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# **Short-term Prediction of Non-linear Added Resistance in head sea with Probability Density Function Method**



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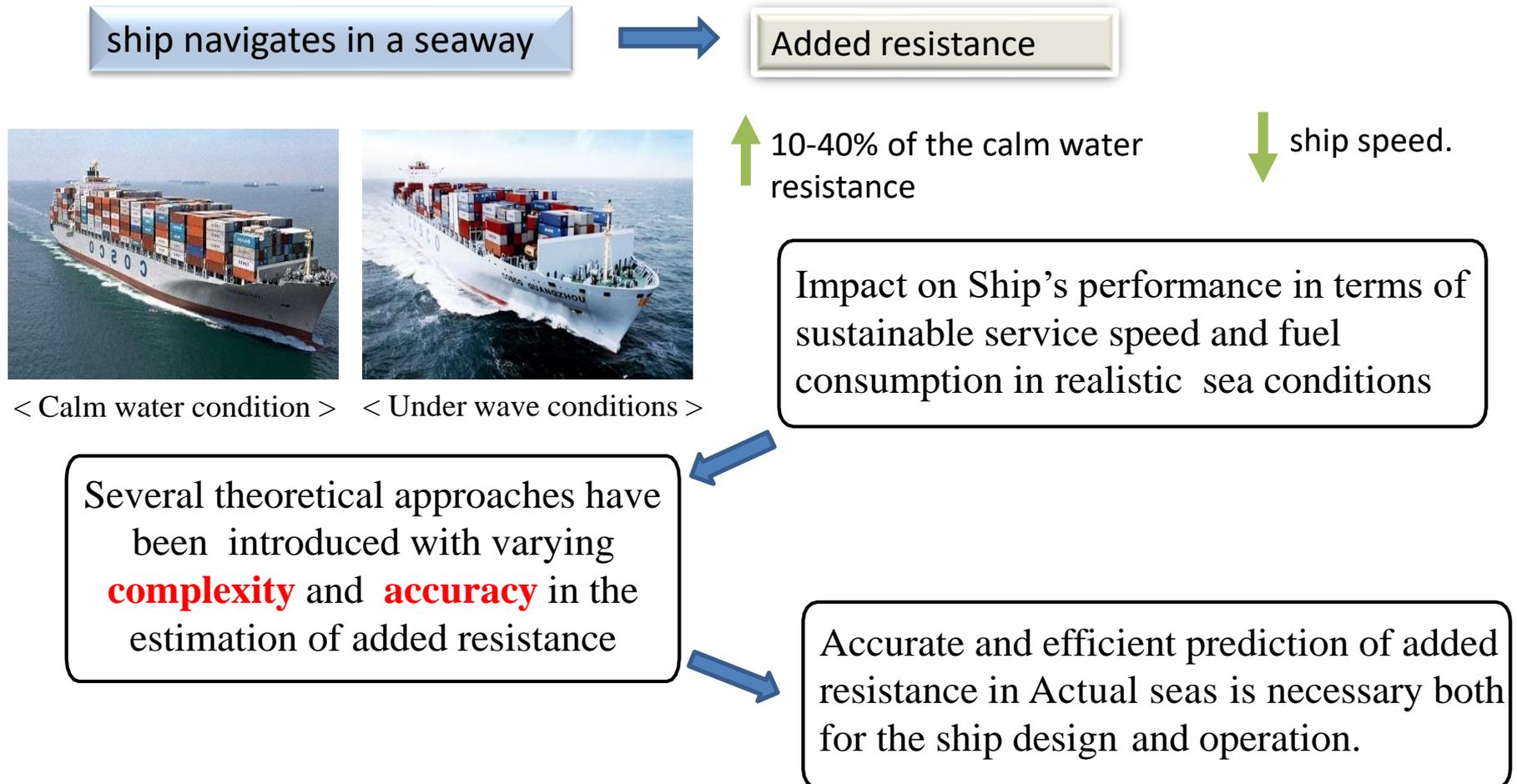
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# Introduction



Estimating the Added resistance in actual sea conditions is an important issue for the shipping industry in the design of energy-efficient ships.



# Problem statement



- ✓ A ship's performance is highly dependent on sea conditions affected by waves. **Incident wave heights** is a key parameter to determine increase in resistance in waves.
- ✓ Theoretically, Added wave Resistance  $\propto (H_w)^2$ .  
In actual sea conditions, non-linearity in the Added Resistance with respect to wave heights exists due to the non-linearity of the fluid near the ship hull because of non-linear hydrodynamic effects e.g., Effect of sea environment, wave-body interactions.
- ✓ Spectral analysis is not complete enough  the added resistance in actual irregular seas appropriately because the **added resistance transfer function is dependent on wave height**.

**Developed a probabilistic method considering non-linearities of added resistance with respect to wave height in head sea conditions using CFD numerical calculations in regular waves**

## Objectives

- Comparison of Conventional Spectral method and Proposed Probability density function method.
- Validate the Non-Linear PDF method with Experimental and CFD numerical simulations in long crested irregular waves.

# Research Background



**Dalzell (1974) and Hirayama, Wang (1993)** : Added Resistance transfer function in regular waves → not coincide with that estimated from irregular waves.

**Yasukawa et al. (2016)** : Spectral analysis is incomplete enough to estimate the added resistance in actual irregular seas and that the transfer function is dependent on wave height

**Kobayashi (2007) and Kuroda et al. (2016, 2018)** : Added resistance in irregular seas using the higher-order approximation response model in regular waves.

{ Limitation → Partially satisfactory in mild sea conditions, discrepancies in harsh seas. }

**Seo et al. (2014), Lee et al. (2016)** Predicted added Resistance using potential-based methods.

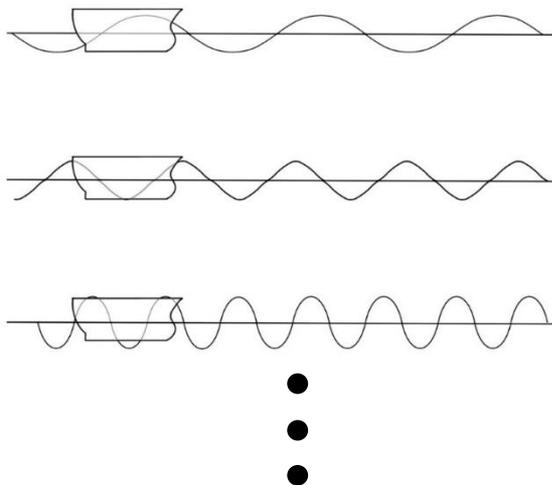
**Lang, X. et al (2020)** : Propose a wave height correction factor in the conventional formula (based on potential theory results), to estimate the added resistance due to actual sea states.

{ Limitation → Not able to consider non-linearity and viscosity effects }

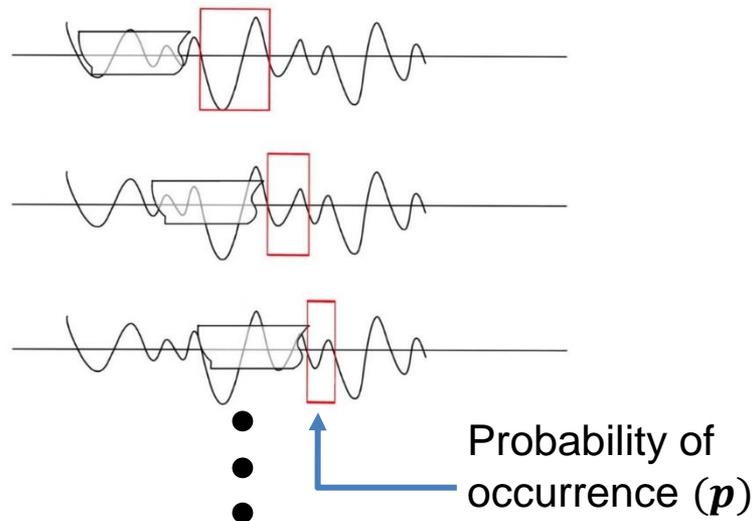
# Introduction of methods



## Conventional < Spectral method >



## Proposed < Probability density function method >



Treat as superposition of component waves

Statistically treatment of individual wave components

$$R_{AW}(H_{1/3}, T_{01}) = 2 \int_0^{\infty} \frac{R_{WU}(\omega, V)}{\zeta_a^2} S(\omega | H_{1/3}, T_{01}) d\omega$$

$$R_{AW}(H_{1/3}, T_{01}) = \int_0^{\infty} \int_0^{\infty} R_{WN}(H, T, V) \mathbf{p}(H, T | H_{1/3}, T_{01}) dT dH$$

Linear response function

Wave spectrum

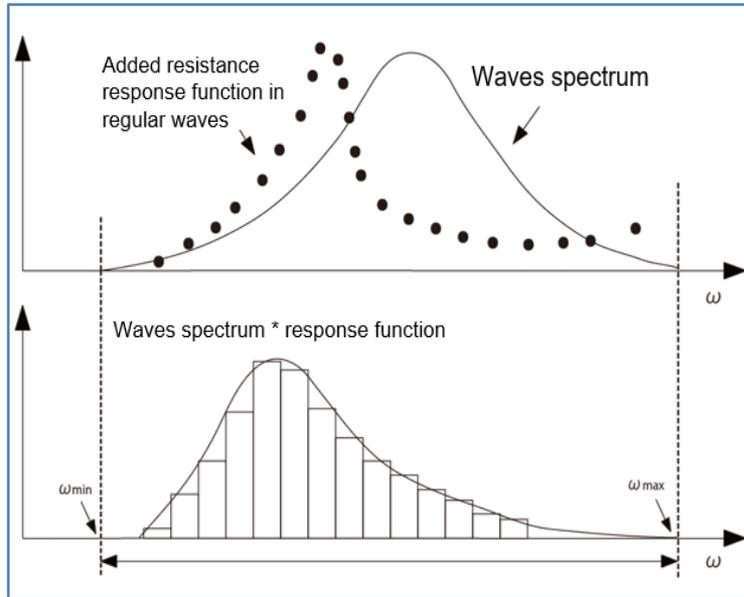
Obtained by Potential based calculation method based on linear theory

$\left(\frac{H}{2}\right)^2 \mathbf{C}(s, V) R_{WU}(\omega, V)$

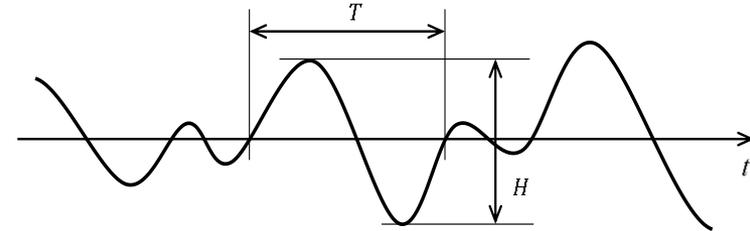
Response function **taking into account non-linearity** with respect to the wave height calculated using CFD data in regular waves.



## Conventional < Spectral method >



## Proposed < Probability density function method >



$H$  → wave height of one wave in the steady irregular wave

$T$  → wave period of the one wave.

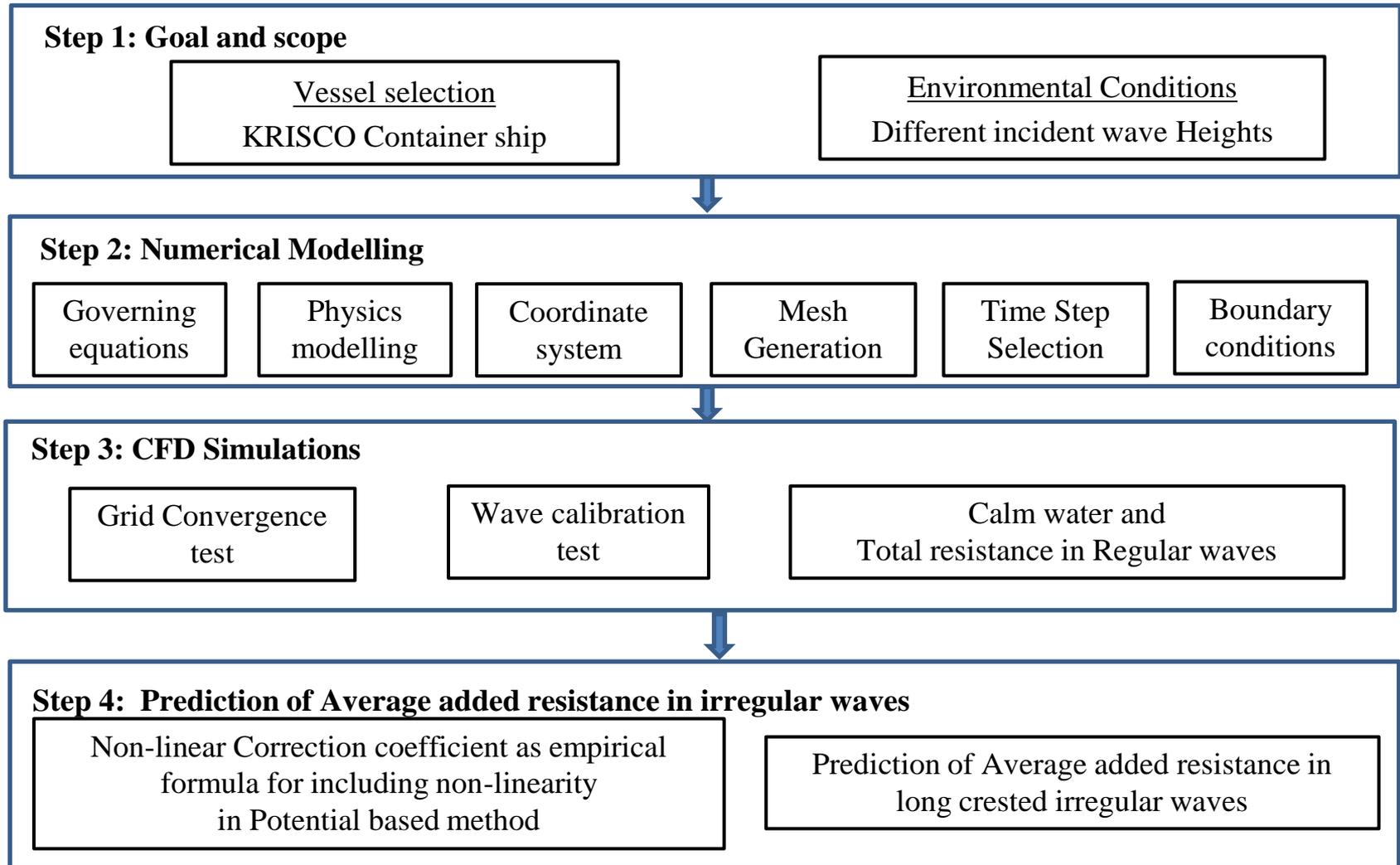
- Joint probability Density function of wave height and period assuming the linearity and narrow bandness of the wave (Longuet-Higgins 1975, 1983).

$$✓ f(\zeta, \eta | H_{1/3}, T_{01}) = \frac{1}{8\sqrt{2\pi\nu}} \left(1 + \frac{\nu^2}{4}\right) \left(\frac{\zeta}{\eta}\right)^2 \exp\left\{-\frac{\zeta^2}{8} \left[1 + \left(1 - \frac{1}{\eta}\right)^2 \frac{1}{\nu^2}\right]\right\}$$

Dimensionless  $\zeta = 4 \frac{H}{H_{1/3}}, \eta = \frac{T}{T_{01}}, \nu = \left(\frac{m_0 m_2}{m_1^2} - 1\right)^{\frac{1}{2}}$

$$✓ p(H, T | H_{1/3}, T_{01}) = f(\zeta, \eta) \frac{\partial(\zeta, \eta)}{\partial(H, T)} = \frac{8}{\sqrt{2\pi\nu}} \left(1 + \frac{\nu^2}{4}\right) \frac{T_{01}}{H_{1/3}^3} \left(\frac{H}{T}\right)^2 \exp\left\{-2 \left(\frac{H}{H_{1/3}}\right)^2 \left[1 + \left(1 - \frac{T_{01}}{T}\right)^2 \frac{1}{\nu^2}\right]\right\}$$

# Flowchart of predicting average added resistance in short term sea conditions



# CFD simulation cases for added resistance calculation for different wave heights



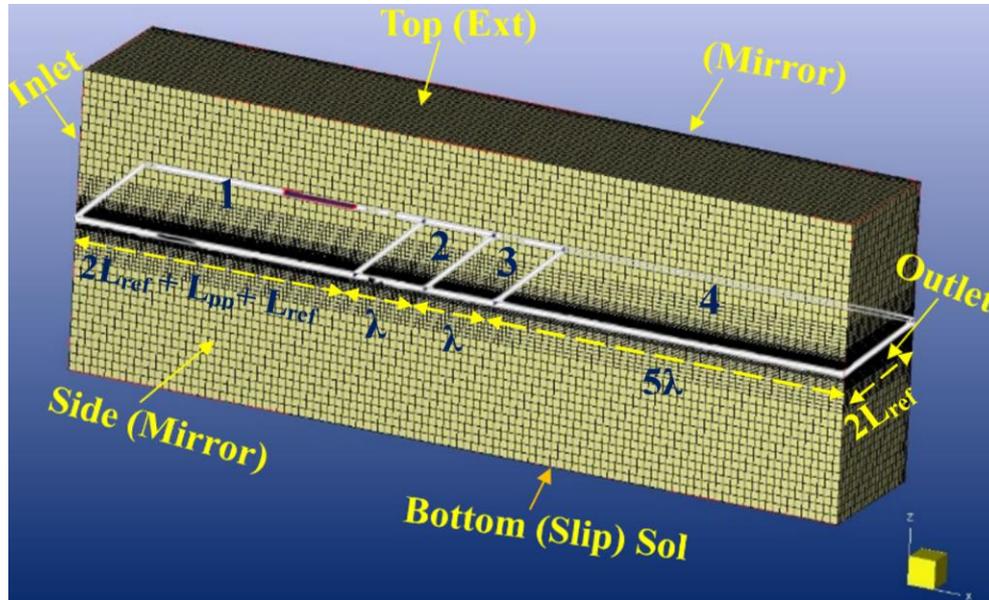
KCS	Full scale(m)	Model scale (m)
Length between perpendiculars LPP [m]	230	3.2
Length at load water line LWL [m]	232.5	3.2348
Breadth B [m]	32.2	0.448
Draft d [m]	10.8	0.1503
Displacement Volume $\nabla$ [m <sup>3</sup> ] $\times 1$	52030	0.1401
Wetted Surface Area S <sub>0</sub> [m <sup>2</sup> ] $\times 1$	9424	1.8242
Trim [m]	0	
Longitudinal center of buoyancy LCB [%Lpp] Aft+	1.48	
KG from Keel [m]	14.33	0.1994
K <sub>xx</sub> /B	0.4	
K <sub>yy</sub> /LPP, k <sub>zz</sub> /LPP	0.25	

No.	$\lambda/L_{pp}$	F <sub>n</sub>	H <sub>WS</sub>
1-3	0.5	0.14, 0.20, 0.26	2
4-6	0.5	0.14, 0.20, 0.26	5
7-9	0.5	0.14, 0.20, 0.26	8
10-12	1.0	0.14, 0.20, 0.26	2
13-15	1.0	0.14, 0.20, 0.26	5
16-18	1.0	0.14, 0.20, 0.26	8
19-21	1.0	0.14, 0.20, 0.26	10
22-24	1.5	0.14, 0.20, 0.26	2
25-27	1.5	0.14, 0.20, 0.26	5
28-30	1.5	0.14, 0.20, 0.26	8
31-33	1.5	0.14, 0.20, 0.26	10

➤ Head sea waves ( $\alpha = 180\text{deg.}$ )

# Boundary Conditions in RANSE based Software

## “FINE Marine”



Meshed Domain with hull surface refinements

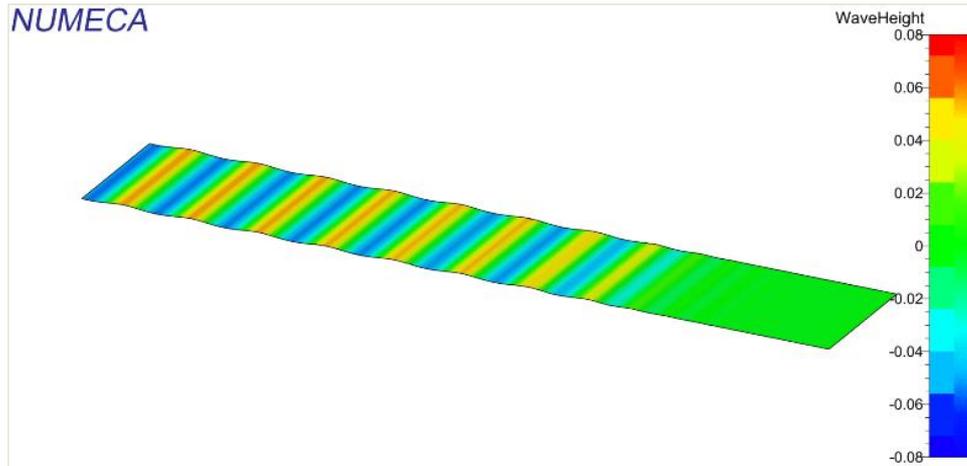
- Domain with Box refinements to generate the proper regular waves and its damping zone.
- Stokes wave that takes into account the second non-linear term from the inlet boundary plane

Item	Scheme used
Grid system	Unstructured, non-conformal, fully hexahedral grid
Spatial discretization	Finite volume method
Advection term	QUICK 3 <sup>rd</sup> -order upwind difference
Viscous diffusion term	2 <sup>nd</sup> order central difference
Time marching	Backward difference, sub-iteration with virtual time
Coupling between pressure and velocity	Projection method solving Poisson's equation
Free surface capturing	VOF method
Turbulence model	K $\omega$ -SST
Body surface boundary condition	Logarithmic function as wall function

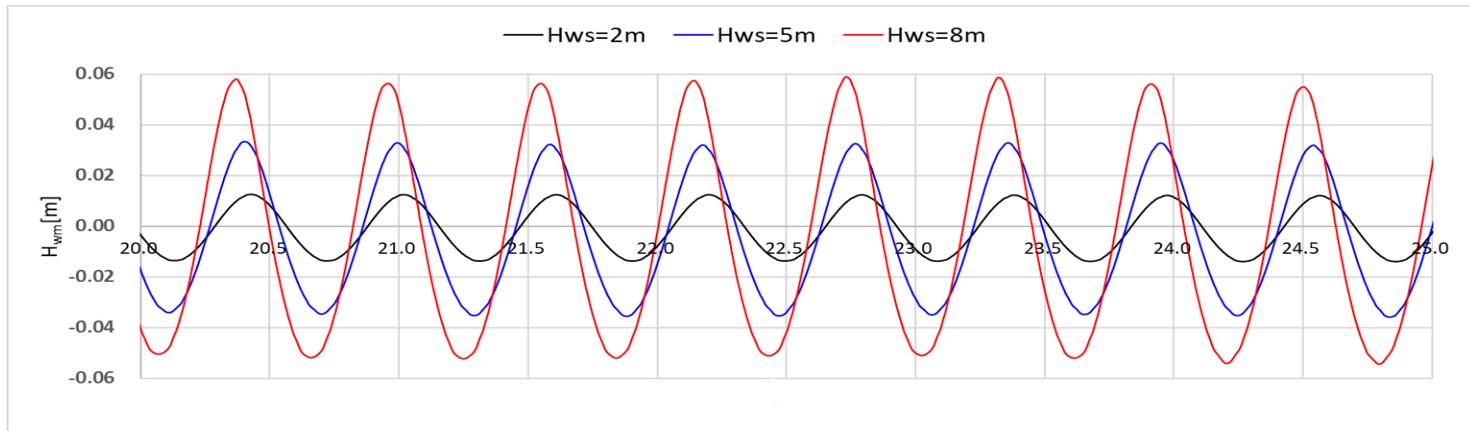
# Wave Calibration



wave profile for ( $\lambda/L_{pp} = 0.5$ ,  $H_{ws} = 8m$ )



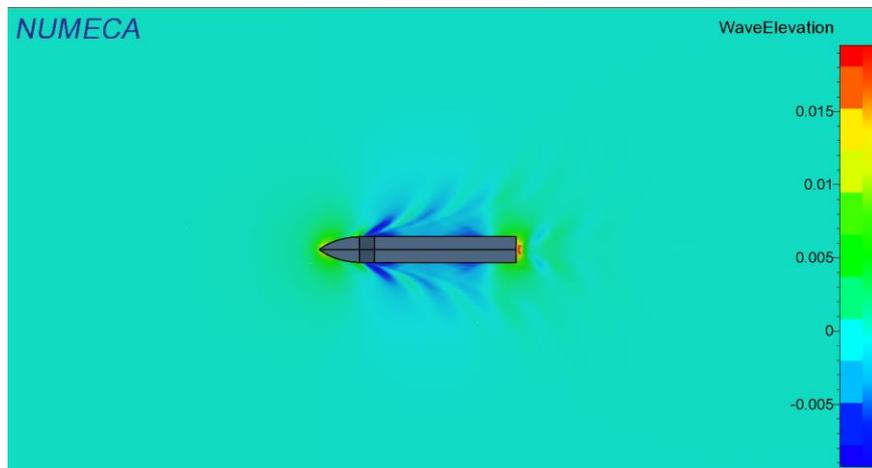
- ✓ Damping zone is created so that waves do not reflect and alter the added resistance calculation
- ✓ Mesh is good enough for modeling the waves properly



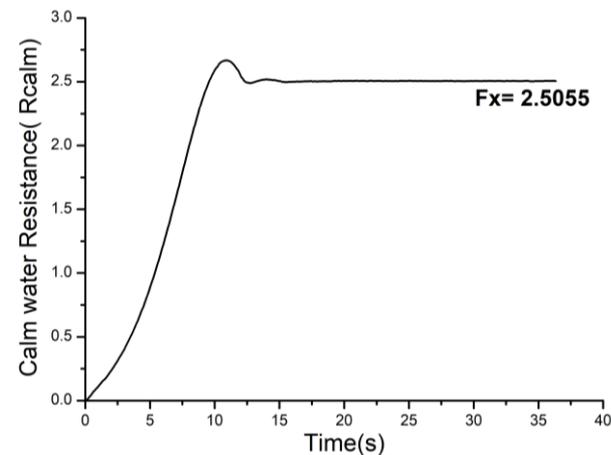
Simulated wave elevation for model wave height



## ➤ Calm water Resistance results

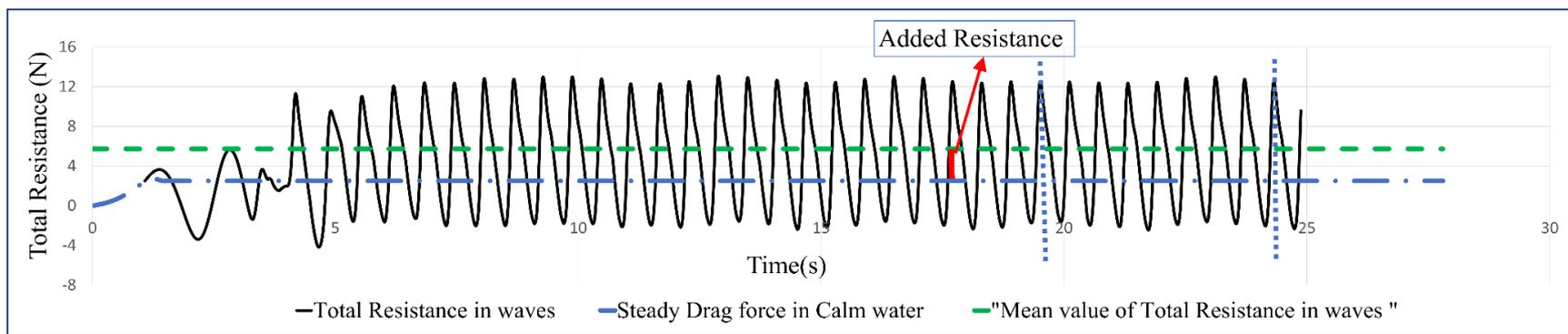


kelvin wave pattern on free surface



Convergence history of resistance in calm water ( $Fn = 0.2$ )

## ➤ Added Resistance results in Waves



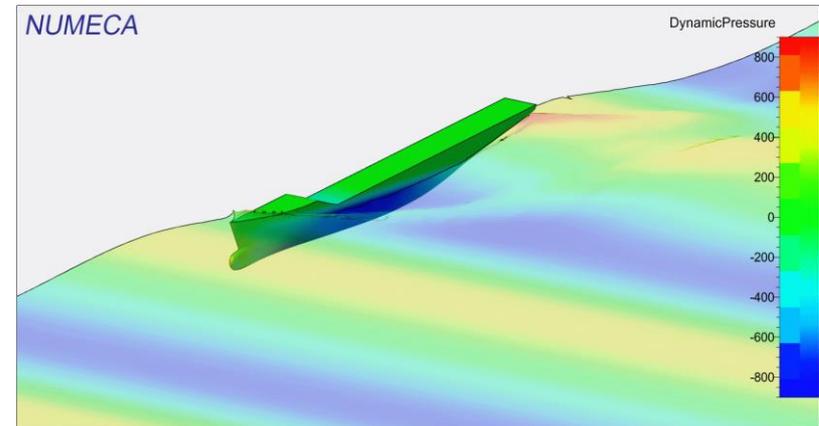
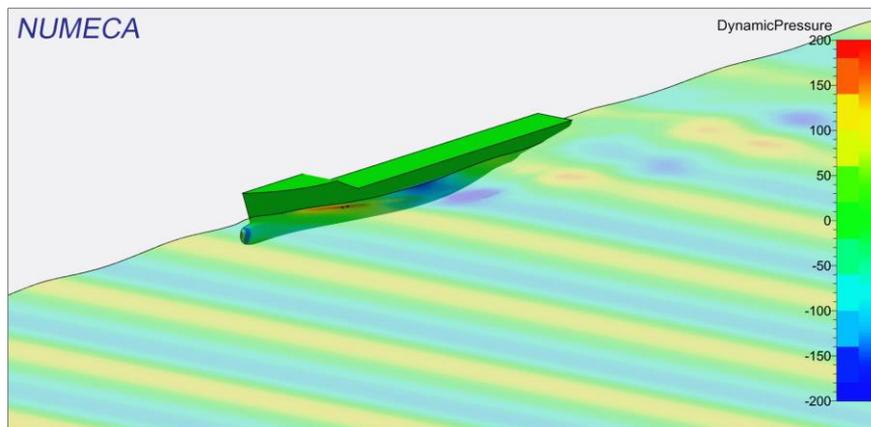
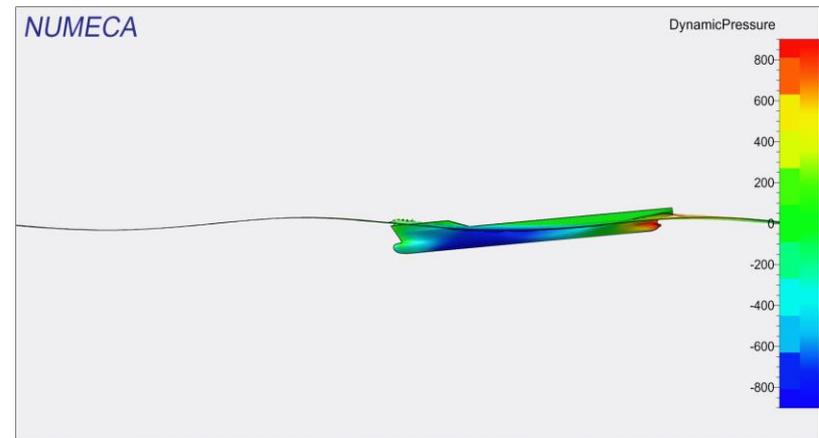
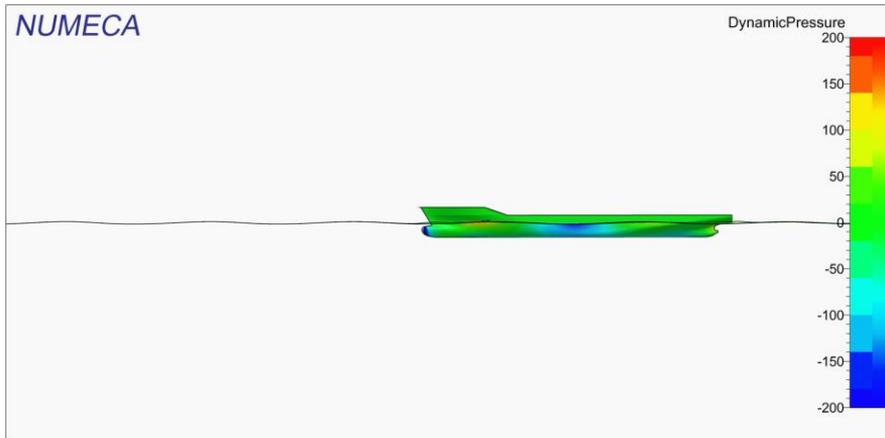
Average of minimum 7 oscillations of converged solution for all the cases i.e., two vertical blue color dotted lines

# Dynamic Pressure contours in short and long waves with different wave heights

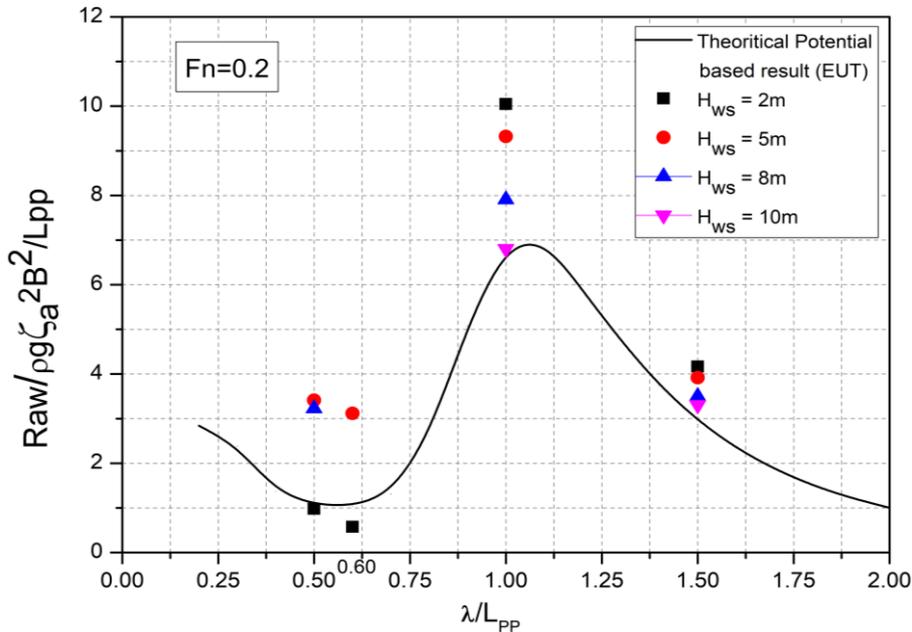


**Fn=0.20, Hws=2m,  $\lambda/Lpp=0.5$**

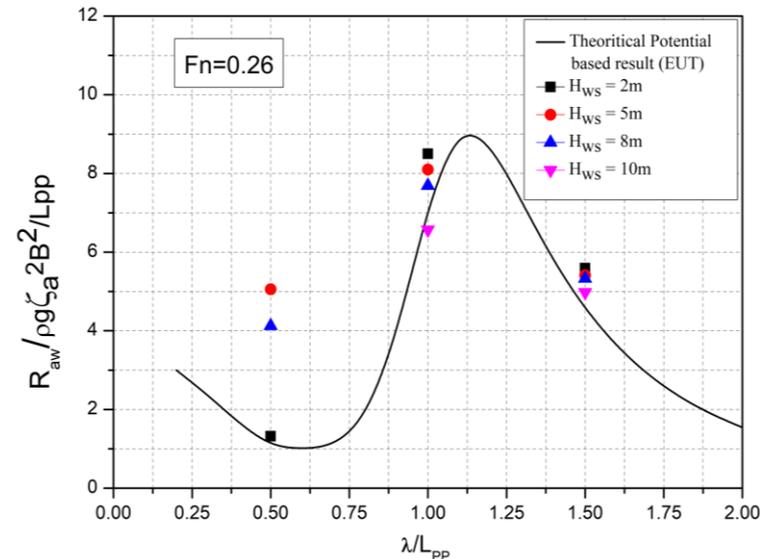
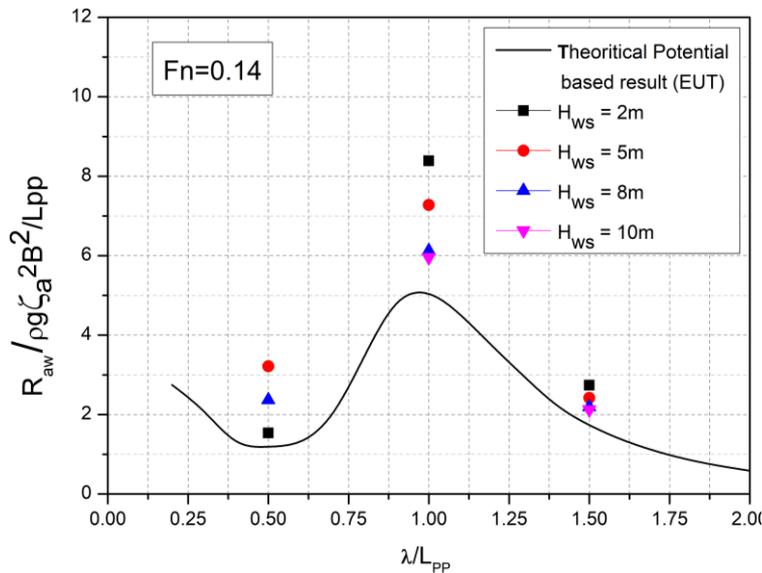
**Fn=0.20, Hws=10m,  $\lambda/Lpp=1.5$**



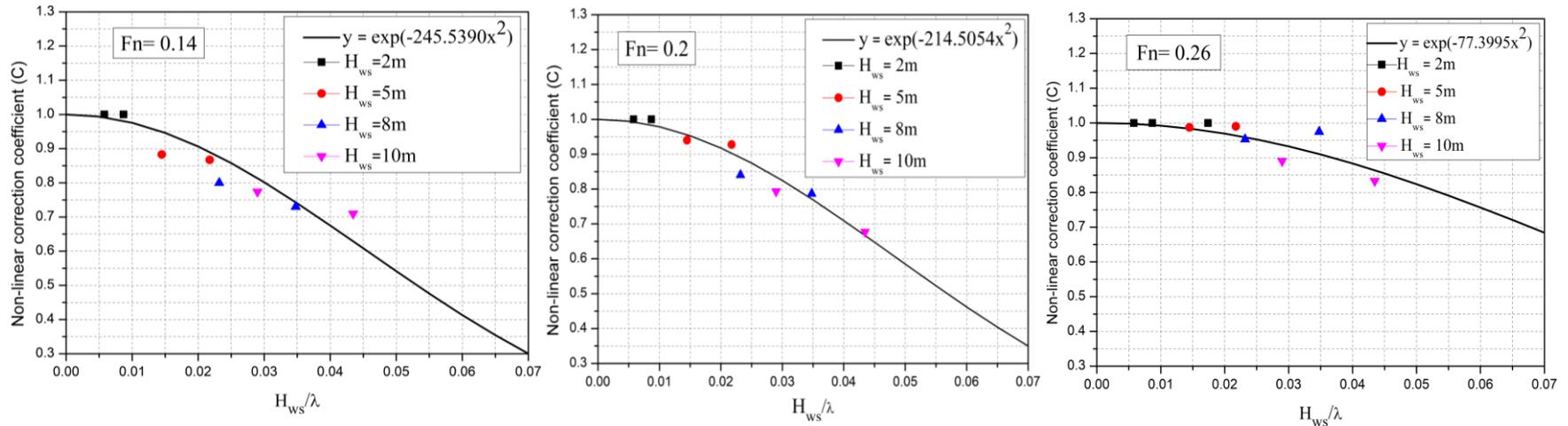
# Comparison of theoretical and CFD calculation results



- Non-linear effects can be seen: At  $\lambda / L_{pp} \geq 1.0$ , the added resistance coefficient decreases as the wave height increases,
- Similar trends were seen in other  $Fn$ 's.



# Approximation curve of non-linear effect correction coefficient

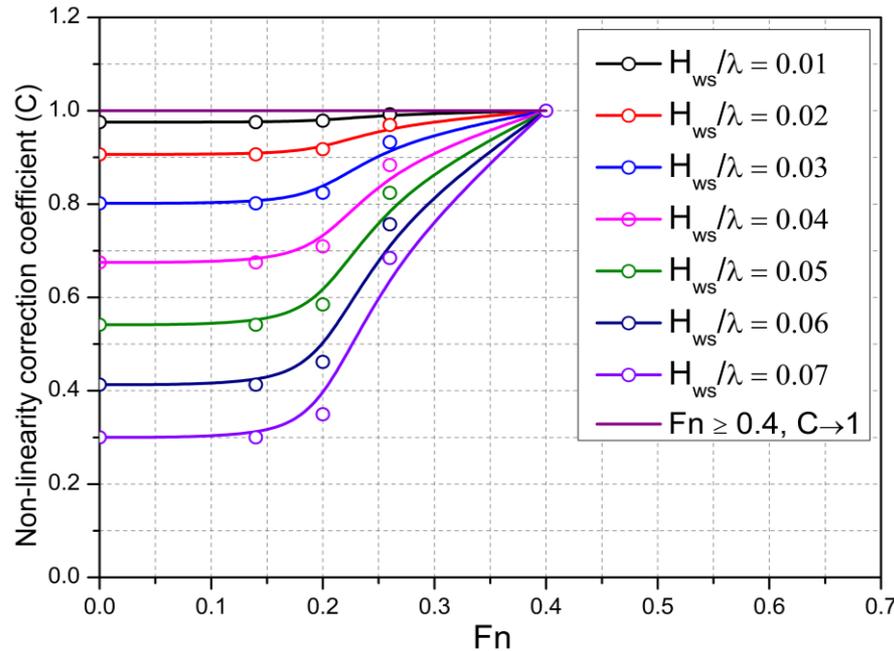


- In order to qualitatively see the non-linearity due to wave height (wave degree), the ratio with the resistance increase coefficient of other wave heights was obtained based on the wave resistance increase coefficient at  $H_{ws} = 2$  m at each Fn., and arranged by wave degree  $H_{ws}/\lambda$

$$H_{ws}(\omega, H_{ws}) = H_{ws}(\omega, 2) C(s, v)$$

$$C(s, V) = \frac{R_{WN}(H, T, V)}{H_{ws}^2 (R_{WN}(\alpha, T, V) / \alpha^2)}$$

# Relationship between non-linear effect correction coefficient and ship speed



- ✓ The target ship KCS is asymptotic to 1 at  $Fn = 0.4$  because it is not expected to operate at speeds higher than  $Fn = 0.4$ .
- ✓ The coefficient  $C$  can be expressed as a function of the wave steepness ( $H_{ws}/\lambda$ ) and the ship speed ( $Fn$ )

## Non-linearity correction coefficient

1)  $s \leq 0.13$

$$C(s, V) = C(s, F_n) = \begin{cases} 1 - (1 - C_0) \exp\{\alpha(F_n - 0.14)^4\} & , F_n \geq 0.14 \\ C_0 & , F_n < 0.14 \end{cases}$$

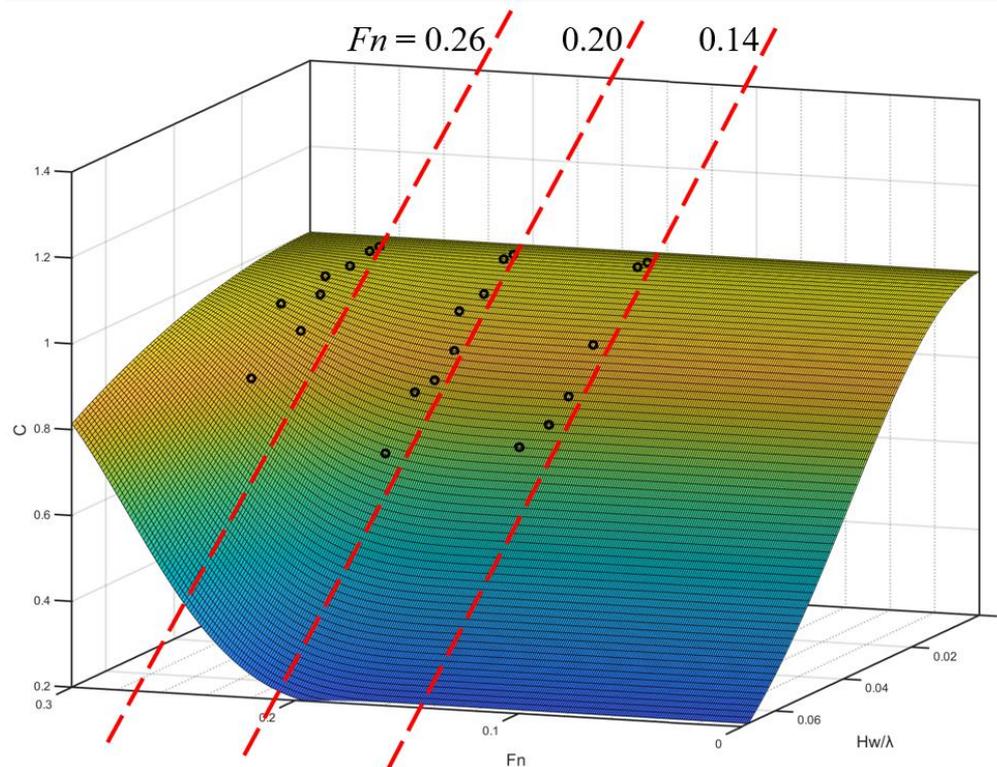
2)  $s > 0.13$

$$C(s, V) = C(s, F_n) = C_0$$

$$\alpha = 10^5 (1.7385s^2 + 0.0273s - 0.0330)$$

$$C_0 = \exp(-245.5390s^2)$$

# 3D plot of correction coefficient vs CFD data



- ✓ Execution of non-linearity correction coefficient empirical formula, in which cut sections in the 3D-plot which shows the approximation curves of non-linear effect correction coefficient.
- ✓ The formula is able to track the impact of the variation of different wave heights on added resistance in actual sea conditions and can deal with different speeds and wave steepness.

# Average Added wave resistance in long crested irregular waves



- < spec > Spectral method based on the linear assumption
- < pdf\_L > Pdf method based on the linear assumption
- < pdf\_NL > Pdf method considering wave height non-linearity

$$\langle \text{spec} \rangle \quad R_{AW}(H_{1/3}, T_{01}) = 2 \int_0^{\infty} \frac{R_{WU}(\omega, V)}{\zeta_a^2} S(\omega | H_{1/3}, T_{01}) d\omega$$

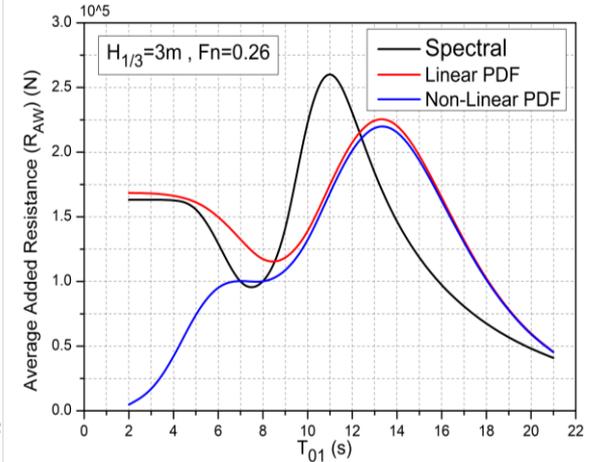
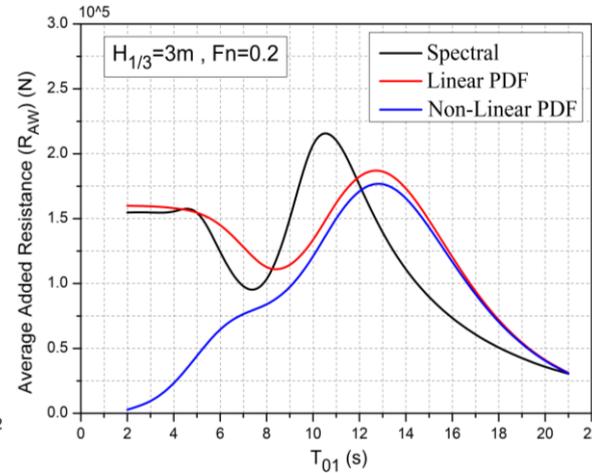
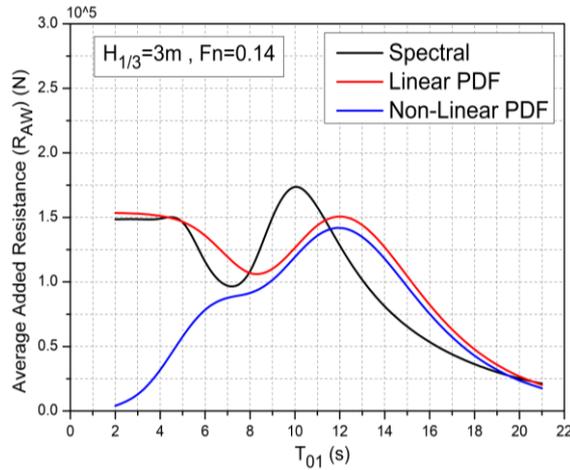
$$\langle \text{pdf}_L \rangle \quad R_{AW}(H_{1/3}, T_{01}) = \int_0^{\infty} \int_0^{\infty} \zeta_a^2 R_{WU}(\omega, V) \mathbf{p}(\mathbf{H}, \mathbf{T} | H_{1/3}, T_{01}) dT dH$$

$$\langle \text{pdf}_{NL} \rangle \quad R_{AW}(H_{1/3}, T_{01}) = \int_0^{\infty} \int_0^{\infty} R_{WN}(H, T, V) \mathbf{p}(\mathbf{H}, \mathbf{T} | H_{1/3}, T_{01}) dT dH$$

- ✓ The JONSWAP wave energy spectrum approximated by  $H_{1/3}$  and  $T_{01}$  is used to calculate the added resistance in head waves at different sea states where  $H_{1/3}$  is the significant wave height,  $T_{01}$  is mean wave period and  $\gamma = 3.3$ .



# Comparison of average resistance increase in irregular waves



➤ Spectral and linear PDF



Differ in peak positions, but they are almost identical in small regions where  $T_{01} \leq 5s$  and qualitatively the same in the large regions.

➤ Non-linear PDF



Significantly different in small regions where  $T_{01} \leq 5s$ . The average added wave resistance is much smaller than the linear calculation.

➤ The difference in peaks in spectral and PDF method.

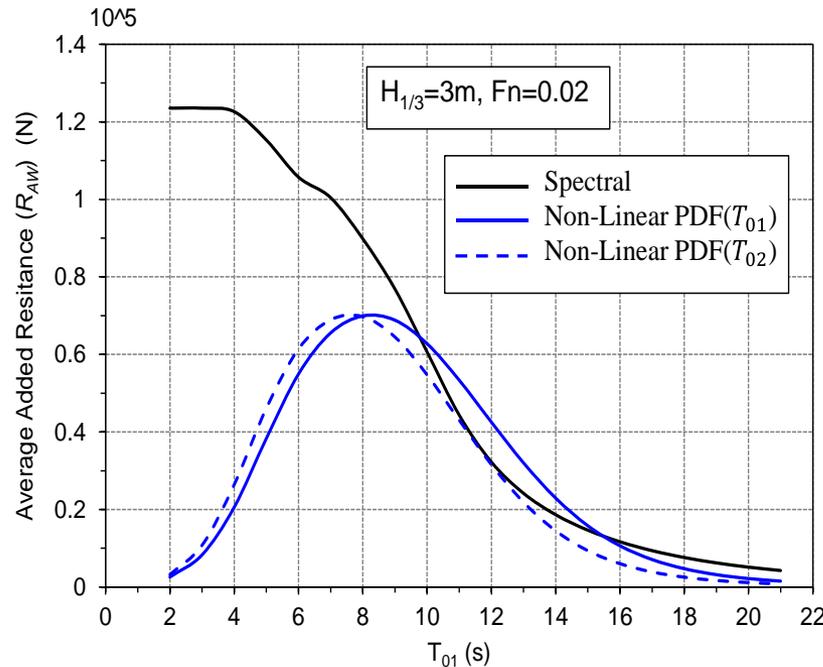


1. Difference between the zero-up cross mean wave period ( $T_{02}$ ), energy mean wave period ( $T_{01}$ )  
Non-linear PDF method      wave Pdf function
2. Effect of encounter wave frequency

# Comparison of average resistance increase in irregular waves

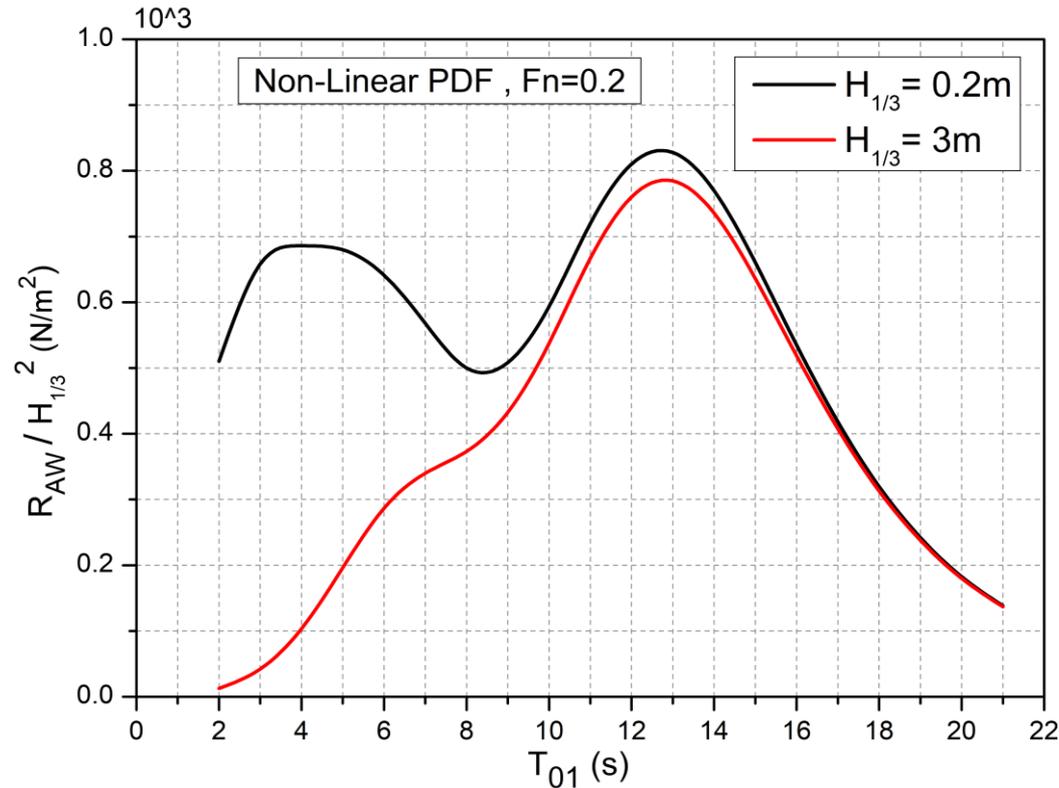


- ◆ Comparison of average added resistances in irregular waves by a spectral method and nonlinear pdf method at  $Fn = 0.02$ .



- No influence of the encounter wave frequency at  $Fn = 0.02$
- Tail shape of the non-linear PDF method is similar to that of the spectral method without ship forward speed, and the peak position of the non-linear PDF method shifts to lower period by changing the base period from  $T_{01}$  to  $T_{02}$ .

# Comparison of added wave resistance with given wave height by Pdf\_NL



- Non-linear probability density function method yields the same results for the linear calculation on smaller wave heights (i.e., when the wave steepness is smaller). This shows that the probability density function method is also valid for calculations in the linear domain.
- Similar trends are seen at different Froude numbers like ( $Fn = 0.14, 0.26$ ).



- The proposed method enables to apply conventional results of regular wave conditions for irregular wave conditions.
- Since the conventional spectral method is not applicable for non-linear responses as it is completely based on linear hydrodynamic theory, in this sense, the proposed method is **novel**.
- Non-linearity is evident in regions where  $T_{01} \leq 5s$  and the average added wave resistance is smaller than linear calculations. The tendency is the same even if the speed of the ship is different, and the method of calculating the proposal is reasonable. This demonstrates the validity of the probability density function method if the spectrum method is positive.
- The non-linear PDF method gives a nonlinearity-corrected average added wave resistance according to the region of mean wave period even at different ship speeds. In the linear domain, it gives almost the same results as the linear calculation, so there is no inconsistency in the calculation methods.



- Resolve the difference in peaks in the spectral and PDF method.
- Validate the Non-linear PDF method with CFD numerical simulations in long crested irregular waves.
- Include the directional spectrum of the waves and predict the average added wave resistance in short-crested irregular waves.

# THANK YOU