## The Reynolds Number Effect on the Microbubble Drag Reduction

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Numerical simulations of turbulent microbubble flows in a channel are carried out in order to make clear the drag reduction mechanism. The Reynolds number effect is examined by paying attention to the local flow modulation. Initially, the relation between the friction coefficient and the energy dissipation is theoretically derived. Based on the theory, the effect of the local velocity gradient around bubbles is investigated in the force coupling simulation. In order to treat the boundary condition rigorously, not only the force monopole but also the force dipole which is related to the shear are taken into account. Transient microbubble simulations are performed at the friction Reynolds number  $Re_{\tau}$  of 150. Similarly to the results reported by Xu *et al.*(2002), the drag reduction occurs when neglecting the force dipole term. On the other hand, when considering the force dipole term, the skin friction increases due to the increase of the energy dissipation near the bubbles. The direct numerical simulation with rigid bubbles is also performed at the same Reynolds number and the skin friction increases for the same reason. The front-tracking simulations with deformable bubbles are also carried out. The skin friction increases at the lower Reynolds number of  $Re_{\tau} = 180$ , while it decreases at the higher Reynolds number of  $Re_{\tau} \sim 1 \times 10^3$ which is same magnitude as that in typical experiments. The physical mechanism to reduce the energy dissipation is discussed.

## **1** Introduction

About 80% of the total propulsion resistance of a ship like a tanker is due to the skin friction with the surrounding water. It will be a great contribution to the environment by reducing the frictional drag. Among the several devices proposed for reducing the skin friction, the microbubble injection method is considered the most suitable for ships because of the high efficiency and no environmental contamination.

Over the last three decades, a lot of experiments have been performed on the microbubble drag reduction. McCormick and Bhattachryya (1973) found that the skin friction on the submerged body is reduced by injecting the microbubbles produced by the electrolysis. Madavan et al. (1984) reported the efficiency of the microbubble drag reduction reaches as much as 80%. Although the mechanism of the microbubble drag reduction is important, it has not been fully understood yet because the microbubbles obstruct detailed measurements. Using the numerical simulations of the microbubble flow has been expected to give basic advantage of investigating the drag reduction. However, it is not easy to reproduce the drag reduction by numerical simulations. Until 2002, no one had succeeded to simulate the microbubble drag reduction. We have simulated the microbubble flow by the Direct Numerical Simulation (DNS) since 2000 when the research project of "Smart Control of Turbulence" started (see Kawamura and Kodama, 2001 & 2002; Sugiyama et al., 2002). We employed the Front-Tracking (F-T) formulation generalized by Unverdi and Tryggvason (1992) in order to resolve not only the turbulence vortices but also the local flow in the vicinity of the bubble surface. So far, the fully developed situation has been assumed and the friction Reynolds  $Re_{\tau}$  has been set to  $O(10^2)$ , which is one or two orders of the magnitude smaller compared with typical experiments. The results unsuccessfully showed that the skin friction increased with increasing void fraction. Recently, Xu et al. (2002) made the first success to simulate the microbubble drag reduction by the Force Coupling Method (FCM). In their simulation, bubbles are initially concentrated near the wall and the drag reduction transiently occurs when the bubbles disperse by turbulence. We also investigated the transient effects of the bubble concentration on the skin friction in the Eulerian-Lagrangian (E-L) method (see Sugiyama et al., 2003; Kodama et al., 2003). We analyzed the profile of the momentum balance and concluded that the spatial development of the bubble concentration, which generates wall-normal velocity, might be important for the drag reduction. Elghobashi and Ferrante (2003) simulated the spatially-developing turbulent boundary layer. They also employed the E-L method and discussed the physical mechanisms of the drag reduction under different orientations of the gravitational acceleration.

In the present study, the objective is numerically to make clear the drag reduction mechanism by investigating the local interaction between liquid and gas phases. Although the local flow modulation around the bubbles has been enabled to be captured due to the advances in numerical and experimental techniques, it has not been clear what variable should be payed attention to yet. In the present study, the dissipation theory (Batchelor,1967; Levich, 1949) is extended to the microbubble channel flow in order to see the the local flow modulation affecting the skin friction. Then, the effects of the Reynolds number and the treatment of the local boundary condition are examined based on the dissipation theory. Similarly to the computation reported by Xu *et al.*(2002), The transient microbubble flows are simulated. Three types of the simulation methods are employed: (1) the FCM with rigid bubbles, (2) the DNS method with rigid bubbles and (3) the F-T method with deformable bubbles. Especially, the Reynolds number in the F-T simulation is set to  $Re_{\tau} \sim 1000$ , which is comparable to that of the experiment by Moriguchi and Kato (2002).

# 2 Effect of energy dissipation on skin friction

According to Einstein's theory shown by Batchelor (1967), the effective viscosity around a particle or bubble is given by a volume integral of energy dissipation. The effective viscosity theory was originally based on the Stokesian dynamics with no inertia. This theory was extended to moderate Reynolds number flows by Ryskin (1980). The drag force on a rising bubble is also given by the volume integral of the energy dissipation. This relation has been employed for potential flows (Levich, 1949), boundary layer flows (Moore,1963) and moderate Reynolds number flows (Ryskin and Leal, 1984). These dissipation theories are useful to see the effect of the local modulation of the velocity field around bubbles on the effective viscosity or the drag force. In this section, the dissipation theory is extended to the microbubble flow and the relation between the energy dissipation and the skin friction is derived.



Figure 1: Schematic figure of analysis on relation between skin friction and energy dissipation

### 2.1 Basic equations

The schematic figure of the present analysis is shown in Fig.1. Streamwise, wall-normal, and spanwise directions are denoted by x, y and z, respectively. The basic equations are formulated based on the three assumptions: (1) Both gas and liquid are incompressible, (2) x and z directions are periodic and (3) The driving pressure gradient is constant. The conservation equations of mass and momentum are expressed as,  $\circ$ Mass conservation equation:

$$\nabla \cdot \mathbf{u} = 0,\tag{1}$$

 $\circ$ Momentum conservation equation:

$$\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot \mathbf{T} + \rho \mathbf{g} - \sigma \kappa \mathbf{n} \delta(\phi).$$
<sup>(2)</sup>

where **u** is the velocity vector,  $\rho$  the density, D/Dt the substantial time derivative, **T** the stress tensor, **g** the gravity vector,  $\sigma$  the surface tension coefficient,  $\kappa$  the curvature on the bubble surface and **n** the unit normal vector on bubble surface. The positive direction of **n** is from bubble inside to its outside. D/Dt and **T** are expressed as,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{u} \cdot \nabla), \quad \mathbf{T} = -p\mathbf{I} + 2\mu\mathbf{S}, \tag{3}$$

where p is the pressure, **I** the unit dyadic,  $\mu$  the viscosity, **S** the strain rate tensor,  $\delta$  the Dirac delta function and  $\phi$  the length from bubble surface.  $\phi$  has negative and positive values inside and outside the bubble, respectively. **S** is expressed as,

$$\mathbf{S} = \frac{1}{2} \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right). \tag{4}$$

p is decomposed into,

$$p = \tilde{p} - \left(-\frac{dP}{dx}\right)x,\tag{5}$$

where the second term in the RHS is related to the driving pressure gradient (-dP/dx), which is spatially constant. The periodic boundary condition is imposed upon the first term  $\tilde{p}$  in the x and z directions.

#### 2.2 Relation between friction coefficient and energy dissipation

Indicator functions  $I_G$  and  $I_L$  are introduced. If a point **x** belongs to liquid phase,  $I_G(\mathbf{x})$  and  $I_L(\mathbf{x})$  equal 0 and 1, respectively. Phase weighted averages  $(\langle \cdot \rangle_L \text{ and } \langle \cdot \rangle_G)$  are defined as,

$$\langle \phi \rangle_{L,G} = \frac{1}{L_x L_y L_z} \int_0^{L_x} dx \int_0^{L_y} dy \int_0^{L_z} dz \ \phi(\mathbf{x}) I_{L,G}(\mathbf{x}).$$
(6)

A friction coefficient  $C_f$ , which is a dimensionless variable, is introduced. The dynamic pressure is scaled by a liquid density  $\rho_L$  and a mean velocity of the mixture fluid  $(\langle u \rangle_L + \langle u \rangle_G)$ .  $C_f$  is defined as,

$$C_f^{Top} + C_f^{Bot.} = \frac{\tau_w^{Top} + \tau_w^{Bot.}}{\frac{1}{2}\rho_L(\langle u \rangle_L + \langle u \rangle_G)^2}.$$
(7)

Considering the volume integals of  $\nabla \cdot (\mathbf{e}_x \cdot \mathbf{T})$  and  $\nabla \cdot (\mathbf{u} \cdot \mathbf{T})$  over whole domain, a relation among  $C_f$ , energy dissipation, turbulent kinetic energy, buoyancy and surface energy is given by,

$$C_{f}^{Top} + C_{f}^{Bot.} = \underbrace{\frac{4L_{y}(\nu_{L} \langle \mathbf{S} : \mathbf{S} \rangle_{L} + (\mu_{G}/\rho_{L}) \langle \mathbf{S} : \mathbf{S} \rangle_{G})}{(\langle u \rangle_{G} + \langle u \rangle_{L})^{3}}}_{\text{Dissipation}} + \underbrace{\frac{(\rho_{G}/\rho_{L})L_{y}}{(\langle u \rangle_{G} + \langle u \rangle_{L})^{2}} \begin{bmatrix} \frac{1}{\langle u \rangle_{G} + \langle u \rangle_{L}} \frac{d \langle \mathbf{u} \cdot \mathbf{u} \rangle_{G}}{dt} - 2\frac{d \langle u \rangle_{G}}{dt} \end{bmatrix}}_{\text{Gas turbulent kinetic energy}} + \underbrace{\frac{L_{y}}{(\langle u \rangle_{G} + \langle u \rangle_{L})^{2}} \begin{bmatrix} \frac{1}{\langle u \rangle_{G} + \langle u \rangle_{L}} \frac{d \langle \mathbf{u} \cdot \mathbf{u} \rangle_{L}}{dt} - 2\frac{d \langle u \rangle_{L}}{dt} \end{bmatrix}}_{\text{Liquid turbulent kinetic energy}} - \underbrace{\frac{2L_{y}}{(\langle u \rangle_{G} + \langle u \rangle_{L})^{2}} \begin{bmatrix} \frac{\langle \mathbf{u} \rangle_{L} \cdot \mathbf{g}}{\langle u \rangle_{G} + \langle u \rangle_{L}} - \alpha_{L}g_{x} \end{bmatrix}}_{\text{Liquid buoyancy}} + \underbrace{\frac{2\sigma}{\rho_{L}L_{x}L_{z}(\langle u \rangle_{G} + \langle u \rangle_{L})^{3}} \sum_{l}^{N_{b}} \frac{dS_{Bub^{(l)}}}{dt}, \\ \text{Surface energy}$$
(8)

where  $\alpha_G$ ,  $\alpha_L$ ,  $g_x$  and  $S_{Bub.(l)}$  correspond to volume fractions of gas and liquid phases, a streamwise component of the gravity and a surface area of the *l*th bubble, respectively.  $\alpha_G$ ,  $\alpha_L$  and  $g_x$  are given by,

$$\alpha_G = \langle 1 \rangle_G, \quad \alpha_L = \langle 1 \rangle_L, \quad g_x = \mathbf{g} \cdot \mathbf{e}_x. \tag{9}$$

Although Eq.8 is derived for the flow with deformable bubbles, it is also applicable to the flow with rigid bubbles by dropping the surface energy term. In the statistically steady situation, the second, third and sixth terms of the RHS are negligible. These terms will temporally decrease if the transient effect, e.g. the large scale density change due to the bubble dispersion, is important for the drag reduction as obtained by Xu *et al.*(2002) and Sugiyama *et al.*(2003).

In case of the bubbly flow, the contribution of the gas phase in the dissipation term is negligible and the volume integral of  $\mathbf{S} : \mathbf{S}$  is taken only over the liquid phase. If the velocity distribution does not change by introducing the bubbles, the dissipation term becomes smaller due to the decrease of the liquid volume. In the laminar flow based on the Einstein's theory, however, the magnitude of the strain rate usually becomes larger near the bubble surface and the skin friction increases. Similarly to the dissipation theory for the effective viscosity (Batchelor, 1967) and the drag force (Levich, 1949), Eq. 8 is independent of the pressure. Thus, Eq.8 is considered useful not only for the numerical simulation but also for the experiment when analyzing the local flow modulation on the skin friction. It is because we can avoid measuring the multi-dimensional distribution of the pressure field which is much more difficult to capture than that of the velocity field.

## **3** Force coupling simulation

Xu et al.(2002) employed the Force Coupling Method (FCM) when they numerically obtained the microbubble drag reduction. They assume the bubble is spherical and rigid. In the FCM, the no-slip boundary

condition is not locally satisfied. Instead of the local satisfaction of the boundary condition, the body force from the bubble to the liquid is introduced based on the Stokesian dynamics. The body force is given by the volume integral of the stress field and able to be decomposed into several modes, e.g. a force monopole, a force dipole and so on. The schematic figure to explain the force monopole and the force dipole is shown in Fig.2. The force monopole is related to the translational motion of the bubble and equivalent to the reaction force against the drag force on the bubble. Xu *et al.*(2002) considered the force monopole in their simulation. The force dipole is related to the velocity gradient around the bubble. According to the Einstein's theory shown by Batchelor (1967), the force dipole contributes to the increase of the skin friction due to the increase of the energy dissipation. Although the method to include the force dipole in the FC computation was generalized by Lomholt *et al.*(2002), Xu *et al.*(2002) neglected it.

As described in §2, the skin friction can be expressed by the energy dissipation. Based on the Einstein's theory, the force dipole is considered important for the modulation of the energy dissipation due to the presence of the bubbles. In this section, the effect of the energy dissipation on the skin friction is discussed by performing the FC simulation of the microbubble flow. The present simulation condition is similar to that reported by Xu *et al.*(2002) who succeeded in the drag reduction computation. In the present simulation, not only the force monopole but also the force dipole are considered.



Figure 2: Schematic figure of force monopole and force dipole

#### 3.1 Simulation method

We assume the bubble is spherical and rigid. The basic equations are almost same as those shown by Maxey and Patel(2001). Governing equations for the bubble motion are based on those given by Lomholt *et al.*(2002) and Xu *et al.*(2002). Governing equations for the liquid consist of the mass conservation equation given by Eq.1 and the momentum one expressed as,

$$\rho_L \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu_L \nabla^2 \mathbf{u} + \mathbf{F}_p, \qquad (10)$$

where  $\mathbf{F}_p$  represents the body force due to the bubble motion. Considering the force monopole and the force dipole,  $\mathbf{F}_p$  is expressed as,

$$F_{pi} = \sum_{l=1}^{N_b} \left( \underbrace{F_i^{(l)} \Delta(\mathbf{x} - \mathbf{Y}^{(l)}, \sigma_m)}_{\text{Force monopole}} + \underbrace{F_{ij}^{(l)} \frac{\partial}{\partial x_j} \Delta(\mathbf{x} - \mathbf{Y}^{(l)}, \sigma_d)}_{\text{Force dipole}} \right), \tag{11}$$

where  $\mathbf{Y}^{(l)}$  is the centroid position of the *l*th bubble.  $\Delta$  is the Gaussian function and  $\sigma_m$  and  $\sigma_d$  are envelope scales for the force monopole and the force dipole, respectively, as shown by Lomholt *et al.*(2002). In the present study, the calculation method for  $F_i^{(l)}$  is based on that shown by Xu *et al.*(2002) and that for the antisymmetric part of  $F_{ij}^{(l)}$  is based on that shown by Lomholt *et al.*(2002).

The symmetric part of  $F_{ij}^{(l)}$  is expressed by the averaged strain rate around the bubble. It is related to the Einstein's effective viscosity. Based on the steady creeping flow, the symmetric part of  $F_{ij}^{(l)}$  is approximated by the surface integral in the following way,

$$\frac{1}{2} \left( F_{ij}^{(l)} + F_{ji}^{(l)} \right) = \frac{5\mu_L a^{(l)}}{3} \oint_{|\mathbf{X} - \mathbf{Y}^{(l)}| = a} d^2 \mathbf{x} S_{ij}.$$
(12)

We checked the Couette flow with a spherical particle by use of Eq.12 and confirmed that the relation between the skin friction and the particle volume fraction showed good agreement with the Einstein's theory.

#### 3.2 Simulation condition

The effect of non-uniform distribution of the initial bubble position on the skin friction is investigated under a similar condition to the computation of the periodic flow with no buoyancy reported by Xu *et al.* (2002). The effect of the local velocity gradient around bubbles on the skin friction is discussed.

Before introducing the bubbles, a fully developed single-phase turbulent channel flow at the friction Reynolds number  $Re_{\tau}$  of 150 is computed. The size of the simulation domain is set to  $L_x \times L_y \times L_z =$  $2\pi h \times 2h \times \pi h$  divided by  $N_x \times N_y \times N_z = 64 \times 64 \times 64$  grid points, in the streamwise (x), wall-normal (y) and spanwise (z) directions, respectively. The periodic boundary condition is imposed in x and z directions. The fourth-order finite difference method is employed to solve the partial differential equations. The discretization of variables is taken on the staggered grid. The time integral of the flow is evaluated in the second-order Adams-Bashforth method. The interpolation from the liquid phase to the particle surface is approximated in the third-order Lagrangian method. In the single phase simulation, the profiles of the mean velocity and the turbulent intensities obtained by the present program code showed good agreement with the DNS result obtained by Kasagi et al. (1992) (see Sugiyama et al., 2002). The bubble radius a and the bubble number  $N_b$ are set to 0.25h and 30, respectively. The mean void fraction  $\alpha_0$  corresponds to 5%. The density ratio is set to  $\rho_G/\rho_L = 0.25$  by considering the numerical stability of the simulation in §4. The initial bubble location is the layer near the top wall. The length between the centroid of the initial bubble position and the top wall is set to 0.28. The x and z position of the initial bubble location is randomly placed. Simulation conditions are listed in Table 1. The simulation method in case 1 is same as that performed by Xu et al. (2002). In case 2, not only the force monopole but also the force dipole (Eq.11) are introduced. The boundary condition on the bubble surface in case 2 is considered more rigorously treated than that in case 1.

Table 1: Conditions of the channel with rigid bubbles

Case	$\alpha_0(\%)$	Simulation method
0	0	-
1	5	FCM without force dipole (Eq.11, $\S3$ )
2	5	FCM with force dipole $(Eq.11, \S3)$
3	5	DNS $(\S4)$

#### 3.3 Results and discussion

The mean void fraction profile is shown in Fig.3 for different times. In both cases 1 and 2, the void fraction peak decreases and the profile becomes uniform as the bubbles disperse. As reported by Xu *et al.*(2002), the drag reduction transiently occurred when the bubbles dispersed due to the turbulence in the FC simulation. Authors also recognized the transient drag reduction due to the larger scale density change in the E-L simulation (see Sugiyama *et al.*,2003 and Kodama *et al.*,2003).

The temporal evolution of the relative friction coefficient  $C_f/C_{f0}$  on the top wall is shown in Fig.4.  $C_{f0}$  is  $C_f$  with no bubble (case 0). As shown in Fig.4,  $C_f$  for case 1, corresponding to the result neglecting the force dipole, decreases in time. This result is consistent with that reported by Xu *et al.*(2002) who also neglected the force dipole. On the other hand,  $C_f$  for case 2 increases in time. Such a drag increase results from introducing the force dipole.

The typical instantaneous distribution of the skin friction and the bubbles is shown in Fig.5. The red color corresponds to the high skin friction. Initially, the bubbles move along the flow and the skin friction distribution is not affected by the force dipole. As the flow evolves, the difference of the skin friction appears. As shown in Fig.5, the local increase of the skin friction near the bubble is recognized in case 2.

Taking into account the energy dissipation, the balance of the skin friction is discussed. Based on Eqs.1 and 10, the dissipation theory shown in §2 is applied to the FCM. The total friction coefficient  $C_f^{Top} + C_f^{Bot}$  is expressed as,

$$C_{f}^{Top} + C_{f}^{Bot} = \underbrace{\frac{8\nu_{L}h\langle\mathbf{S}:\mathbf{S}\rangle}{\langle u\rangle^{3}}}_{\text{Dissipation}} + \underbrace{\frac{2h}{\langle u\rangle^{3}} \left[\frac{d\langle\mathbf{u}\cdot\mathbf{u}\rangle}{dt} - \frac{d\langle\mathbf{u}\rangle\cdot\langle\mathbf{u}\rangle}{dt}\right]}_{\text{Fluid motion}} \underbrace{-\frac{4h}{\rho_{L}\langle u\rangle^{3}} \left[\langle\mathbf{u}\cdot\mathbf{F}_{p}\rangle - \langle\mathbf{u}\rangle\cdot\langle\mathbf{F}_{p}\rangle\right]}_{\text{Bubble force}}.$$
 (13)

The bubble force term in Eq.13 does not appear in Eq.8 which is derived for the real flow. This terms comes from the treatment of the bubble boundary. In the FCM, the gas and liquid phases are not distinguished

when the flow field is solved based on Eq.10. The bubble force term in Eq.13 accounts for the bubble and liquid interaction. Therefore, the dissipation term in Eq.13 is not completely equivalent to the dissipation term in Eq.8 for the real flow. We check the numerical residual error of Eq.13 and confirm it is small enough. The temporal evolution of the total friction coefficient  $C_f^{Top} + C_f^{Bot}$  and the dissipation term in Eq.13 are shown in Fig.6. As shown in Fig.6,  $C_f^{Top} + C_f^{Bot}$  is strongly dependent on the dissipation term. Namely,  $C_f^{Top} + C_f^{Bot}$  is mostly affected by the total energy dissipation in the flow. When neglecting the force dipole in the FC simulation (case 1), the fluid motion term in Eq.13 initially decreases due to the transient density change caused by the bubble dispersion. The decrease of the fluid motion term transfers to the decrease of the dissipation around the bubble becomes large due to the shear flow modulation. Initially, the bubble force term in Eq.13 increases due to the force dipole. The increase of the bubble force term causes the increase of the dissipation term.

### 4 Direct Numerical Simulation with rigid bubbles

As described in §3, the skin friction increases due to the force dipole. In the force coupling simulation, however, the local no-slip boundary condition is not fully satisfied on the bubble surface even if the force dipole is introduced. Moreover, the expression of the force dipole given by Eq.12 is based on the "steady" solution for the creeping flow. In the unsteady flow such as the turbulent flow, Eq.12 might not be appropriate approximation for the expression of the force dipole. In order to treat the boundary condition more rigorously than the FC simulation, the DNS with rigid bubbles is performed under the same condition as shown in §3.

#### 4.1 Simulation method

We assume the bubble is spherical and rigid, similarly to the analysis in  $\S3$ . The present simulation method is based on that explained by Sugiyama *et al.*(2001). Empirical equations for the bubble motion, such as the drag force, the added inertia and so on, are not used. The bubble and liquid flows are solved on the rectangular grid. Governing equations for the liquid consist of the mass conservation equation given by Eq.1 and the momentum one expressed as,

$$\rho_L \frac{D\mathbf{u}_L}{Dt} = -\nabla p + \mu_L \nabla^2 \mathbf{u}_L. \tag{14}$$

The bubble motion is based on the Newton's law and tracked in the Lagrangian way. The integral of the liquid stress is directly taken over the bubble surface. The translational and rotational motions of the lth bubble are expressed as,

$$\frac{4}{3}\pi a^{(l)3}\rho_G \frac{d^2 \mathbf{Y}^{(l)}}{dt^2} = \oint_{|\mathbf{Y}^{(l)} - \mathbf{x}| = a^{(l)}} d^2 \mathbf{x} \left(-p\mathbf{I} + 2\mu_L \mathbf{S}_L\right) \cdot \mathbf{n} + \frac{4}{3}\pi a^{(l)3}(\rho_G - \rho_L)\mathbf{g},\tag{15}$$

$$\frac{8}{15}\pi a^{(l)5}\rho_G \frac{d\mathbf{\Omega}_G^{(l)}}{dt} = \oint_{|\mathbf{Y}^{(l)} - \mathbf{x}| = a^{(l)}} d^2 \mathbf{x} \left(\mathbf{Y}^{(l)} - \mathbf{x}\right) \times \left\{\left(-p\mathbf{I} + 2\mu_L \mathbf{S}_L\right) \cdot \mathbf{n}\right\},\tag{16}$$

where  $\Omega_G$  is the angular velocity of the *l*th bubble. The bubble velocity is imposed on the grid point belonging to the bubble inside.

#### 4.2 Simulation condition

The accuracy of the spatiotemporal discretization is same as that described in §3.2 The integral of the stress in Eqs.15 and 16 is sampled by  $2048(=32(\theta)\times64(\phi))$  points. The liquid stress on the bubble surface is interpolated by the third-order Lagrangian method. The conditions of the grid and the flow properties are same as those shown in §3.3. The present simulation condition corresponds to case 3 in Table 1.

#### 4.3 Results and discussion

The mean void fraction profile is shown in Fig.7 for different times. Similarly to Fig.3 for the FC simulation, the bubbles disperse due to the turbulence. The short time behavior of the bubble dispersion on the skin friction is discussed.

The temporal evolution of the relative friction coefficient  $C_f/C_{f0}$  on the top wall is shown in Fig.8. As shown in Fig.8,  $C_f$  for the present DNS (case 3) increases in time. Such a drag increase is caused by the increase of the effective viscosity. The skin friction suddenly changes at the time zero when the bubbles are introduced as compared with the result of the FCM shown in Fig.4. It is considered because the noslip boundary condition is suddenly satisfied in the present DNS, while the boundary condition is gradually satisfied in the FCM as the flow evolves.

The instantaneous distribution of the skin friction and the bubbles is shown in Fig.9 for cases 1 and 3. As shown in Fig.9, the local increase of the skin friction near the bubble is recognized in case 3, corresponding to the present DNS. This tendency is same as that in case 2, corresponding to the FC result with the force dipole (see Fig.5).

The dissipation theory shown in §2 is applied to the present DNS. Considering Eq.14 and the present simulation condition, Eq.8 is simplified as,

$$C_{f}^{Top} + C_{f}^{Bot.} = \underbrace{\frac{8h\nu_{L} \langle \mathbf{S} : \mathbf{S} \rangle_{L}}{(\langle u \rangle_{G} + \langle u \rangle_{L})^{3}}}_{\text{Dissipation}} + \underbrace{\frac{2h(\rho_{G}/\rho_{L})}{(\langle u \rangle_{G} + \langle u \rangle_{L})^{2}} \left[ \frac{1}{\langle u \rangle_{G} + \langle u \rangle_{L}} \frac{d \langle \mathbf{u} \cdot \mathbf{u} \rangle_{G}}{dt} - 2 \frac{d \langle u \rangle_{G}}{dt} \right]}_{\text{Bubble motion}} + \underbrace{\frac{2h}{(\langle u \rangle_{G} + \langle u \rangle_{L})^{2}} \left[ \frac{1}{\langle u \rangle_{G} + \langle u \rangle_{L}} \frac{d \langle \mathbf{u} \cdot \mathbf{u} \rangle_{L}}{dt} - 2 \frac{d \langle u \rangle_{L}}{dt} \right]}_{\text{Liquid motion}}.$$
(17)

The first term is related to the statistically steady fluid motion. On the other hand, the second and third terms is related to the statistically unsteady one because of containing the time derivative. The total friction coefficient balance expressed by Eq.17 is investigated. The temporal evolution of the dissipation, the bubble and liquid motions terms is shown in Fig.10. Figure 10 (a) and (b) correspond to the steady term and the unsteady terms, respectively. The alternative long and short dash line corresponds to the result of the single phase flow. As shown in Eq.17, the negative change accounts for the contribution to the drag reduction. As shown in Fig.10(a), the dissipation term shows positive change and contributes to the drag increase under the present condition. The drag increase shown in Fig.9 mainly caused by the increase of the dissipation term. On the other hand, as shown in Fig.10(b), the liquid motion terms contributes to the drag reduction. The negative change of the liquid motion term is considered to result from the transient density change due to the bubble dispersion. When the bubbles moves toward the higher flow region, the liquid loses its speed. The bubble dispersion effect to reduce the skin friction might be observed in the simulations performed by Xu et al.(2002) in the FCM and Sugiyama et al.(2003) in the E-L method. Considering the numerical scheme on the local velocity modulation around bubbles, however, the energy dissipation is considered underestimated in the both simulations. It is because Xu et al.(2002) neglected the force dipole term which should contribute to the augmentation of the strain rate near the bubble surface. Moreover, in the E-L simulation, the energy dissipation near the bubble surface is not resolved since the grid size is much larger than the bubble diameter.

In order to see the local modulation of the energy dissipation around the bubble,  $(\mathbf{S} : \mathbf{S})^*$ , corresponding to the deviation of  $\mathbf{S} : \mathbf{S}$ , is introduced.

$$(\mathbf{S}:\mathbf{S})^{*}(x,y,z) = (\mathbf{S}:\mathbf{S})(x,y,z) - \frac{1}{L_{x}L_{z}} \int_{0}^{L_{x}} dx' \int_{0}^{L_{z}} dz' \ (\mathbf{S}:\mathbf{S})(x',y,z').$$
(18)

The typical instantaneous distribution of  $(\mathbf{S} : \mathbf{S})^*$  and the bubbles is shown in Fig.11. The red color corresponds to  $(\mathbf{S} : \mathbf{S})^* \ge 1 \times 10^3$  and the blue one to  $(\mathbf{S} : \mathbf{S})^* \le -1 \times 10^3$ . Figure 11(a) and (b) correspond to the top view at y/h = 1.69 on the *xz*-plane and the side view at z/h = 1.45 on the *xy*-plane. As shown in Fig.11,  $(\mathbf{S} : \mathbf{S})^*$  is high around the bubbles. The local increase of the energy dissipation around the bubble causes the drag increase.

The local change of the  $(\mathbf{S} : \mathbf{S})^*$  is conditionally sampled with respect to the nearest length from the bubble centroid  $\lambda$ . The conditionally averaged  $(\mathbf{S} : \mathbf{S})^*$  is denoted by  $(\widehat{\mathbf{S} : \mathbf{S}})^*(\lambda)$ . The relation between  $(\widehat{\mathbf{S} : \mathbf{S}})^*$  and the nearest bubble position  $\lambda$  is shown in Fig.12.  $\lambda$  is normalized by the bubble radius a. In calculating  $(\widehat{\mathbf{S} : \mathbf{S}})^*$ ,  $\Delta\lambda$  is set to 0.1a. The sampling is taken from  $t^+ = 0$  to  $t^+ = 150$ . We confirm the profile in the region of  $\lambda/a < 4$  is almost independent of  $t^+$  in the sampled period mentioned above. As shown in Fig.12,  $(\widehat{\mathbf{S} : \mathbf{S}})^*$  shows negative value in the region of  $\lambda/a < 1$ , corresponding to the bubble inside. It is because the  $\mathbf{S}$  is assumed zero inside the bubble due to the rigid bubble and  $\mathbf{S} : \mathbf{S}$  inside the bubble must be smaller than that around the bubble in the xz-plane. In the region of the bubble outside  $(\lambda/a > 1)$ ,  $(\widehat{\mathbf{S} : \mathbf{S}})^*$  shows the positive value in the vicinity of the bubble. The effective length indicating the positive  $(\widehat{\mathbf{S} : \mathbf{S}})^*$  is about one bubble radius from the bubble surface. The positive change of  $(\widehat{\mathbf{S} : \mathbf{S}})^*$  contributes to the increase of the skin friction as expected by Eq.17. From Fig.12, the increase of the skin friction shown in Fig.8 is considered to result from the local increase of the energy dissipation near the bubbles.

The drag increase result is obtained by the FC simulation in §3.3 and the DNS in §4.3. This result is reasonable in view of the dissipation theory derived in §2. The present simulation condition is limited so that we are not sure whether the skin friction will reduce or not under the different condition. However, we can conclude that we must choose the simulation method that fully resolves the energy dissipation near the bubble surface when investing the skin friction change.

## 5 Large Eddy Simulation with deforming bubbles

We have performed the Front-Tracking (F-T) simulation with deformable bubbles. The bubble has the free slip boundary. As expected from Eq.8, the drag reduction is easier to be obtained in the F-T simulation than that for the rigid bubbles. It is because that the vorticity generation on the bubble surface, which makes the increase of the energy dissipation, is considerably smaller with the free slip boundary than that with the no-slip one. Moreover, according to Frankel and Acrivos (1970), the effective viscosity decreases due to the bubble deformation. However, we have not obtained the drag reduction in the F-T method yet. Although we carried out the transient simulation in which the bubbles were initially concentrated near the wall, the skin friction increased at  $Re_{\tau} = 180$ . This result is inconsistent with that reported by Xu *et al.*(2002), while it is consistent with the simulations for the rigid bubbles at  $Re_{\tau} = 150$  described in §3.3 and §4.3, in which the boundary condition is more rigorously treated. The friction Reynolds numbers  $Re_{\tau}$  in our previous simulation have been set to less than 200, which is considerably smaller than that in typical experiments. In this section, the F-T simulation with deformable bubbles is performed at  $Re_{\tau} = 1100$ , which is comparable to that in the experiment performed by Moriguchi and Kato (2002).

#### 5.1 Numerical method

The numerical method used in this study is almost the same as in the previous studies (Kawamura and Kodama, 2001, 2002; Sugiyama *et al.*, 2003) except that the dynamic Smagorinsky model (Germano *et al.*, 1991; Lilly, 1992) is introduced. This code adopts the front-tracking approach to deal with deforming bubbles. The liquid and gas phases are separately solved as incompressible viscous fluids with satisfying the continuity of velocity and stresses at the interface. The previous report (Kawamura and Kodama, 2002) should be referred to for more detailed description and validation of the code.

The grid spacing is assumed to be sufficiently small for resolving bubble surfaces. Therefore the difference between the grid-filtered and the test-filtered velocity fields is related only to the sub-grid scale eddies and can be modeled by subgrid scale models for single phase turbulent flows.

### 5.2 Condition of simulation

A fully developed turbulent channel flow containing bubbles is investigated by the present numerical method. Since we have removed the restriction in the Reynolds number by introducing LES approach, we can set the same condition as in the experiment. In this study, we set the condition after the experiment by Moriguchi and Kato (2002) in which the channel height 2h was 10mm. The bulk mean velocity  $U_m$  was 5 m/s, the mean bubble diameter D is about 1.0 mm, and the mean void fraction  $\alpha$  is 1%. The corresponding non-dimensional parameters are  $Re_{\tau}=1,100$ , D/h=0.2, the Froude number  $F_d = U_m/\sqrt{gh} = 16$ , and the Weber number  $We = \rho U_m^2 D/\sigma = 340$ . But since a preliminary simulation indicated that breakup of bubbles can happen, the Weber number in the LES computation had to be decreased to 34. The breakup of bubbles was also confirmed by experimental observation using a high speed camera.

Before introducing the bubbles, a fully developed single-phase turbulent channel flow at the Reynolds number  $Re_{\tau}=1,100$  was computed. The size of the computational domain was set to 6.4*h*, 3.2*h* and 2*h*, in the streamwise, wall-normal and spanwise directions respectively. A periodic boundary condition was used in the streamwise and spanwise directions. The x-, y- and z- axes are taken in the streamwise, wall-normal and spanwise directions respectively. The number of the grid points was  $256 \times 128 \times 128$ . The computational domain was decomposed into eight blocks in the streamwise direction, and each block of  $32 \times 128 \times 128$  grid points was computed on a node of a parallel computer system. At the non-dimensional time  $t^+ = u_{\tau}^2 t/\nu = 0$ , 98 bubbles were introduced at positions shown in Fig.13(a) and (c). Initially, the centers of all the bubbles are located on the plane y = 0.2h. The flow is driven by the mean pressure gradient in the streamwise direction, which is controlled at each time step to maintain the constant volume flow rate.

#### 5.3 Results and discussion

The results of the computation up to  $t^+ = 480$  is presented in this paper. The initially concentrated bubbles are dispersed by turbulent eddies. The variation of the void fraction profile to  $t^+ = 400$  shown in Fig.14 and an instantaneous bubble distribution  $t^+ = 400$  shown in Fig.13(b) and (d) indicate that bubble distribution is still developing at  $t^+ = 400$ .

The time histories of the friction coefficient in Fig.15 shows that the friction of the top wall was decreased by 3% between  $t^+ = 200$  and 400. This drag reduction rate is comparable with the measurement by Moriguchi and Kato (2002). The decrease of the friction on the top wall starts at about  $t^+ = 50$ , while it is supposed that turbulence modification effect has not reached the bottom wall at  $t^+ = 400$ . As indicated in §4, the change in the drag can be caused by the change in the energy dissipation rate or by the transient terms. However, it has been confirmed that this drag reduction in the LES is mainly due to the reduction in the energy dissipation. Fig.16 shows the time histories of the liquid phase energy dissipation rate  $\langle \mathbf{S} : \mathbf{S} \rangle_L$  in the single phase and multiphase computations. The decrease in the sum of the top wall and bottom wall friction coefficients is very well correlated to the decrease in the energy dissipation rate.

Figure 17 compares the liquid phase velocity profiles in the single- and multiphase LES and the mean streamwise velocity of bubble centroids in wall units. The bubble velocity is smaller than the liquid velocity, and the difference increases as the distance from the initial bubble position increases. Probably the slower bubble velocity is due to the transient effect. The difference in the mean liquid phase velocity in the single- and multiphase flows is very small in the near wall region  $y^+ < 100$ , but the liquid velocity in the multiphase LES is significantly larger in the region  $y^+ > 200$  than in the single phase flow. This fact suggests that microbubble drag reduction may occur in the outer region. This assumption does not contradict the drag increase in the prior DNS at low Reynolds numbers.

Figure 18 shows the comparison of the Reynolds shear stress  $\overline{u'v'}$  for the single- and multiphase with the measurement by Kitagawa *et al.*(2004). Since the multiphase flow LES is still in the transient stage, the liquid phase Reynolds shear stress is influenced by the mean motion of the bubbles going away from the wall. However, the correlation  $\overline{u'v'}$  for the bubble centroid velocity is much smaller than that for the liquid phase, and this qualitatively agrees with the measurement.

The decrease in the correlation  $\overline{u'v'}$  is probably explained by averaging effect due to the finite bubble size. Bubbles respond to the force integrated over the surface, and the integration cancels the contribution of eddies relatively smaller than bubbles. This tendency is also seen in the streamwise component of the turbulent intensity shown in Fig.19. The intensity  $\overline{u'u'}$  of bubble centroid is smaller than that of liquid phase, and the difference increases as the wall is approached or the relative eddy scale to the bubble size is decreased.

The presence of relatively large and dull bubbles limits the freedom of the turbulent motion in the liquid phase. This effect probably obstructs the development of large scale eddy motion and leads to net decrease in the turbulent momentum transfer. The experimental evidence that small bubbles are not necessarily effective (Kawamura *et al.* 2004) also supports this assumption.

## 6 Conclusions

Extending the energy dissipation theory to the microbubble channel flow, the relation between the energy dissipation and the skin friction is derived as Eq.8. In the statistically steady situation, the skin friction is expressed by the energy dissipation. The transient microbubble flows are numerically simulated in order to the energy on the skin friction. The low Reynolds number flow ( $Re_{\tau} = 150$ ) with rigid bubbles is simulated in the Force Coupling (FC) and Direct Numerical Simulation (DNS) methods. The transient bubbles dispersion is confirmed to contribute to the drag reduction, while the local velocity gradient around the bubble to the drag increase. Combining these effects, the drag increase result is obtained by the FC simulation in §3.3 and the DNS in §4.3. The drag increase result is reasonable in view of the present energy dissipation theory.

It has been shown that both the frictional drag and the total energy dissipation decreased in the large eddy simulation of channel flow with deformable bubbles at  $Re_{\tau} = 1, 100$ . Although the simulation is still in a transient stage, the statistics of the turbulent motion of the bubble centroids qualitatively agreed with the measurement. It has been shown that the turbulent motion of bubbles is smaller than that of the surrounding liquid, and it is suggested that the restriction of the liquid eddy motion by the presence of "dull" bubbles can lead to the decrease in the turbulence production. The fact that the increase in the mean velocity was observed only in the outer region ( $y^+ > 200$ ) supports the hypothesis that the presence of bubbles increases the energy dissipation in the near wall region but can decrease in the outer region.

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Figure 3: Profiles of the void fraction profile in ydirection at  $Re_{\tau} = 150$  ( $t^+ = 0, 37.5, 75, 112.5$  and 150)



Figure 4: Temporal evoluton of relative friction coefficient on top wall at  $Re_{\tau} = 150$  ( $C_{f0}$ : friction coefficient with no bubble, Case 1: FCM without force dipole; Case 2: FCM with force dipole)



Figure 5: Distribution of skin friction and bubbles at  $Re_{\tau} = 150$  (Case 1: FCM without force dipole; Case 2: FCM with force dipole)



Figure 6: Temporal evolution of total friction coefficient and energy dissipation in microbubble flow obtained by FCM at  $Re_{\tau} = 150$ 



Figure 7: Profiles of the void fraction in y-direction of case 3 at  $Re_{\tau} = 150$  ( $t^+ = 0, 37.5, 75, 112.5$  and 150)



Figure 8: Temporal evoluton of relative friction coefficient on top wall at  $Re_{\tau} = 150$  ( $C_{f0}$ : friction coefficient with no bubble, Case 3: DNS)



Figure 9: Distribution of skin friction and bubbles at  $Re_{\tau} = 150$  (Case 1: FCM without force dipole; Case 3: DNS for rigid bubbles)



Figure 10: Temporal evolution of dissipation and unsteady terms on total friction coefficient at  $Re_{\tau} = 150$ obtained by DNS



Figure 11: Instantaneous distributions of  $(\mathbf{S} : \mathbf{S})^*$  and bubbles on *xz*- and *xy*-planes in case 3 ( $Re_{\tau} = 150$ )



Figure 12: Relation between  $(\mathbf{S} : \mathbf{S})^*$  (corresponding to conditionally averaged  $(\mathbf{S} : \mathbf{S})^*$  with respect to the nearest length from bubble  $\lambda$ ) and  $\lambda/a$  at  $Re_{\tau} = 150$ 



(d) Side view at  $t^+ = 400$ 

Figure 13: Distribution and deformation of bubbles in the LES of channel flow with deforming bubbles  $(Re_{\tau} = 1, 100, \alpha = 1.0\%)$  The flow is from left to right.



Figure 14: Profiles of the void fraction in the LES at  $t^+ = 0, 100, 200 \text{ and } 400.$ 



the top and bottom walls in the LES of channel flow



Figure 16: Time history of the energy dissipation term of Eq.14 in the LES of channel flow



Figure 17: Mean velocity profiles in the LES of channel flow



Figure 18: Profiles of the Reynolds shear stress  $\overline{u'v'}$ in the LES of channel flow



Figure 15: Time history of the friction coefficient on Figure 19: Profiles of the streamwise component of the turbulent fluctuation