Proc. 6th Symp. Smart Control of Turbulence
Tokyo, March 6 - 9, 2005

Feedback control achieving sublaminar friction drag

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Effects of an idealized feedback control are studied by means of direct numerical simulation (DNS). The control input is a body force directly reducing the Reynolds shear stress near the wall. The DNS of turbulent pipe flow under a constant flow rate at $Re_b = 5300$ (i.e, $Re_\tau \simeq 180$ for uncontrolled flow) shows that the skin friction can be reduced even to a sublaminar level. This is caused by the reversal of the sign of Reynolds shear stress, which results in a negative value of "the turbulent contribution to skin friction" [Fukagata et al., *Phys. Fluids* **14**, L73 (2002)]. The turbulence structure is also largely modified with this control. The quasi-streamwise vortices completely vanish and alternating spanwise roller-like structure forms instead. It is, however, mathematically proved that the net power required to drive the flow, i.e., the summation of pumping and actuation powers, cannot be reduced to a sublaminar level.

I. INTRODUCTION

Recently, an exact relationship was discovered between the skin friction drag and the Reynolds shear stress [1] (hereafter called as the FIK identity). In fully-developed pipe flows, for instance, the FIK identity reads

$$C_f = \frac{16}{Re_b} + 16\int_0^1 2r(\overline{u_r'u_z'})\, rdr\ . \tag{1}$$

Throughout the paper, all the quantities are nondimensionalized by using twice the bulk mean velocity, $2U_b^*$, and the pipe radius, R^*, whereas the dimensional quantities are expressed by the superscript "$*$". The skin friction coefficient, $C_f = 2\overline{\tau_w}^*/(\rho^* U_b^{*2})$ (where τ_w^* is the wall shear), can be decomposed into two contributions. The first term in the right-hand-side (RHS) of Eq. (1) is the laminar contribution, which is identical to the laminar friction drag. Here, the bulk Reynolds number, Re_b, is defined as

$$Re_b = \frac{2U_b^* R^*}{\nu^*}\ . \tag{2}$$

The second term of Eq. (1) is the turbulent contribution, which is expressed by a weighted integration of the Reynolds shear stress. The subscript to the velocity, u, represents the direction in the cylindrical coordinates, i.e., r (radial), θ (azimuthal), and z (longitudinal). The overbar $(\bar{\cdot})$ and prime $(\cdot')$ denote, respectively, the mean and fluctuation components of the Reynolds decomposition.

With the opposition control [2], the Reynolds shear stress in the near-wall layer is directly suppressed. Due to the change near the wall, the Reynolds shear stress is indirectly suppressed in the region far from the wall, too [3]. The amount of indirect suppression is determined by that of the direct suppression. Therefore, suppression of the Reynolds shear stress in the near-wall layer is primarily important for the reduction of frictional drag. This argument is supported by the recent studies on the suboptimal control aiming at suppression of the near-wall Reynolds shear stress [4] and the theoretical analysis of an idealized case, where the velocity fluctuations are perfectly damped out in the near-wall layer [5].

In the cases of velocity damping, the theoretical maximum drag reduction is attained when the flow is relaminarized. The FIK identity, however, indicates that a sublaminar friction drag can be attained if the turbulent contribution (i.e., the second term of Eq. (1)) can be made negative. Therefore, in the present study, we investigate the possibility of such a sublaminar drag. We perform DNS of turbulent pipe flow with a feedback body force, which ideally reduces the Reynolds shear stress in the near-wall layer.

II. FEEDBACK CONTROL

We consider an incompressible flow in a cylindrical pipe. The flow is driven by a time-dependent mean pressure gradient, $(-dP/dz)(t)$, adjusted so as to keep the flow rate constant. The governing equations, i.e., the continuity and momentum equations, are

$$\nabla \cdot \mathbf{u} = 0 \tag{3}$$

and

$$\frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \left[-\mathbf{uu} - p' \, \mathbf{I} + \frac{2}{Re_b} \mathbf{s} \right] - \frac{dP}{dz} \, \mathbf{e}_z + \mathbf{f} \, , \tag{4}$$

where p' is the pressure fluctuation (divided by the density), $\mathbf{e}$ and $\mathbf{I}$ are the unit directional vector and the unit dyadic, respectively, $\mathbf{s}$ is the strain rate tensor, i.e.,

$$\mathbf{s} = \frac{1}{2} \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] \, , \tag{5}$$

and $\mathbf{f}$ denotes the body force.

In the present study, a zero-mean feedback body force in the wall-normal direction is applied as an idealized control input, i.e.,

$$\mathbf{f} = f_r \, \mathbf{e}_r \, , \tag{6}$$

with $\overline{f_r} = 0$ and $f_r' \neq 0$. We assume that the body force works toward away from the wall ($f_r' < 0$) in the high-speed region ($u_z' > 0$) and toward the wall ($f_r' > 0$) in the low-speed region ($u_z' < 0$). The magnitude of the body force is proportional to the streamwise velocity fluctuations, $u_z'(r, \theta, z, t) = u_z(r, \theta, z, t) - \overline{u_z}(r, t)$. Thus, the feedback control law reads

$$f_r'(r, \theta, z, t) = -\alpha(y) u_z'(r, \theta, z, t) \, . \tag{7}$$

Here, $\alpha(y)$ is an arbitrary envelope function (y is the distance from the wall, i.e., $y = 1 - r$). In the present study, we use the following smooth function, which takes finite values in the region near the wall and zero far from the wall:

$$\alpha(y^{+u}) = a \, y^{+u} \exp\left(-\frac{y^{+u}}{b^{+u}} \right) \, . \tag{8}$$

TABLE I: Number of grids (N_r, N_θ, N_z) and grid spacings (Δr, $R\Delta\theta$, Δz) used in DNS of pipe flow.

N_r	N_θ	N_z	Δr^{+u}	$(R\Delta\theta)^{+u}$	Δz^{+u}
48	128	256	0.95 - 6.11	9.03	14.4

TABLE II: Studied cases, drag reduction rate (R_D) and net power saving rate (R_P).

	b^{+u} (thickness)	τ^{+u} (time scale)	R_D [%]	R_P [%]
Case 1 (thin, weak)	10	0.76	48	48
Case 2 (thin, strong)	10	0.38	83	-63
Case 3 (thick, weak)	20	0.72	91	-16

The parameters, a and b^{+u}, determine the control amplitude and the thickness of controlled layer. The superscript "$+u$" denotes the wall unit of the uncontrolled flow. The envelope function, $\alpha(y)$, has a dimension of inverse time. Therefore, we define a time scale of control, τ^{+u}, as

$$\tau^{+u} = \frac{1}{\max(\alpha^{+u})} \ . \tag{9}$$

With the present control, the transport equation of the Reynolds shear stress reads

$$\frac{\partial(\overline{u_r' u_z'})}{\partial t} = R_{rz} - \frac{\alpha}{2}\overline{(u_z')^2} \ , \tag{10}$$

where R_{rz} denotes the RHS of transport equation without body force. It is clear that the body force of the present control always works to reduce the Reynolds shear stress.

III. DNS RESULTS

The governing equations are solved by using the energy conservative finite difference method on the cylindrical coordinates [6]. The computational domain has a length of $20R$ and the periodic boundary condition is applied at the both ends. The bulk Reynolds number, Re_b, is kept at 5300. The friction Reynolds number of the uncontrolled flow, $Re_{\tau u} = u^*_{\tau u} R^*/\nu^*$, is about 180, where u_τ is the friction velocity and the subscript "u" denotes the quantities of the uncontrolled flow. Specifications of the grid system used in the present simulations are shown in Table I. Although the grid system is not extremely fine, it has been shown in the previous study [3] that it is sufficient for the evaluation of drag reduction rate.

Table II shows the studied cases. In Case 1, a relatively weak control input is applied in a relatively thin layer near the wall. A stronger control input and wider layer than those in Case 1 are considered in Cases 2 and 3, respectively. For grid dependency check, Case 3 is studied also by using a finer grid system ($N_r = 96$, $N_\theta = 128$, $N_z = 512$). The results are found to be almost the same as those with the coarser grid system.

Figure 1a shows the time trace of the skin friction drag coefficient, C_f, normalized by that of the uncontrolled flow, C_{fu}. In all the cases examined, C_f drastically reduces just after the onset

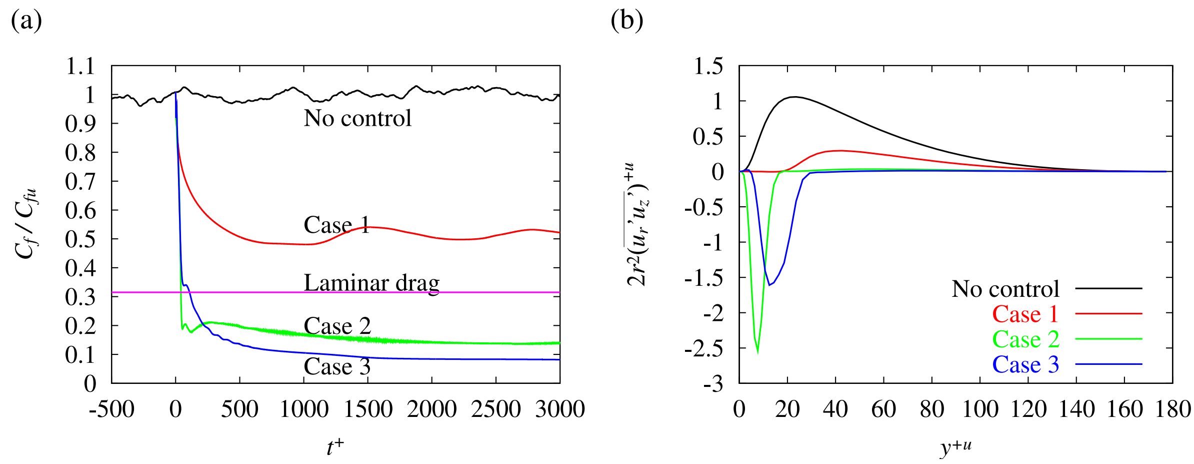

FIG. 1: Effects of present control: (a) Time trace of normalized skin friction; (b) Weighted Reynolds shear stress.

of control. After $t^{+u} \sim 1000$ the flows reach the quasi-steady states. The drag reduction rate, R_D, is defined by using C_f at the quasi-steady state, as

$$R_D = \frac{C_{fu} - C_f}{C_{fu}} \,. \tag{11}$$

The computed drag reduction rate is shown in Table II. In Case 1, the drag is still larger than the laminar flow (the drag reduction rate at relaminarization is $R_D = 67\%$ for the pipe flow at $Re_b = 5300$). In Cases 2 and 3, the drag reduction rates are higher than that of relaminarization. Namely, sublaminar drag is attained. It is worth noting that when the control was terminated in the sublaminar case, the flow was relaminarized after a transient period.

Figure 1b shows the weighted Reynolds shear stress, $2r^2\overline{u_r' u_z'}$, which is the integrand of the turbulent contribution term in the FIK identity (i.e., Eq. (1)). In Case 1, the Reynolds shear stress is nearly zero in the near-wall layer and indirectly suppressed in the region far from the wall. This situation is similar to the case of near-wall damping [5]. In Cases 2 and 3, in contrast, the Reynolds shear stress is largely negative in the near-wall layer and nearly zero in the region far from the wall. By this negative Reynolds shear stress, the turbulent contribution term in the FIK identity results in a negative value, which yields the sublaminar friction drag.

The flow structure is dramatically modified in the cases where the sublaminar friction drag is achieved. As shown in Fig. 2, the well-known coherent structure, which exists in the uncontrolled turbulent flow, totally vanishes by the present control. Self-organized roller-like structure appears instead. The structure is nearly periodic in the longitudinal direction and nearly homogeneous in the azimuthal direction.

IV. NET POWER

Discussions are extended on the net power required at the steady state after the present control is applied. The ideal control power required to keep the sublaminar drag, W_a, is the power input due to the body force. The pumping power is denoted as W_p. By using these

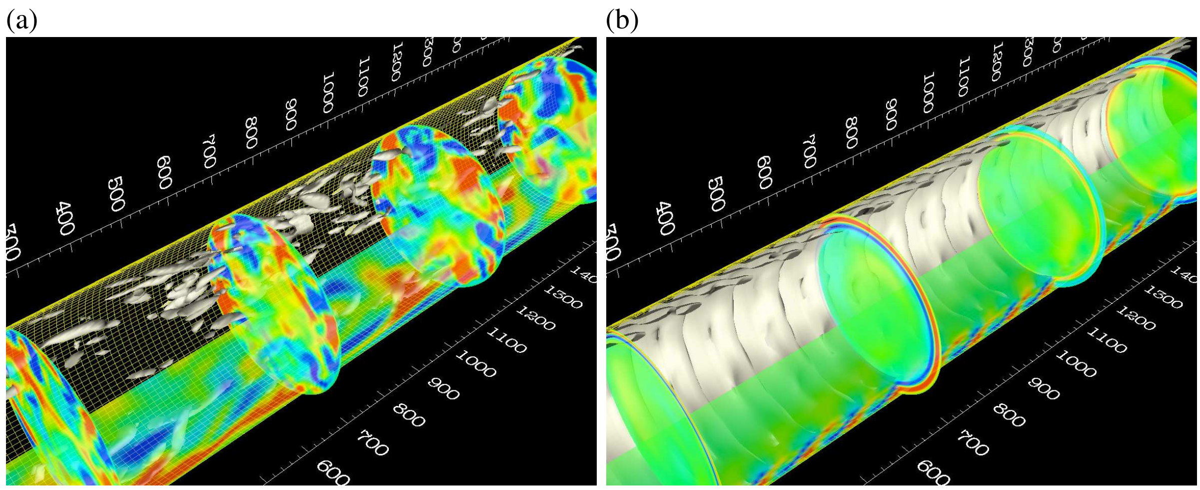

FIG. 2: Flow structure. (a) uncontrolled; (b) controlled (Case 3). White, vortex core; red, high-speed region; blue, low-speed region.

quantities, the net power saving rate can be defined as

$$R_P = \frac{W_{pu} - (W_p + W_a)}{W_{pu}} \, . \tag{12}$$

Again, the subscript "u" denotes the quantity of the uncontrolled flow.

The net power saving rate, R_P, computed in the present DNS is shown in Table II. In Case 1, the power required for actuation, W_a, is negligibly small as compared to the pumping power, W_p. Therefore R_P takes an approximately the same value as the drag reduction rate, R_D. In Cases 2 and 3, however, the net power saving rate takes negative values. This means that the actuation power required to keep the sublaminar drag is larger than the pumping power saved by the drag reduction.

The present DNS results suggest that sublaminar drag can be achieved by adding an appropriate body force to the flow, but the net power required to drive the flow may not reduced below the laminar level. The latter argument can be mathematically proved. Here, only the result is presented. In the present case, the energy balance (per unit volume) can be written as

$$W_p + W_a = \frac{2}{Re_b} + \frac{2}{Re_b} \int_0^1 \left(\frac{d\overline{u}_C}{dr} \right)^2 r dr + \frac{2}{Re_b} \int_0^1 \overline{\mathbf{s}' : \mathbf{s}'} \, r dr \, . \tag{13}$$

The first term in the RHS is the dissipation of the laminar profile. The second term, i.e., the dissipation due to the deviation of mean velocity from the laminar flow, $\overline{u}_C(r) = \overline{u_z}(r) - (1 - r^2)$, and the third term, i.e., the dissipation from the fluctuation velocities, are always positive. Therefore, the dissipation rate takes the minimum value when the flow has the laminar mean velocity profile. Equivalently the net power, $W_p + W_a$, cannot be reduced below the laminar level.

V. CONCLUSIONS

Direct numerical simulation was performed for an idealized feedback control of turbulent pipe flow. The flow rate was set constant and the bulk Reynolds number was 5300. Based on the implication by the FIK identity [1], wall-normal body forces proportional to the streamwise velocity fluctuations were added in the near-wall layer so as to reduce the Reynolds shear stress.

With the present control, the Reynolds shear stress became largely negative in the near-wall layer. As a result, the turbulent contribution to the skin friction [1] became negative and sublaminar skin friction drag was achieved. In such cases, the flow structure changed dramatically and self-organized roller-like structure appeared.

In the cases of sublaminar drag, however, the power required for control exceeded the pumping power saved by the drag reduction. It was mathematically proved that the net power required cannot be reduced below the laminar level.

Acknowledgments

The authors are grateful to Prof. Thomas R. Bewley (UCSD), Dr. Yasuo Kawaguchi (AIST, Japan), Prof. Shu Takagi, Prof. Yuji Suzuki and Dr. Kaoru Iwamoto (The Univ. of Tokyo) for helpful comments and fruitful discussions. This work was supported through the Project for Organized Research Combination System by the Ministry of Education, Culture, Sports and Technology of Japan (MEXT).

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