response of the feed system submitted to perturbations developing in the chamber.

Many factors are clearly favorable to a strong coupling between combustion and pressure modes. It is important to avoid the occurence of such interactions because they have detrimental consequences (vibrations, structural fatigue, enhanced heat fluxes to the chamber walls) and may lead to failure, loss of the system and in some cases spectacular accidents. Control of combustion and suppression of instability may rely on passive or active methods. In general, one tries to change the balance between gain and losses. This is usually achieved by augmenting the removal of energy from the oscillation with resonant cavities or acoustic liners or by modifying the chamber design to avoid the most destructive modes of oscillation. In rocket engines, this is achieved by placing baffles near the injection backplane. It is also possible to envisage active control solutions involving feedback stabilization concepts. Early ideas of this type were explored theoretically during the 1950th by Tsien [6], Marble and Cox [7], Crocco and Cheng [5] but practical demonstrations had to wait until the 1980th and full scale application has been achieved only recently.

Combustion dynamics is a complex process and design tools for instability control are not yet available. However, this situation is evolving rapidly with research focusing on the development of detailed numerical models for combustion dynamics and control. These new developments rely heavily on experimental data and on the broad insight gained on the process during the previous years.

All the complexities of this problem cannot be covered in the space allocated to this article. Four aspects will be examined in what follows. A few elementary processes in combustion/acoustics coupling are described in Section 2 with various experiments on premixed flames responding to an acoustic modulation imposed upstream. This topic is also reviewed in Refs. [4] and [8]. It is shown in particular that a flame impinging on a solid boundary and perturbed from upstream operates like an amplifier and radiates an intense sound field. This typifies one possible source of instability. It is also shown that a similar mechanism may be observed when adjacent flame sheets impinge on each other. Simulation of combustion dynamics is reviewed in Section 3 and illustrated in Section 4 with calculations of a premixed turbulent flame in an unstable combustion process. Active control of combustion instabilities is treated in Section 5. This section deals with principles, describes some applications and reviews low order models for active control simulation. Multidimensional simulations of active control is envisaged in Section 6. It is shown that MDS of AC could be used to advance the state of the art in this field.

2 Flame interactions as a source of combustion instability

The contents of this section are meant to illustrate mechanisms which may act as sources of instabilities. In the wide variety of possible interactions two situations are examined because they typify more complex processes which could take place in larger scale turbulent combustors. Both situations involve perturbed laminar flames. These examples are selected for the following reasons :

- Laminar cases provide simple and well controlled configurations for detailed observations and they may be used to uncover novel coupling phenomena.
- Interactions may be modeled from basic principles and do not depend on closure schemes like those required for descriptions of turbulent combustion.
- Data may be used to validate computational tools for combustion dynamics.

In addition, some practical applications feature quasi-laminar flames (domestic boilers, radiant heaters and industrial dryers). Systematic experiments on laminar flame dynamics have been carried-out in the case of a premixed flame anchored on a cylindrical burner (see Ref. [9]). This has allowed direct comparisons between observed flame dynamics and simulations. Recent studies in which the flame impinges on a flat plate feature remarkable interactions. The phenomena analyzed in detail in Ref. [10] are only summarized.

In the first experiment, a premixed flame is formed from an axisymmetric burner comprising a cylindrical tube, 16 cm in length, followed by a convergent nozzle. The exhaust diameter is 22 mm(Figure 2 (a)). A driver unit placed at the bottom of the burner modulates the flow around its mean value $\bar{U}$.

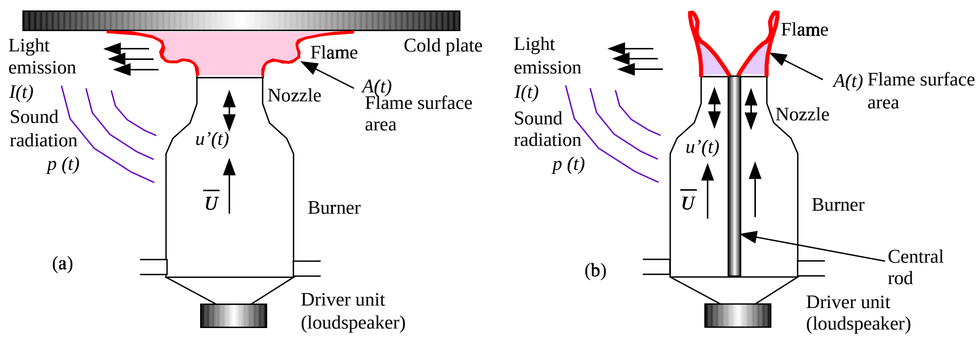

Figure 2: (a) Experimental setup for perturbed flame/plate interactions. (b) Experimental setup for flame/flame interactions. $\bar{U}$ is the mean bulk velocity and u' the fluctuating velocity component at the burner outlet.

A well controlled harmonic velocity perturbation u' is generated at the burner outlet. This wrinkles the surface of the flame front $A(t)$ as shown in Figure 3.

Sound pressure measurements were carried out for a range of driving frequencies f_e. The overall pressure level at 25 cm away from the flame is plotted in Figure 4 as a function of the driving frequency f_e. The sound generated in the case of a perturbed impinging jet flame is always 10 to 20 dB louder than the sound produced by the same upstream perturbation but without combustion or in the absence of the plate. The additional noise is generated by combustion only when the flame interacts with the plate. This mechanism is investigated in further detail in Ref. [10] .

Sequences of phase locked images of the flame pattern over a cycle of excitation $T_e = 1/f_e$ are processed to obtain the trace of the flame front in the plane of symmetry of the burner. They suggest that the flame front is strongly wrinkled by the modulation of the flow. The perturbation produced at the nozzle outlet is convected along the flame front away from the burner. When it collides the plate a large portion of the flame surface vanishes in a short period of time (see Figure 3). The sudden collapse of the flame surface area is responsible for the significant noise output from the flame [11].

The flame interactions with the plate define sources of sound and it can be shown that the radiated pressure p_∞ in the far-field is given by the time retarded rate of change of the flame surface area $A(t)$ (see for example Refs. [12] and [13]) :

$$p_\infty(r,t) = \frac{\rho_\infty}{4\pi r}\left(\frac{\rho_u}{\rho_b} - 1\right) S_L \left[\frac{dA}{dt}\right]_{t-\tau} \tag{3}$$

In this expression, ρ_∞, ρ_u, ρ_b are respectively the far-field air, the reactant gas and the burned gas densities. τ is the travel time of sound propagation over the distance r from the sources to the microphone. In a first approximation

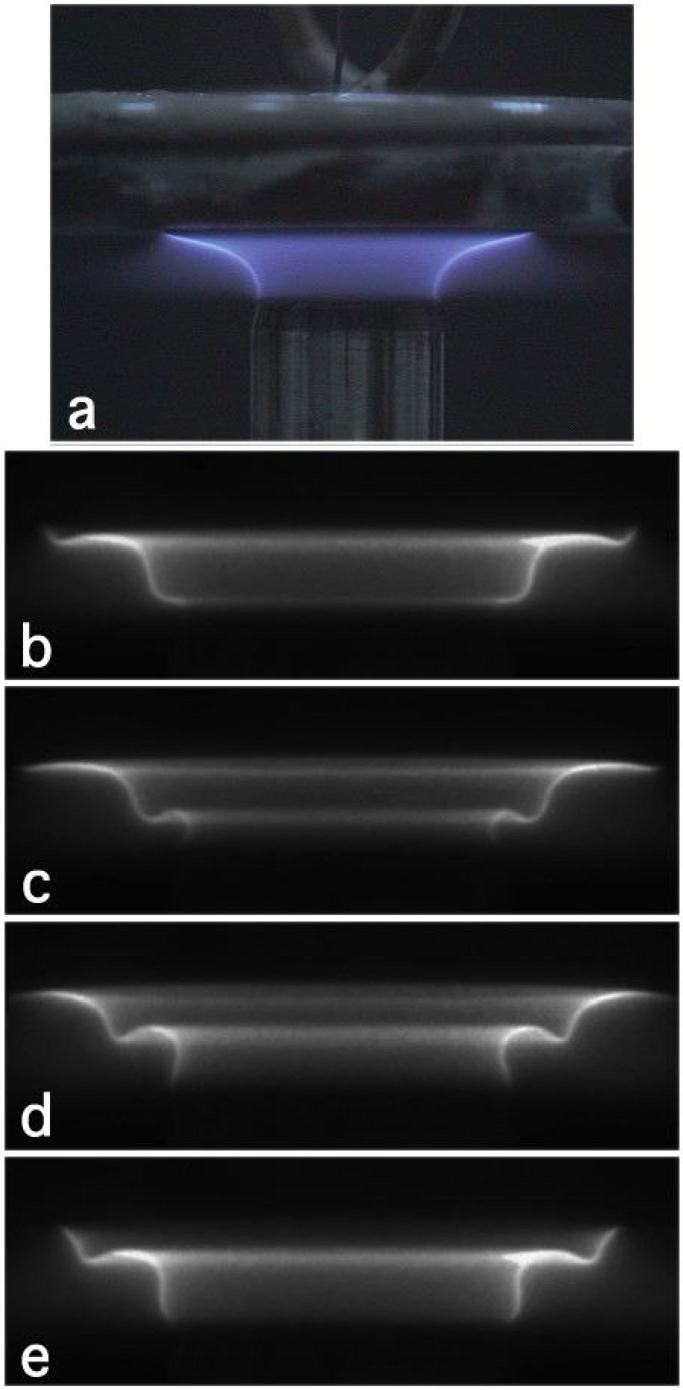

Figure 3: Typical interactions between a perturbed flame and a plate. The first photograph shows a direct view of the flame in the absence of perturbation

the emission of the flame $I(t)$ can be considered to be proportional to the volumetric rate of consumption of gas reactants $Q(t)$ [14]. Assuming a constant flame speed S_L, $I(t)$ is thus also proportional to the total flame surface area $A(t)$ [10]. The far-field pressure signal p_∞ should also be proportional to the rate of change of $I(t)$.
Measurements of the flame light emission $I(t)$, direct estimations of the flame surface area $A(t)$ (using image sequences) and measurements of the far-field pressure signals p_∞ are indeed well correlated. The fast rate of extinction of the flame surface at the cold boundary observed corresponds to the significant acoustic pressure decrease measured by the microphone.

This experiment and many others indicate conclusively that periodic flame-wall quenching can be a strong source

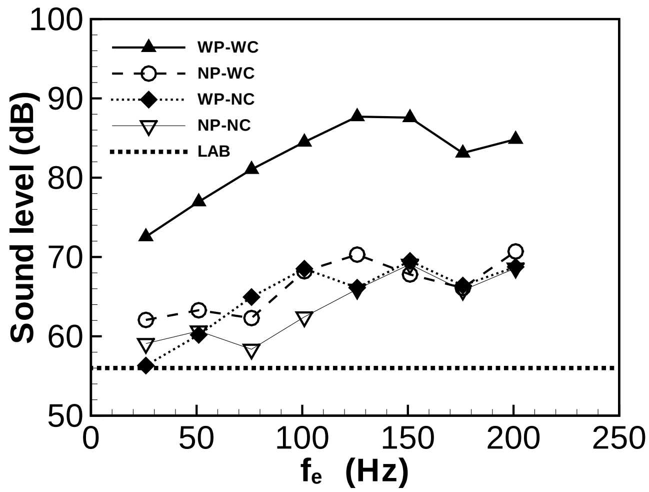

Figure 4: Overall sound level as a function of the driving frequency. Mean axial velocity: $\bar{U} = 1.44$ m/s. Equivalence ratio: $\Phi = 0.95$. Nozzle-to-plate distance $z = 7.6$ mm. The convention adopted in the caption is WP: with plate, NP: without plate, WC: with combustion, NC: without combustion, and LAB: mean background noise in the laboratory (from Schuller, *et al.* [10]).

of noise. In a confined system (in the acoustic sense) where a periodic unsteady flame-wall interaction takes place, the noise emitted in this situation could be a source of acoustic energy which would feed the natural modes of oscillation of the system. This could lead to a resonant feedback process driven by the flame-wall interaction mechanism. Such self-sustained oscillations constitute a simple but remarkable example of the coupling between combustion, acoustics and solid boundaries (Durox, *et al.* [15]).
The second example typifies a similar mechanism in which two flames interact inducing a rapid change of flame surface area. Here again the fast change of this quantity produces an intense radiation of sound. The configuration is that shown in Figure 2 (b). A central rod is now located inside the exhaust nozzle. The flame is stabilized on the central body and on the burner rim and its shape is like that of a liquid sheet in a fountain. Under weak perturbations from upstream, adjacent sheets near the flame tip come close together and impinge on each other producing a rapid destruction of flame surface area accompanied by an intense radiation of sound (Figure 5).

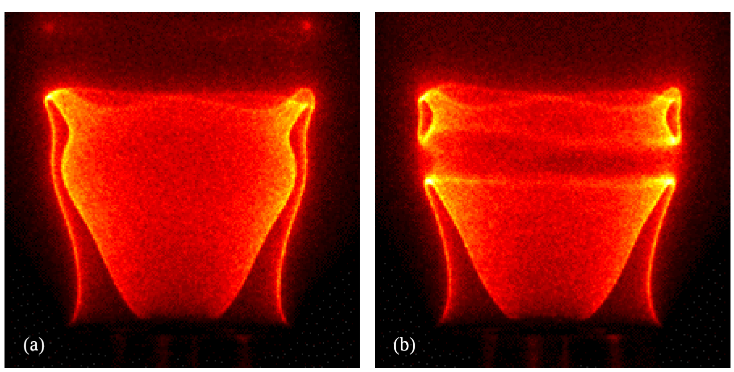

Figure 5: Mutual flame interactions leading to a rapid destruction of flame surface area. The steady flame is stabilized on an annular burner.

Here again, the rapid destruction of flame surface constitutes a strong source of pressure. This is potentially another driving mechanism of combustion oscillations as demonstrated in Schuller, *et al.* [16]. Mutual interactions of the type illustrated here can also be found in more complex turbulent flow configurations like those prevailing in gas turbine combustors and there are numerical simulations of such flows showing this type of phenomena.

3 Modeling of combustion dynamics

The modeling of combustion dynamics is a central question if one wishes to develop control methods. Ideally, the system which one wishes to control should be described by a set of differential equations of the standard form $\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\boldsymbol{u}$ or more generally by a set of equations of the form $\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u})$. To obtain this kind of representation one follows the scheme defined in the first of the following two procedures :

- Acoustics is considered as the central mechanism and one writes a wave equation for the reacting flow. This is then used to derive a unified framework for the analysis of combustion oscillations. Even though this is an attractive approach, it tends to hide the difficult problem of describing the response of the flame to the wave motion.

- Combustion is taken as the central process and the modeling is aimed at the description of the flame dynamical response to acoustic waves. The analysis emphasizes the fluid mechanics, the flame motion in a field of perturbations, the differential response of the cold and hot streams of gases ... The modeling of premixed combustion dynamics has most often relied on a kinematic equation (G-equation) to represent the flame motion but much of the current effort is focused on the large eddy simulation of combustion dynamics. This second approach does not yield a simple description of the type required for control applications ($\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u})$) but it is possible to couple a multi-dimensional simulation of the flame with an external control loop (see Section 6).

These two procedures are now briefly outlined.

The “unified analytical framework” of combustion oscillations established by Culick [6] and his co-workers uses an eigenfunction projection method equivalent to the Galerkin procedure initially introduced in this field by Zinn and Lores [17]. One first formulates a boundary value problem for acoustic perturbations in a reactive medium

$$\boldsymbol{\nabla}^2 p' - \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} = h(\overline{\boldsymbol{v}}, \overline{p}, \boldsymbol{v}', p'; \ldots), \quad \text{with} \quad \boldsymbol{n} \cdot \boldsymbol{\nabla} p' = -f \tag{4}$$

Inhomogeneous Neumann boundary conditions are used in this formulation but other conditions could be defined as well.

The acoustic pressure field is expanded in terms of the normal modes of the system:

$$p'(\boldsymbol{x}, t) = \overline{p} \sum_{n=1}^{\infty} \eta_n(t) \psi_n(\boldsymbol{x}) \tag{5}$$

The eigenfunctions $\psi_n(\boldsymbol{x})$ satisfy a homogeneous Helmholtz equation and corresponding homogeneous boundary conditions :

$$\boldsymbol{\nabla}^2 \psi_n + (\omega_n^2 / c^2)\psi_n = 0, \quad \text{with} \quad \boldsymbol{n} \cdot \boldsymbol{\nabla} \psi_n = 0 \tag{6}$$

The amplitudes η_n then satisfy a set of second order differential equations :

$$\frac{d^2 \eta_n}{dt^2} + \omega_n^2 \eta_n = F_n \quad \text{where} \quad F_n = -\frac{c^2}{\overline{p} E_n^2}\left[\int_V h \psi_n dV + \int_A f \psi_n dA\right] \tag{7}$$

This formulation focuses on the wave motion and provides a framework for the determination of the modal amplitudes η_n. The central difficulty is to specify the source terms arising from the spatially distributed coupling between acoustics and combustion and formally represented in the previous equations by the functions $h(\overline{\boldsymbol{v}}, \overline{p}, \boldsymbol{v}', p'; \ldots)$ and f. A similar difficulty is found when one wishes to develop a suitable representation of control source terms. Because the formulation is linear, these terms generally appear as additive sources, h becomes $h + h_c$ and f becomes $f + f_c$ but these linear forms do not describe the subtle interactions which take place between control sources and unstable motion. The unified framework is however a valuable tool for theoretical investigations and may be used to devise “low order models” of combustion dynamics and control.

The second procedure emphasizes flow and combustion dynamics but there is no unique framework. The general objective is to obtain a faithful rendering of the nonsteady motion. The flame motion constitutes the central issue while the system acoustics define the boundary conditions. For premixed systems, one general method has consisted in describing the flame as a front. This may be done with a kinematic equation describing the motion of this front, the so-called G-equation. The flame is defined as one iso-level $G = G_0$ of a function G describing the propagation of a front featuring a normal velocity S_d. One considers for example that $G = 0$ in the fresh gases and $G = 1$ in the reacted mixture and one has to solve the first order partial differential equation :

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} G = S_d \, |\boldsymbol{\nabla} G| \tag{8}$$

In most analytical studies using this formulation the change in density accross the flame front is neglected. Various arguments are used to justify an assumption which is obviously not quite right but is often made in combustion and has the advantage of simplifying the analysis.
Because the theoretical approach has limitations and in particular cannot portray the flame dynamics in most cases, recent efforts have focused on the numerical modeling of combustion dynamics. Again, there is no unique procedure but it is useful to identify some modeling levels and combined approaches:

- Direct simulation. The flame motion is calculated with a DNS method, acoustics are treated with characteristic boundary conditions possibly involving time domain impedance models ...
- Large eddy simulation of the flame motion, acoustics and corresponding boundary conditions.
- Large eddy simulation for the flame motion, analytical or linearized Euler description of the wave motion outside the flame zone.
- Low order models for the flame zone, analytical or linearized Euler description of waves.

In most practical cases the flow and flame motion are turbulent and the LES approach is the most suitable. Large eddy simulations (LES) and subgrid scale modeling (SSM) methodologies are now well established for nonreactive flows. Combustion LES is less mature and constitutes a relatively new field not much older than about a dozen years. One motivation for the development of LES is that it provides a route to an improved modeling of combustion and is specifically suited to the description of nonsteady dynamical combustion phenomena which one wishes to suppress. Important advances have been made in the numerical modeling of flame dynamics using large-eddy simulations (see Refs. [18-26]). but there are many difficulties in the application of LES to combustion dynamics. Some prominent issues are (1) the correct description of the flame motion and the associated subgrid scale modeling problem, (2) the reduction of numerical dissipation and dispersion to acceptable levels and (3) the treatment of boundary conditions.
One difficulty with combustion LES arises from the fact that chemical conversion of reactants takes place in thin layers. These cannot be captured on the relatively coarse grid used for the simulation. It is therefore necessary to adapt the flame description to the LES framework. In the premixed case, this has been achieved by following two different routes :

- The flame is described as a thin front and its motion is calculated by a front tracking technique or with a G-equation formulation.
- The flame is artificially thickened but its displacement speed is preserved.

The first method based on the theoretical work of Kerstein, *et al.* [27], was explored for example by Menon and Jou [18]. The reactive layer is considered to be infinitely thin with respect to the size of the turbulent structures, and it is represented by a propagative iso-surface separating fresh and reacted gases. This surface is located at an isolevel of a field variable G which satisfies the propagation equation (8) where S_d is a turbulent displacement speed.
The second representation of premixed combustion in the LES framework, consists in artificially thickening the reaction zone, while keeping its displacement speed unchanged. The idea was originally developed by Butler and O'Rourke [28] and O'Rourke and Bracco [29], for the computation of laminar flames in complex geometries. It has been applied in many recent large eddy simulations of premixed combustion (for example, Thibaut and Candel [22], Angelberger, *et al.* [23], Colin, *et al.* [25]). Details may be found in these references and will not be reproduced here. Fine tuning of the thickened flame model have been devised to account for combustion taking place in the small scales Colin, *et al.* [25], Nottin, *et al.* [30]. An efficiency function E is used to incorporate the effects of the unresolved scales on the resolved rate of reaction.
Another issue of importance in combustion dynamics LES is that of boundary conditions. From computational fluid dynamics, it is known that the treatment of inflow, outflow conditions together with conditions at the walls determines to a great extent the quality of the solution (see for example Poinsot and Lele [31]). In combustion instability simulations one has to establish boundary conditions for the fluid flow but simultaneously impose conditions on the various waves involved in the process. The two types of conditions may not be compatible. To illustrate this point consider an outflow boundary. This is usually treated by imposing a constant static pressure. In doing so one makes the implicite assumption that the specific acoustic impedance vanishes at the boundary ($\zeta = Z/(\rho c) = (1/\rho c)(p'/\boldsymbol{v}' \cdot \boldsymbol{n}) = 0$). This condition implies that outflow pressure fluctuations are vanishing. However, this may not be so and one should be able to impose other types of conditions at this boundary. Similarly, one generally imposes the mass flow rate at the inflow. This implies a condition on the specific impedance

$\zeta = 1/M$ where M designates the mean flow Mach number. Again this condition may not suitably render the response of the upstream domain to acoustic waves generated in the system and impinging on the inflow boundary. Another aspect which deserves special attention is related to the modulation of a given system to study the forced response of the flow to external perturbations. One such problem is treated in detail in Ducruix, *et al.* [32] and Kaufmann, *et al.* [33]. Consider for example a sinusoidal velocity perturbation $u = u_0 \cos \omega t$ injected in the system at the upstream boundary. This modulation is introduced at time $t = 0$ in a region which is initially free of perturbations. The system will respond to this perturbation by exhibiting a forced motion but it also features a free motion which compensates the mismatch which exists between the forced motion and the perturbation free initial state. As a consequence, the modulation excites all the natural modes of the system and the amplitudes of the eigenmodes which are close to the perturbation frequency will have finite values of the same order of magnitude as the response at the excitation frequency. If the integration scheme features a reduced numerical viscosity, as it should to deal with acoustic wave problems, the modes excited in the process will be weakly damped and the oscillations of these modes will be sustained for a long period of time. Methods which may be used to deal with this problem are discussed in Ducruix and Candel [34] .

4 Simulation of flame dynamics

Turbulent combustion dynamics is illustrated in this section with unsteady flame calculations in a realistic geometry [32]. The premixed combustor developed by the University of Cambridge is a model scale version of an industrial gas turbine unit. The airflow enters the injector through an annular section and is then divided into two streams which flow through concentric channels (Fig. 6). Vanes are placed in these channels to create a rotating velocity component with an opposite sign (“contra-rotating swirl”). Gaseous fuel is injected in the airflow through cylindrical bars fitted with two 0.45 mm diameter holes. The swirler unit is plugged on a 70 mm in diameter quartz tube. Burnt gases are exhausted from this tube at atmospheric pressure.

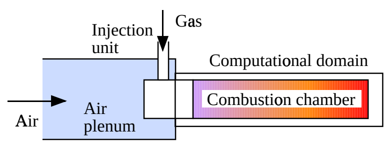

Figure 6: Schematic representation of combustion system and position of the computational domain.

Simulations of the complete system (air supply, fuel supply, swirler unit and combustion chamber) would require an excessive amount of computer ressources. It is therefore assumed that (1) the flow beyond the injection vanes is perfectly premixed, (2) the rotating component u_θ of the flow created by the vanes is proportional to the axial flow velocity and to the tangent of the vane angle and (3) u_θ is a constant in a given section. The system is considered to be axisymmetric and the calculations are carried-out over a sector corresponding to a 10° angle around the symmetry axis. This calculation is an illustration of what can be done in this field. New computer power makes it now possible to develop three-dimensional simulations, even if they still remain costly.

The artificial thickening approach described in the previous section is implemented in AVBP, a LES flow solver developed by Cerfacs [35]. The key idea is to thicken the flame while maintaining its propagation speed. The thickening factor F must be large enough to resolve the flame front on the LES mesh, but small enough to let the flame be wrinkled by the vortices calculated in the simulation. A thickening factor F = 8 was chosen to meet both requirements.

Calculations correspond to an equivalence ratio ϕ = 0.7. Fresh gases are injected through the internal and external channels of the injector. The flow velocity is set equal to 15 m s^{-1}, corresponding to an airflow rate of 60 g s^{-1}.

Figure 7 shows the temperature in the swirler unit near field. A recirculation zone appears in the central part of the chamber and plays an important role in the stabilization of the flame. Burned gases are in contact with fresh gases, providing a continuous source of ignition to the flame.

The stabilization region is near the lower border of the internal channel in a region of reduced flow velocities. Snapshots of the temperature field show that vortices develop and the flame stabilization position moves when these structures are shed. This causes fluctuations in the flame surface which may lead to combustion instabilities. At certain instants in time the flame propagates inwards along the internal channel separation. This has been observed experimentally and could eventually lead to flashback.

Vortices created in the region between the outer jet and the chamber wall (Fig. 7) are convected in the chamber and clearly wrinkle the flame front producing from time to time large pockets of fresh gases which are conveyed into the recirculation region and destabilize the flame. Pockets of fresh gases are also created near the flame extremity.

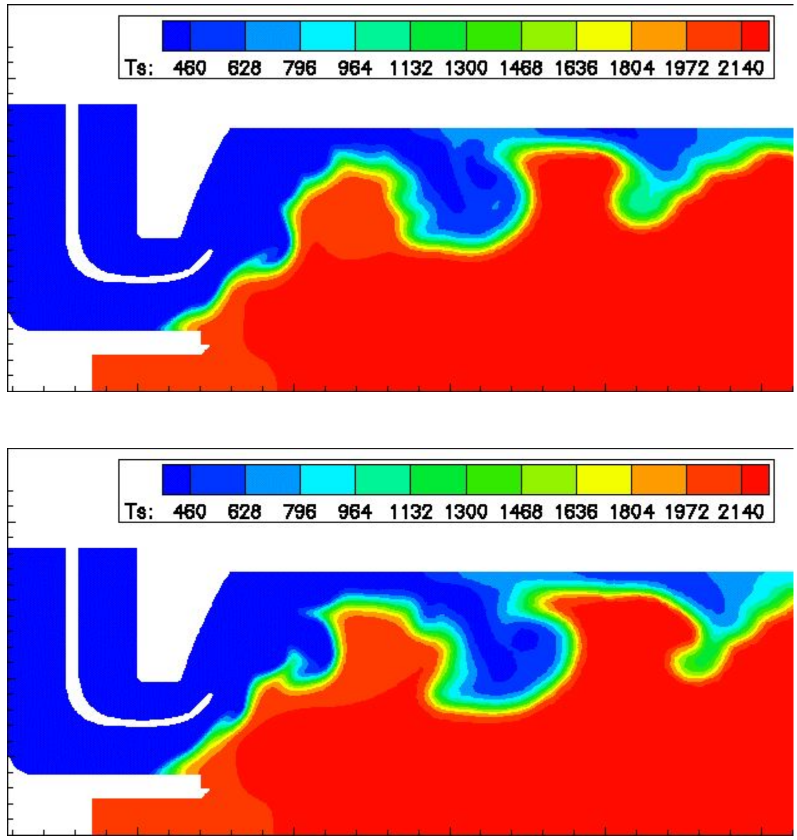

Figure 7: Two snapshots of the temperature field near the injector outlet.

These pockets are convected and burned with a certain delay. Depending on the delay, this phenomenon may participate in the development of acoustic instabilities.
The flame length varies between 8 and 12 cm during the calculation. In the first part of the flame front (0.1$<x<$0.3), the amplitude of the motion is relatively small and the flame remains close to its mean position, whereas large movements can be observed further downstream. Averaged images of the flame would then show a short flame, corresponding to the integration of the flame emission in the first part of the chamber and similar to experimental visualizations. The flame is slightly lifted with respect to the injector channel, even though a small reaction rate exists near the separation wall. It is interesting to note that the stabilization position is not fixed in time. Longer simulations would make it possible to determine frequencies associated with this phenomenon.

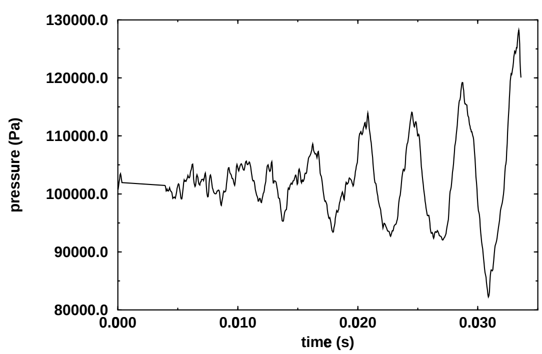

Figure 8: Temporal evolution of the pressure at a point in the middle of the chamber.

Figure 8 shows the temporal evolution of the pressure in the chamber for a time period of 35 ms. After a transition period, the signal is clearly periodic. The growth rate is still positive at the end of the simulation indicating that no limit cycle has been reached at that time. Spectral analysis indicates that the signal frequency is close to 250 Hz. Conditions chosen for the simulation correspond to experimental observations of an instability around 200 Hz. Calculations described in this section indicate that it is possible to simulate the dynamical behaviour of turbulent flames in a gas turbine combustor. Many other calculations can be found in the literature [23], [36].

5 Active control

Methods which can be used to suppress or attenuate combustion instabilities can be divided into two classes. In the first group, one finds passive control methods (PCM),which require a physical understanding of the phenomenon and involve subtle modifications of the combustion system. Active control methods (ACM) belong to the second class and are already proposed in early work on liquid rocket instabilities (Refs. [5-7]). In the 1950's, these authors derived the sensitive time lag (STL) model to analyze rocket motor instabilities. This theory could not provide a detailed explanation of the complicated physical processes involved but it helped design global strategies to deal with the problem. The stabilization method consisted in actively injecting perturbations into the combustor using an actuator in order to decouple the physical processes responsible for the combustion oscillations (see Figure 9).

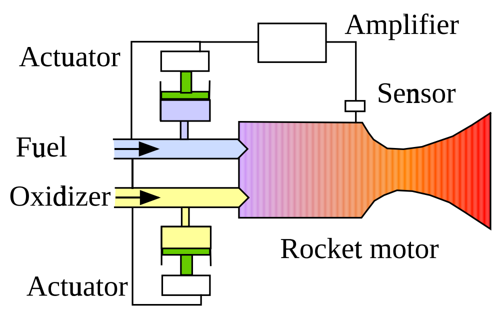

Figure 9: Servostabilization of liquid rocket combustion instability.

Although the principles of active control of instabilities were described in these early studies, the pratical demonstration of the concept was only achieved in the recent past (see McManus, *et al.* [37] for a review). Much of the development of active control strategies has relied on experiments on model scale combustors. These studies have revealed the potential of ACM and many technical limitations of the control system components (sensors, algorithms and actuator). These three features are briefly reviewed in the following. A particular care is given to the question of actuation, which remains critical.

Sensors

A great effort has been made concerning the sensors and many possibilities now exist (pressure transducers and microphones, diodes, photomultipliers, etc.). Their size and resistance to large temperature variations remain an issue, but technological responses have been recently given (see for example Docquier and Candel [38-39] for reviews). One important aspect of the problem is that of integration of the sensor in the system. It is always difficult to place a sensor in a real device because this induces many practical problems and augments the probability of failure. If the sensor requires an optical window, one has to deal with the problem of obscuration due to soot and other deposits. It is also important to think that the sensor will have to be maintained and the problem of access becomes an issue.

Algorithms

The choice of an algorithm determines to a great extent the effectiveness of the controller. One has to deal in the present situation with a highly nonlinear system. It is necessary to cope with rapid changes in the dynamical behavior of the process and be able to identify the system dynamics online. Also, the control scheme should be reliable and robust. Many recent efforts have been made on the theoretical side to use control concepts in generic situations. Low order modeling is exploited in most of these studies (Fung, *et al.* [40], Yang and Fung [41], Yang, *et al.* [42], Koshigoe, *et al.* [43], Candel, *et al.* [2], Annaswamy, *et al.* [44], Annaswamy, *et al.* [45], Hathout, *et al.* [46], Hathout, *et al.* [47], Evesque and Dowling [48], Hathout, *et al.* [49]). Block diagrams are used extensively to formalize the control problem and allow theoretical analysis. A few examples are gathered in Fig. 10. One generally distinguishes the combustion process and the acoustical coupling as in Fig. 10 (a) and (b). One may also consider that the combustor constitutes a plant to be controlled but without a detailed description of its behavior (Fig. 10 c). The block diagram representation of the control loop is less straightforward as may seem. In Fig. 10 (a) and (b) the controller acts on the acoustics of the system but the representation implies that the path between actuator and sensor is also the path which induces the coupled motion in the unstable operation of the system. This is not always the case. The actuator modulates in many circumstances a secondary fuel injection and it is the chemical conversion of the injected fuel which induces an acoustic wave. This wave then combines with the acoustic motion associated with the instability in the system (see Fig. 10 (d)). Many studies make an attempt to look at the problem from the point of view of control theory. This is exemplified in Ref. [50] which uses state space descriptions of combustor and controller in a robust control framework including many disturbances and uncertainties affecting the process.

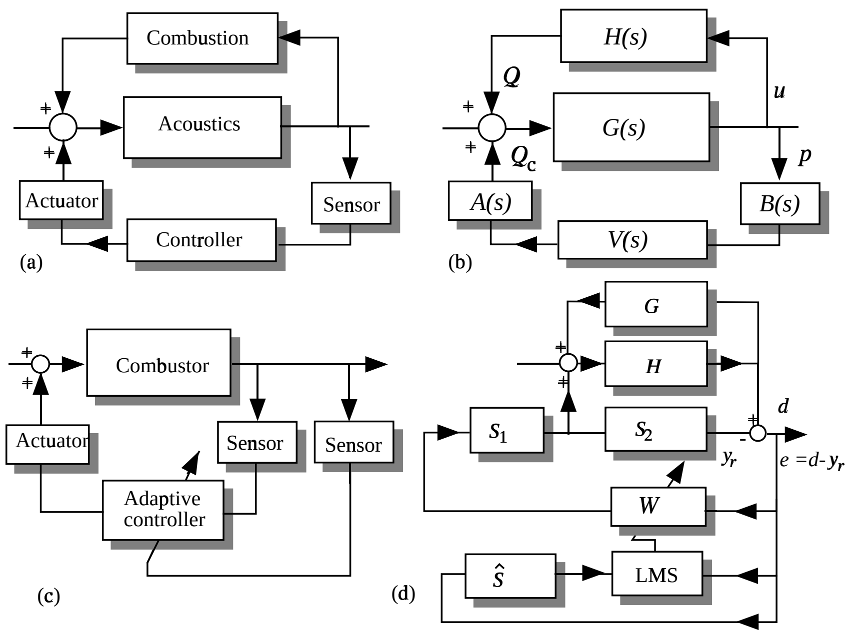

Figure 10: (a) Block diagram representation of combustor coupled to an external control loop, (b) Transfer function description of the system shown in (a), (c) General principle of adaptive instability control, (d) Block diagram representation of adaptive control of combustion H coupled by acoustic feedback G. W is the LMS filter, S_1, S_2 represent the actuator transfer functions and part of the secondary path between actuator and sensor $S = S_1 S_2$, $\hat{S}$ is a filter representing the secondary path S.

Adaptive techniques have also been extensively investigated with much of the recent work focusing on self tuning adaptive regulators most notably by the MIT group (Refs. [44-47]). In general, the theoretical analysis is limited by the assumptions made to describe the flame response and actuator influence on the combustion process.

Actuators

The actuator effectiveness constitutes the most critical issue, since one must face a complex trade-off between bandwidth and power. The design of the actuation system is a key element in the success of the identification and control of the burner. The actuator must be efficient and require a small amount of power, and the modifications in the flow should be significant.
The underlying idea is that if the technique is able to avoid any instability by strongly modifying the flow, the other parameters (stabilization, efficiency, pollutant emission) can be optimized in a second step by controlling the controller. Secondly, the actuator must be able to modulate the selected parameter (pressure, mass flow rate, etc.) up to a few hundred Hertz for gas turbine applications. One may also envision applications requiring a bandwidth of a few thousand Hertz to control higher order transverse modes but nothing has been demonstrated in this range. In the following, the focus is put on flow (liquid or gas) modulation, a technique, which could strongly act on the combustor behavior.

Liquid injection is difficult to modulate but piezoelectric actuators have been used for example in Ref. [51]. Developments of magnetostrictive devices are also being carried out most notably at Georgia Tech. Fuel injection timing is an interesting method of depositing energy at the right moment during an instability cycle (Refs. [52-56]). For gaseous fuel, it is possible to modulate the flow using direct drive valves (DDV) but the bandwidth is usually limited to about 400 Hz (Hermann, *et al.* [57] and Bernier, *et al.* [58-59]). Application of direct drive valves have been made on full scale gas turbine combustors (see Figure 11) and in model scale premixed prevaporized systems as shown in Figure 12.

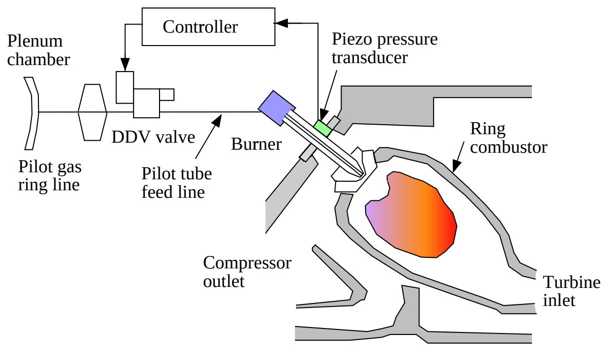

Figure 11: Active control of a gas turbine combustor.

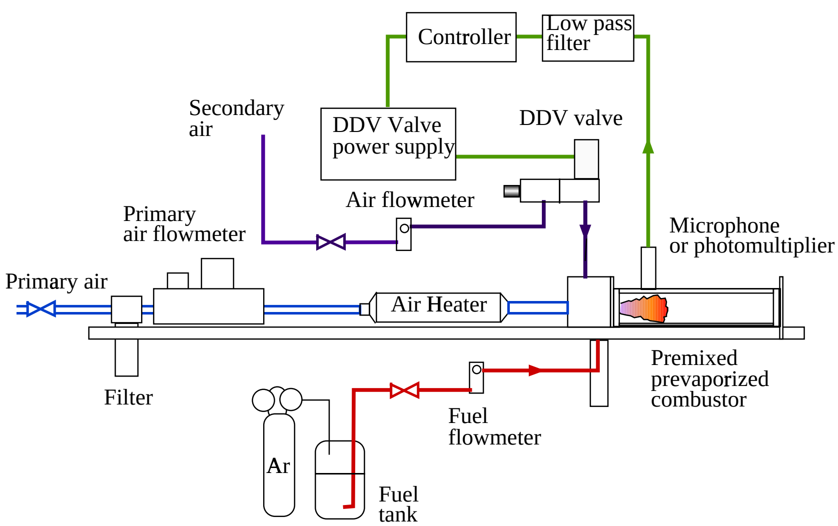

Figure 12: Active control of a lean premixed prevaporized (LPP) combustor.

This last example is used here to illustrate some of the difficulties which appear when the actuator operates on the gaseous flows. Experimental results were obtained on the laboratory scale LPP burner shown in Figure 12 operating at atmospheric pressure and at a power of 70kW. A fuel-air premixing injector designed as a model scale of a real gas turbine injector is mounted on a rectangular combustor [59]. Liquid heptane is injected through an atomizer in the preheated air flow coming out of two co-rotating swirler nozzles. The combustor is 0.5 m long and features a square section of 0.1×0.1 m^2.

Pressure oscillations in the chamber are measured with a microphone.

A port placed at 5.5 cm from the injector is connected with a waveguide. The water-cooled microphone is placed at 37cm from the port and the waveguide ends by a 25m long tube, which eliminates reflected waves. In a previous study [59], it was found that, strong instabilities appeared at a frequency f = 400Hz with an amplitude of oscillation of about p = 650 Pa. This unstable behavior is driven by large vortical structures shed from the injector exit plane. The coupling, involving the first longitudinal mode of the combustion chamber, is acoustic (quarter wave mode, around 400Hz).

The actuator system designed for this experiment comprises a high frequency valve modulating a secondary source of air. The modulated air flow is injected through four radial high speed jets in the vicinity of the atomization zone and impacts the spray at its basis.
Preliminary studies indicated that the secondary jets profoundly modified the spray characteristics like the size distribution and the droplets ballistics. The mean secondary air flow corresponds to 4% of the total mass flow rate in the burner. Modulation of this flow is obtained through a MOOG D633 high bandwidth electromagnetic directly driven proportional valve.
It is first necessary to verify that this system can be used to inject perturbations up to typical instability frequencies (f = 400Hz). It is then important to determine the transfer function H_u of the DDV alone, where u is the DDV position transducer value. Figure 13 shows the transfer function between the DDV control and position sensor signals obtained by excitating the system with a filtered white noise signal (f ¡ 800 Hz). The upper subfigure shows the amplitude, the middle one gives the phase shift ϕ and the bottom subfigure displays the time delay $\tau = -\partial\phi/\partial\omega$ deduced from the phase. The DDV clearly behaves like a low pass filter. Between 380Hz and 450Hz, the amplitude of the response drops rapidly, but it still seems possible to operate in this range. For f = 450 Hz, the signal amplitude is 20 dB lower than in the low frequency region, showing that the actuator does not respond to high frequency modulations.
near
The DDV also induces a phase shift but the corresponding delay is not constant and depends on frequency (1 ms $< \tau_{DDV} <$3.5 ms). One arrives to the conclusion that the DDV acts like a low pass filter and that it introduces a time delay, which is of the order of the instability period at least in this case.
It is next possible to estimate the overall characteristics of the whole system. This is achieved by determining the transfer functions between the DDV command and microphone signals. The corresponding transfer functions are displayed in Fig. 14 In this figure the transfer function obtained through identification (ITF) appears as a solid line and is compared to the transfer function obtained from spectral estimates (DTF), plotted as a dashed line.
Amplitude and phase cease to be coherent above 500Hz, because the DDV is unable to modulate the secondary air flow beyond this limit. The transfer function amplitude decreases with increasing frequencies, showing that the whole system behaves like a low-pass filter with a cut-off frequency f = 450Hz. This seems to be essentially due to the DDV. The phase shift is almost linear with frequency, yielding a nearly constant time delay. The typical delay value is in a range 11 ms $< \tau_{ACT,MIC} <$ 12 ms. It is long compared to the instability period, and contributions to this delay should be carefully analyzed and optimized.
Delays from the various system components shown in Fig. 12: DDV (as already mentionned) but also actuator, injector, combustion chamber and microphone. An analysis of the different orders of magnitude proposed in Ref. [59] shows that the time lag associated with the sensor (microphone) and actuator are non negligible and could clearly be optimized.
This identification was successfully applied in Ref. [59] to active control of combustion instabilities using an adaptive algorithm developed in Mettenleiter*, et al.* [60-61]. The mean pressure oscillation amplitude was reduced by about 50% (from 650 Pa to 400 Pa, the noise level being 130 Pa). This modest reduction is mainly due to the fact that the actuator operates at a frequency, which is slightly greater than its cut-off frequency and as a consequence that its effectiveness is degraded. This underlines the difficulties associated with the bandwidth limitations of practical actuators. Moreover time delays introduced in the control path prevent the control algorithm from acting efficiently on the flow features.

6 Multi-dimensional simulation of active control

Active control strategies have been essentially developed on an empirical basis. Much of the early work relies on experimental testing of various control schemes. Low order modeling is also explored in many recent studies but the description of the combustion dynamics is usually not faithful. A limited number of studies is directed at the multidimensional simulation (MDS) of control. One objective is to see whether active control can be included in a dynamical simulation of the flow. Such simulations could then be used to test and improve control algorithms before any practical application (Menon [62],Mettenleiter*, et al.* [63]). Numerical simulation complements experimentation and it could be useful for the development of flow control methods. The aim in this section is to underline problems arising when one wishes to couple a flow solver with a control algorithm and to discuss results of a multidimensional simulation of active control. The application is based on research directed at the control of vortex driven instabilities found in solid segmented rocket motors (see Refs[64-66]). The controller employed in this simulation relies on an adaptive scheme (Refs. [60] and [67]). The conceptual principle is shown in Figure 15. The control system is used to reduce low frequency pressure oscillations in large scale segmented solid rocket motors (SSRM). These oscillations correspond to one of the first longitudinal acoustic modes of the motor. The

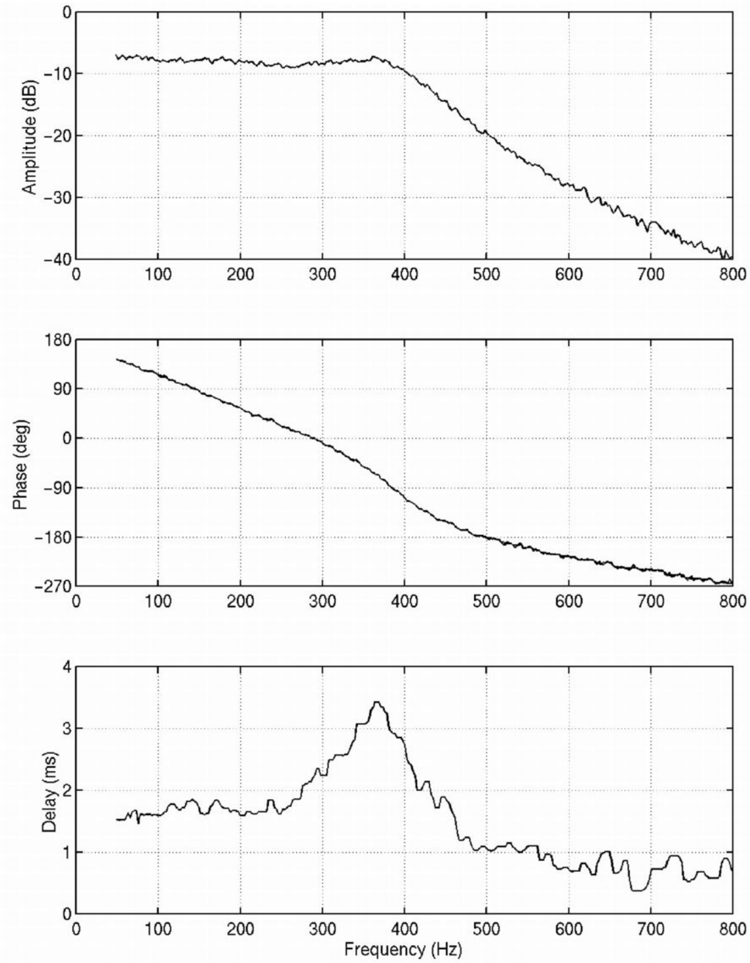

Figure 13: Transfer function between DDV command signal and sensor position when actuated by a filtered ($f <$ 800Hz) white noise. Upper curve: amplitude. Middle curve: phase shift. Bottom curve: delay (τ_{DDV}) (from Ref. [59].

resonant frequency is low because the longitudinal dimension is quite large. Mechanisms driving the process are linked with internal flow instabilities. Under strong resonant coupling large scale periodic vortices develop in the turning flow induced by propellant burning. Vortices may also be shed from the baffles partitionning the propellant segments or from the edge of the propellant grain. This last process is simulated in this study. A model scale geometry is considered in the calculations (the "C1-case"). Vortices shed from the propellant edge are convected downstream, impinge on the nozzle producing a pressure signal which feeds energy into one of the longitudinal acoustic modes. The controle scheme uses a sensor located near the shedding region. The sensor signal is used as input to a controller which drives an actuator placed in the motor head.

Similar arrangements are found in most combustion control investigations and the simulation developed in the special case of a solid rocket motor has a generic nature. The flow solver was used in many previous studies to analyze vortex instabilities in solid propellant propulsion. It serves as a platform for the present simulation.

One aspect which deserves special care is the definition of the "software actuator". One has to devise a numerical description of the physical actuator. From sytematic testing of various possible schemes it is found (Refs. [61] and [63]) that the actuator could be described as a spatial distribution of sources. The sources are modulated by the controller signal and inject a reactive fluid into the flow. This may be used to introduce perturbations in the flowfield. In fact, it is quite natural to use such a representation because the practical implementation in a real motor would involve a controlled injection of an evaporating and/or reacting substance inducing a distribution of

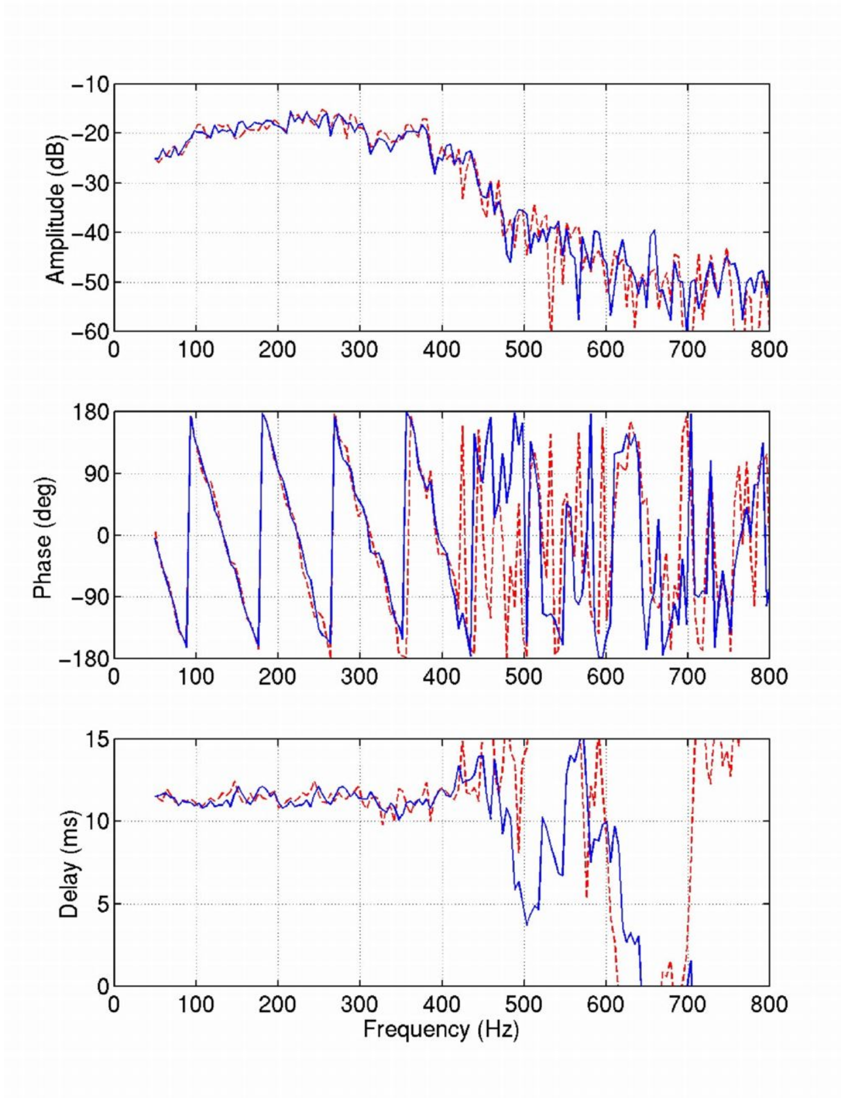

Figure 14: Transfer functions between DDV command signal and combustion chamber pressure when actuated by a filtered ($f < 800$Hz) white noise. Upper curve: amplitude. Middle curve: phase shift. Bottom curve: delay ($\tau_{ACT,MIC}$). Solid line: ITF. Dashed line: DTF Bernier, *et al.* [59].

sources of mass, momentum and energy. A series of open loop tests not shown here indicate that the numerical sources operate as expected.

Problems related to the coupling of the flow simulation module with the control algorithm must be treated next. There is a large mismatch between the time stepping Δt_{fs} of the flow solver and the sampling period of the controller Δt_c. Usually Δt_{fs} is quite small to obtain an accurate numerical flow and acoustic solution. On the other hand Δt_c is much larger because the numerical controller operates at frequencies which are a few times larger than the frequency of the process which is to be controlled. Then, $\Delta t_c >> \Delta t_{fs}$ and coupling clearly requires precautions. Detailed studies indicate that the input and output of the control routine should be low-pass filtered to prevent a growth of high frequency perturbations which after a short duration would alter the calculation. The simulation methodology is illustrated by a calculation of vortex driven instabilities in a small rocket motor and control of these instabilities using an adaptive algorithm.

In the present application, the controller input is a pressure signal provided by a sensor located near the nozzle while the actuator is placed near the motor head. A typical flow configuration is shown in Figure 16.
Shedding takes place from the solid propellant edge. Vortices interact with the nozzle producing a resonant pres-

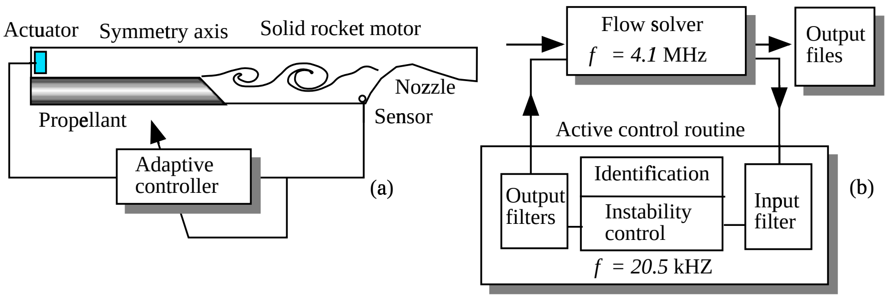

Figure 15: Active control of a solid rocket motor. (a) Principle, (b) Coupling between the flow solver Sierra and the adaptive controller RAC. (Adapted from Ref. [61-63]).

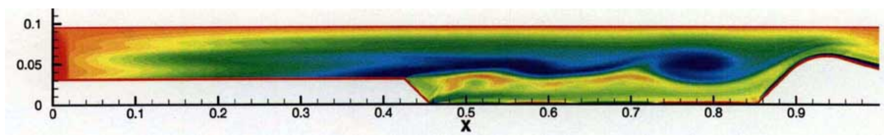

Figure 16: Simulated vorticity field inside the rocket motor. Controller is off. (Adapted from Refs. [61] and [63]).

sure field in the motor. When the controller is on, vortices are still shed from the propellant edge but their number at a given instant is larger, the frequency is shifted to a higher value and there is no resonance, the amplitude is lower and the level of pressure oscillations is notably reduced (Figure 17).

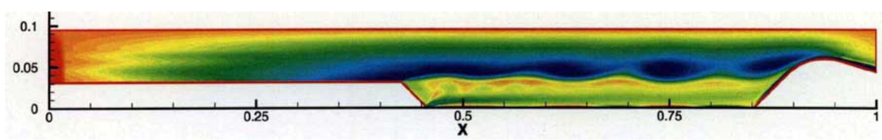

Figure 17: Simulated vorticity field inside the rocket motor. The controller is operating. It modifies the vorticity field and reduces the level of pressure oscillations inside the motor. (Adapted from Refs. [61-63]).

This calculation indicates that multidimensional simulations of active control may be used to examine control strategies and modifications of the flow under control.

7 Conclusions and perspectives

This article has focused on active control of combustion dynamics. This topics is technologically important and it also raises many scientific problems. Fundamental aspects of combustion dynamics have been extensively investigated and much has been learned from well controlled laboratory experiments like those described in this article. This knowledge has allowed progress in the numerical modeling of combustion dynamics based on large eddy simulation (LES) but the numerical prediction of combustion instability remains a challenging problem.

One objective for research in this area will be to advance subgrid models for combustion LES. This could be achieved by combining modeling, simulation and experimentation oriented towards LES needs. Progress in the treatment of boundary conditions is essential if one wishes to deal with the coupling with acoustics which governs the dynamical combustion phenomena. Integration of computational strategies describing the flow and the wave motion is needed to develop complete models for combustion instability. Studies of upstream and downstream component dynamics are also needed to obtain a complete description of the combustor response when it is placed in its environment.

Active control of combustion instability has also progressed in many ways. While initial demonstrations were carried out on small-scale systems, control has also been implemented in full-scale gas turbines. Much of the current research effort has focused on the system analysis using modern control theory. Simplified instability models have been devised to this purpose and control strategies have been explored. Control analysis has been pursued on the basis of simple low order essentially linear instabilty models. This research has been mostly theoretical with limited demonstrations on practical devices. One challenge for the future will be to apply these theoretical strategies (state feedback, robust control, model based adaptive control ...) to real systems. This can only be envisaged through the improvement of the actuator efficiency.

The multi-dimensional simulation (MDS) of active control is a nascent but important area of research. The objective is to allow complete (and realistic) software studies of control concepts. MDS of active control combines the difficulties of computational fluid dynamics with those of coupling a flow solver with a control scheme. Early developments of such a capability were limited but recent efforts have uncovered and resolved some of the difficulties. It has been found that a validated software description of the actuator is essential and that coupling of the flow solver and controller needs to be treated with care. The mismatch between sampling frequencies of the Navier-Stokes flow solver and controller requires suitable filtering of the input and output of the controller. Further progress in MDS of AC may be foreseen with advances in combustion dynamics simulation tools and active control methodologies. Advances in active control simulation are important but not sufficient. Further progress is needed on the technological level to deal with the current limitations of sensors and actuators and to resolve the difficult question of integration in practical devices.

Acknowledgments

Part of this work was supported by DGA, CNES and the European Brite Euram project Aciacoc. The kind invitation extended by Professor Koichi Hayashi is gratefully acknowledged.

References

1. Candel, S., 24th Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, 1992, pp. 1277-1296.

2. Candel, S., Huynh Huu, C. and Poinsot, T., Some modeling methods of combustion instabilities in *Unsteady Combustion* (F. E. C. Culick, M. V. Heitor and J. H. Whitelaw, Eds.), Kluwer, Dordrecht, 1996, pp. 83-112.

3. Culick, F. E. C., Combustion instabilities in propulsion systems in *Unsteady combustion* (F. E. C. Culick, M. V. Heitor and J. H. Whitelaw, Eds.), Kluwer, Dordrecht, 1996, pp. 173-241.

4. Candel, S., Proceedings of the Combustion Institute 29: 1-28 (2002).

5. Crocco, L. and Cheng, S. L., *Theory of combustion instability in liquid propellant rocket motors.* Butterworths Science Publication, London, 1956.

6. Tsien, H. S., Journal of the American Rocket Society 22: 256-263 (1952).

7. Marble, F. E. and Cox, D. W., jr, Journal of the American Rocket Society: 63-81 (1953).

8. Ducruix, S., Schuller, T., Durox, D. and Candel, S., Jounal of Propulsion and Power 19: 722-734 (2003).

9. Ducruix, S., Durox, D. and Candel, S., Proceedings of the Combustion Institute 28: 765-773 (2000).

10. Schuller, T., Durox, D. and Candel, S., Combustion and Flame 128: 88-110 (2001).

11. Bragg, S. L., Journal of the Institute of Fuel 36: 12-16 (1963).

12. Abugov, D. I. and Obrezkov, O. I., Combustion, Explosions and Shock Waves 14: 606-612 (1978).

13. Clavin, P. and Siggia, E. D., Combustion Science and Technology 78: 147-155 (1991).

14. Price, R. B., Hurle, I. R. and Sudgen, T. M., Twelfth Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, 1969, pp. 1093-1102.

15. Durox, D., Schuller, T. and Candel, S., Submitted to Proceedings of the Combustion Institute 29: 69-76 (2002).

16. Schuller, T., Durox, D. and Candel, S., Combustion and Flame 135: 525-538 (2003).

17. Zinn, B. T. and Lores, E. M., Combustion Science and Technology 4: 269-278 (1972).

18. Menon, S. and Jou, W., Combustion Science and Technology 75: 53-72 (1991).

19. Piana, J., Veynante, D., Candel, S. and Poinsot, T., Direct numerical simulation analysis of the G-equation in premixed combustion in *Proceedings of the Second ERCOFTAC workshop on direct and large-eddy simulation* Grenoble, 1996).

20. Piana, J., Ducros, F. and Veynante, D., Large eddy simulations of turbulent premixed flames based on the G-equation and a flame wrinkling description in *Proceedings of the 11th Symposium on Turbulent Shear Flows* (F.

Durst, N. Kasagi, B. E. Launder, F. W. Schmidt and K. Suzuki, Eds.), Springer-Verlag, Berlin, 1997, pp. 21.13-21.18.

21. Veynante, D. and Poinsot, T., Reynolds average and large eddy simulation modeling for turbulent combustion in *New Tools in Turbulence Modelling* (O. Metais and J. Ferziger, Eds.), Springer Verlag, New York, 1997, pp. 105-140.

22. Thibaut, D. and Candel, S., Combustion and Flame 113: 51-65 (1998).

23. Angelberger, C., Veynante, D. and Egolfopoulos, F., Flow, Turbulence and Combustion 65: 205-222 (2001).

24. Weller, H. G., Tabor, G., Gosman, A. D. and Fureby, C., Proceedings of the Combustion Institute 27: 899-907 (1998).

25. Colin, O., Ducros, F., Veynante, D. and Poinsot, T., Physics of Fluids 12: 1-21 (2000).

26. Hawkes, E. R. and Cant, R. S., Proceedings of the Combustion Institute 28: 51-58 (2000).

27. Kerstein, A. R., Ashurst, W. T. and Williams, F. A., Physical Review A37: 2728-2731 (1988).

28. Butler, T. D. and O'Rourke, P. J., Sixteenth, The Combustion Institute, Pittsburgh, 1976, pp. 1503-1515.

29. O'Rourke, P. J. and Bracco, F. V., Journal of Computational Physics 33: 185-203 (1979).

30. Nottin, C., Knikker, R., Boger, M. and Veynante, D., Proceedings of the Combustion Institute 28: 67-73 (2000).

31. Poinsot, T. J. and Lele, S. K., Journal of Computational Physics 101: 104-129 (1992).

32. Ducruix, S., Poinsot, T. and Candel, S., Large eddy simulations of combustion instabilities in a swirled combustor in *Proceedings of the IUTAM Colloquium on Turbulent Mixing and Combustion* Kingston, Canada, 2001).

33. Kaufmann, A., Nicoud, F. and Poinsot, T., Combustion and Flame 131: 371-385 (2002).

34. Ducruix, S. and Candel, S., Accepted, AIAA Journal: (2004).

35. Schonfeld, T., Angelberger, C., Legier, J. P. and Ducruix, S., *Numerical simulation of compressible reactive flows on unstructured grids,* 99-, (1999).

36. Chakravarthy, V. K. and Menon, S., Flow, turbulence and combustion 65: 133-161 (2000).

37. McManus, K. R., Poinsot, T. and Candel, S., Progress in Energy and Combustion Science 19: 1-29 (1993).

38. Docquier, N. and Candel, S., Sensor requirements for combustion control in *Applied Combustion Diagnostics* (K. Kohse Hoinghaus and J. B. Jeffries, Eds.), Taylor and Francis, New York, 2002, pp.

39. Docquier, N. and Candel, S., Progress in Energy and Combustion Science 28: 107-150 (2002).

40. Fung, Y. T., Yang, V. and Sinha, A., Combustion Science and Technology 78: 217-245 (1991).

41. Yang, V. and Fung, Y. T., Journal of Propulsion and Power 8: 1282-1289 (1992).

42. Yang, V., Sinha, A. and Fung, Y. T., Journal of Propulsion and Power 8: 66-73 (1992).

43. Koshigoe, S., Komatsuzaki, T. and Yang, V., Journal of Propulsion and Power 15: 383-389 (1999).

44. Annaswamy, A. M., Fleifil, M., Hathout, J. P. and Ghoniem, A. F., Combustion Science anf Technology 128: 131-180 (1997).

45. Annaswamy, A. M., El Rifai, O. M., Fleifil, M., Hathout, J. P. and Ghoniem, A. F., Combustion Science and Technology 135: 213-240 (1998).

46. Hathout, J. P., Annaswamy, A. M., Fleifil, M. and Ghoniem, A. F., Combustion Science and Technology 132: 99-138 (1998).

47. Hathout, J. P., Fleifil, M., Annaswamy, A. M. and Ghoniem, A. F., Proceedings of the Combustion Institute 28: 721-730 (2000).

48. Evesque, S. and Dowling, A. P., Combustion Science and Technology 164: 65-93 (2001).

49. Hathout, J. P., Fleifil, M., Annaswamy, A. M. and Ghoniem, A. F., Jounal of Propulsion and Power 18: 390-399 (2002).

50. Boe-Shong Hong, Yang, V. and Ray, A., Combustion and Flame 1999: 91-106 (1999).

51. Hermann, J., Gleis, S. and Vortmeyer, D., Combustion Science and Technology 118: 1-25 (1996).

52. Yu, K. H., Wilson, K. J. and Schadow, K. C., Proceedings of the Combustion Institute 26: 2843-2850 (1996).

53. Haile, E., Lacas, F. and Candel, S., Comptes Rendus de l'Academie des Sciences, Serie II b 325: 203-209 (1997).

54. Yu, K. H., Wilson, K. J. and Schadow, K. C., 27th Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, 1998, pp. 2039-2046.

55. Yu, K. H., Active control of engine dynamics : fundamentals and fluid dynamics in *Proceedings of the RTO/VKI Special Course Active control of engine dynamics* (VKI, Rhode Saint Genese, Belgium, 2001).

56. Yu, K. H. and Wilson, K. J., Journal of Propulsion and Power 18: 53-60 (2002).

57. Hermann, J., Orthmann, A., Hoffmann, S. and Berenbrink, P., Combination of active instability control and passive measures to prevent combustion instabilities in a 260 MW heavy duty gas turbine in *Proceedings of the NATO/RTO Symposium on Active Control Technology* (RTO, Braunschweig, Germany, 2000).

58. Bernier, D., Ducruix, S., Lacas, F., Candel, S., Robart, N. and Poinsot, T., Identification of the transfer function in a model gas turbine combustor : application to active control of combustion instabilities in *Proceedings of the 18th ICDERS* Seattle, 2001).

59. Bernier, D., Ducruix, S., Lacas, F., Candel, S., Robart, N. and Poinsot, T., Combustion Science and Technology 175: 993-1013 (2003).

60. Mettenleiter, M., Haile, E. and Candel, S., Journal of Sound and Vibration 230: 761-789 (2000).

61. Mettenleiter, M., Vuillot, F. and Candel, S., AIAA Journal 40: (2002).

62. Menon, S., Combustion Science and Technology 84: 51-79 (1992).

63. Mettenleiter, M., Vuillot, F. and Candel, S., Numerical simulation of active control in unstable solid rocket motors. in *Proceedings of the NATO/RTO Symposium on Active Control Technology* (RTO, Braunschweig, Germany, 2000).

64. Blomshield, F. and Mathes, H. B., Journal of Propulsion and Power 9: 217-221 (1993).

65. Vuillot, F., Journal of Propulsion and Power 1995: 626-639 (1995).

66. Dotson, K. W., Koshigoe, S. and Pace, K. K., Journal of Propulsion and Power 13: 197-206 (1997).

67. Mettenleiter, M. and Candel, S., Developments of adaptive methods for active instability control in *Proceedings of the NATO/RTO Symposium on Active Control Technology* Braunschweig, Germany, 2000).