ABSTRACT
Verification and validation methodology is presented for CFD simulation results from an already developed RANS CFD code applied for specified objectives, geometry, conditions, and available benchmark information. Concepts and definitions are provided for errors and uncertainties and verification and validation. The simulation error and uncertainty equations are derived with modeling and numerical errors being additive and modeling and numerical uncertainties combining by root-sum-square. The concepts, definitions, and equations are fundamental to the verification and validation methodologies.

Verification methodology extends previous work and puts it on a more firm foundation. Iterative and parameter convergence studies are conducted using multiple solutions with systematic parameter refinement to estimate numerical errors and/or uncertainties. Three convergence conditions are possible: (i) converging; (ii) oscillatory; and (iii) diverging. For condition (i), generalized Richardson extrapolation for J input parameters and accounting for the effects of the higher-order terms is used to estimate uncertainties and, when conditions permit, the numerical error itself. For condition (ii), the upper and lower bounds of the solution oscillation are used to estimate uncertainties. For condition (iii), errors and/or uncertainties can not be estimated.

Validation methodology follows Coleman and Stern published in 1997.
An example is provided for a RANS CFD code and results for steady flow for a cargo/container ship.

1. INTRODUCTION
Discussion and methodology for estimating errors and/or uncertainties in computational fluid dynamics (CFD) simulations has reached a certain level of maturity with increased attention and recent progress on common concepts and terminology (AIAA, 1998), advocacy and detailed methodology (Roache, 1998), and numerous case studies (e.g., Mehta, 1998). Progress has been accelerated in response to the urgent need for achieving consensus on concepts and terminology and useful methodology, as CFD is applied to increasingly complex geometry and physics and integrated into the engineering design process. Such consensus is required to realize the goals of simulation-based design and other uses of CFD such as simulating flows for which experiments are difficult (e.g., full-scale Reynolds numbers, hypersonic flows, off-design conditions). In spite of the progress and urgency, the various viewpoints have not converged and current methodology falls short of providing practical procedures and methodology for estimating errors and/or uncertainties in CFD simulations.
subsequent work on validation (Coleman and Stern, 1997) [hereafter referred to as C&S] thereby providing the framework for overall procedures and methodology. The philosophy is strongly influenced by experimental fluid dynamics (EFD) uncertainty analysis (Coleman and Steele, 1999), which has been standardized. Hopefully, CFD verification and validation procedures and methodology can reach a similar level of maturity and user variability can reach similar low levels, as for EFD.

The work is part of a larger program (Rood, 1996) for developing and implementing a strategy for verification and validation of Reynolds-averaged Navier-Stokes (RANS) ship hydrodynamics CFD codes. The program includes complementary CFD and EFD towing-tank investigations and considers errors and uncertainties in both the simulations and the data in assessing the success of the verification and validation efforts. The work also benefited from collaboration with the 21st and 22nd International Towing Tank Resistance Committees (ITTC, 1996 and 1999).

The focus is on verification and validation procedures and methodology for CFD simulation results from an already developed CFD code applied for specified objectives, geometry, conditions, and available benchmark information. The procedures and methodology were developed considering RANS CFD codes, but should be applicable to a fairly broad range of codes such as boundary-element methods and certain aspects of large-eddy and direct numerical simulations.

The present work differs in many respects from recent literature. The presentation is relatively succinct with intention for use for practical applications for which numerical errors and/or uncertainties can not be considered negligible or overlooked. The approach includes both the situations of estimating errors with relatively small uncertainties and of estimating uncertainties only. The concepts and definitions are consistent with those well established for EFD uncertainty assessment. The use of quantitative estimates for errors and the use of uncertainties for those estimates constitute a new approach in verification and validation. The use of Richardson extrapolation (RE) for verification is not new; however, the present generalizations for J input parameters and accounting for the effects of the higher-order terms also constitute a new approach.

The definitions of errors and uncertainties and verification and validation that are used in any approach need to be clearly stated. Table 1 summarizes the present definitions along with those given by the AIAA (1998) and Roache (1998) for comparison. The present and Roache (1998) definitions for errors and uncertainties are consistent with those used for EFD. The AIAA (1998) definitions are from an information theory perspective and differ from those used in EFD, but are not contradictory to the present definitions. The present definitions for verification and validation are closely tied to the present definitions of errors and uncertainties and equations derived for simulation errors and uncertainties. The Roache

<table>
<thead>
<tr>
<th>Errors</th>
<th>Present and Roache (1998)</th>
<th>Error $\delta$ is the difference between a simulation value or an experimental value and the truth</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIAA (1998)</td>
<td>A recognizable deficiency in any phase or activity of modeling and simulation that is not due to lack of knowledge</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncertainties</th>
<th>Present and Roache (1998)</th>
<th>An error $U$ is an estimate of an error such that the interval $\pm U$ contains the true value of $\delta$ 95 times out of 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIAA (1998)</td>
<td>A potential deficiency in any phase or activity of the modeling process that is due to lack of knowledge</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Verification</th>
<th>Present</th>
<th>Verification is defined as a process for assessing numerical uncertainty $U_{SN}$ and, when conditions permit, numerical error $\delta_{SN}$ itself</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roache (1998)</td>
<td>Solving the equations right/mathematics</td>
<td></td>
</tr>
<tr>
<td>AIAA (1998)</td>
<td>The process of determining that a model implementation accurately represents the developer’s conceptual description of the model and the solution to the model</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Validation</th>
<th>Present</th>
<th>Validation is defined as a process for assessing modeling uncertainty $U_{SM}$ by using benchmark experimental data and, when conditions permit, estimating the sign and magnitude of the modeling error $\delta_{SM}$ itself</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roache (1998)</td>
<td>Solving the right equations/science/engineering</td>
<td></td>
</tr>
<tr>
<td>AIAA (1998)</td>
<td>The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Definitions of errors and uncertainties and verification and validation
This paper is organized as follows. Section 2 describes the overall verification and validation procedures. Section 3.1 gives concepts and definitions for errors and uncertainties and verification and validation and the simulation error and uncertainty equations are derived with modeling and numerical errors being additive and modeling and numerical uncertainties combining by root-sum-square. The concepts, definitions, and equations are fundamental to the verification and validation methodologies. Sections 3.1 and 3.2 provide methodology for verification and validation. To aid in application of the methodology, Section 4 provides an example for results from a RANS CFD code for steady flow for a cargo/container ship. Section 5 mentions conclusions and recommendations. Appendices A, B, and C of Stern et al. (1999) provide a detailed description of the simulation error equation, a detailed description of generalized RE, and use of an analytical solution benchmark.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_k$</td>
<td>correction factor</td>
</tr>
<tr>
<td>$D$</td>
<td>benchmark data</td>
</tr>
<tr>
<td>$E$</td>
<td>comparison error</td>
</tr>
<tr>
<td>$P$</td>
<td>order of accuracy</td>
</tr>
<tr>
<td>$r$</td>
<td>parameter refinement ratio</td>
</tr>
<tr>
<td>$R$</td>
<td>solution change (convergence) ratio</td>
</tr>
<tr>
<td>$S$</td>
<td>simulation result</td>
</tr>
<tr>
<td>$S_C$</td>
<td>simulation result corrected for numerical errors</td>
</tr>
<tr>
<td>$T$</td>
<td>truth</td>
</tr>
<tr>
<td>$U$</td>
<td>uncertainty estimate</td>
</tr>
<tr>
<td>$U_D$</td>
<td>uncertainty in data</td>
</tr>
<tr>
<td>$U_E$</td>
<td>uncertainty in comparison error</td>
</tr>
<tr>
<td>$U_I$</td>
<td>iteration uncertainty</td>
</tr>
<tr>
<td>$U_{req}$</td>
<td>programmatic validation requirement</td>
</tr>
<tr>
<td>$U_S$</td>
<td>simulation uncertainty</td>
</tr>
<tr>
<td>$U_{SM}$</td>
<td>simulation modeling uncertainty</td>
</tr>
<tr>
<td>$U_{SMA}$</td>
<td>simulation modeling assumption uncertainty</td>
</tr>
<tr>
<td>$U_{SN}$</td>
<td>simulation numerical uncertainty</td>
</tr>
<tr>
<td>$U_{SPD}$</td>
<td>simulation previous data uncertainty</td>
</tr>
<tr>
<td>$U_V$</td>
<td>validation uncertainty</td>
</tr>
<tr>
<td>$\delta$</td>
<td>error</td>
</tr>
<tr>
<td>$\delta_G$</td>
<td>error contribution from grid size</td>
</tr>
<tr>
<td>$\delta_I$</td>
<td>error contribution from iteration number</td>
</tr>
<tr>
<td>$\delta_P$</td>
<td>error contribution from other parameters</td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>simulation error</td>
</tr>
<tr>
<td>$\delta_{SN}$</td>
<td>simulation numerical error</td>
</tr>
</tbody>
</table>

2. VERIFICATION AND VALIDATION PROCEDURES

The overall CFD verification and validation procedures can be conveniently grouped in four consecutive steps: (1) preparation; (2) verification; (3) validation; and (4) documentation.

**Preparation.** The 1st step is preparation, which involves selection of the CFD code and specification of objectives, geometry, conditions, and available benchmark information. The objectives might be prediction of certain variables at certain levels of validation (e.g., programmatic validation requirements $U_{req}$). The variables can either be integral (e.g., resistance) or point (e.g., mean velocities and turbulent Reynolds stresses) values and the levels of validation may be different for each variable.

**Verification.** The 2nd step is verification, which is defined as a process for assessing simulation numerical uncertainty $U_{SN}$ and, when conditions permit, numerical error $\delta_{SN}$ itself. Iterative and parameter convergence studies are conducted using multiple solutions with systematic parameter refinement to estimate numerical uncertainties. Three convergence conditions are possible: (i) converging; (ii) oscillatory; and (iii) diverging. For condition (i), generalized RE for $J$ input parameters and accounting for the effects of the higher-order terms are used to estimate uncertainties and, when conditions permit, numerical error itself. For condition (ii), the upper and lower bounds of the solution oscillation are used to estimate uncertainties. For condition (iii), errors and/or uncertainties cannot be estimated.

**Validation.** The 3rd step is validation, which is defined as a process for assessing simulation modeling uncertainty $U_{SM}$ by using benchmark experimental data and, when conditions permit, estimating the sign and magnitude of the modeling error $\delta_{SM}$ itself. The comparison error $E$ (which is the difference between the data value $D$ and simulation value $S$) and the validation uncertainty $U_V$, are used in this process. $U_V$ is the combination of all uncertainties in the data and the portion of the simulation uncertainties that can be estimated (i.e., all but those due to modeling assumptions). Validation is achieved at the $U_V$ level for $|E| < U_V$. If $|E| > U_V$, then $E$ can be used to estimate the sign and magnitude of the modeling error.

**Documentation.** The 4th step is documentation, which is detailed presentation of the CFD code (equations, initial and boundary conditions, modeling, and numerical methods), objectives, geometry, conditions, verification, validation, and analysis.
3. VERIFICATION AND VALIDATION METHODOLOGY

Verification and validation methodology is presented for CFD simulation results from an already developed CFD code applied for specified objectives, geometry, conditions, and available benchmark information. First, in Section 3.1, concepts and definitions are provided for errors and uncertainties and verification and validation. The simulation error and uncertainty equations are derived with modeling and numerical errors being additive and modeling and numerical uncertainties combining by root-sum-square. The concepts, definitions, and equations are fundamental to the verification and validation methodologies.

Verification methodology is presented in Section 3.2. The methodology extends previous work (Stern et al., 1996) and puts it on a more firm foundation. Iterative and parameter convergence studies are conducted using multiple solutions with systematic parameter refinement to estimate numerical errors and/or uncertainties. Three convergence conditions are possible: (i) converging; (ii) oscillatory; and (iii) diverging. For condition (i), generalized RE for J input parameters and accounting for the effects of the higher-order terms are used to estimate uncertainties and, when conditions permit, numerical error itself. For condition (ii), the upper and lower bounds of the solution oscillation are used to estimate uncertainties. For condition (iii), errors and/or uncertainties can not be estimated.

Validation methodology is presented in Section 3.3. The methodology follows C&S.

3.1. Concepts and Definitions

Accuracy indicates the closeness of agreement between a simulation or experimental value of a quantity and its true value. Error \( \delta \) is the difference between a simulation value or an experimental value and the truth. Accuracy increases as error approaches zero. The true values of simulation/experimental quantities are rarely known. Thus, errors must be estimated. An uncertainty \( U \) is an estimate of an error such that the interval \( \pm U \) contains the true value of \( \delta \) 95 times out of 100. An uncertainty interval thus indicates the range of likely magnitudes of \( \delta \) but no information about its sign. In some cases, sufficient information is available to make an estimate of the sign and magnitude of \( \delta \) itself, and then the uncertainty considered is the uncertainty in the \( \delta \) estimate. When \( \delta \) is estimated, it can be used to obtain a corrected value of the variable of interest. This approach is used under certain conditions of the methodology, which follows.

Sources of errors and/or uncertainties in results from simulations can be divided into two distinct sources -- modeling and numerical. Modeling errors and/or uncertainties are due to assumptions and approximations in the mathematical representation of the physical problem (such as geometry, mathematical equation, coordinate transformation, boundary conditions, turbulence models, etc.) and incorporation of previous data (such as fluid properties) into the model. Numerical errors and/or uncertainties are due to numerical solution of the mathematical equations (such as discretization, artificial dissipation, incomplete iterative and grid convergence, lack of conservation of mass, momentum, and energy, internal and external boundary non-continuity, computer round-off, etc.).

The simulation error \( \delta_S \) is defined as the difference between a simulation result \( S \) and the truth \( T \). In considering the development and execution of a CFD code, it can be postulated that \( \delta_S \) is comprised of the addition of modeling and numerical errors

\[
\delta_S = S - T = \delta_{SM} + \delta_{SN}
\]  

(1)

A derivation of the simulation error equation (1) is provided in Appendix A of Stern et al. (1999). Modeling errors arise when assumptions are made in the mathematical model, whereas numerical errors arise when the continuous modeled equations are discretized and solved numerically.

The uncertainty equation corresponding to the error equation (1) is

\[
U_S^2 = U_{SM}^2 + U_{SN}^2
\]  

(2)

where \( U_S \) is the simulation uncertainty and \( U_{SM} \) and \( U_{SN} \) are the simulation modeling and numerical uncertainties.

Verification is defined as a process for assessing numerical uncertainty \( U_{SN} \) and, when conditions permit, numerical error \( \delta_{SN} \) itself. Use of iterative and parameter convergence studies are advocated using multiple solutions with systematic parameter refinement thereby providing estimates for \( \delta_{SN} \) and/or \( U_{SN} \). Verification methodology is described in Section 3.2. Analytical benchmarks can be used for the truth and are useful in code development and confirmation of verification procedures and methodology, but can not be used for validation and are restricted to simple equations. Results from the use of analytical benchmarks are provided in Appendix C of Stern et al. (1999).

Validation is defined as a process for assessing modeling uncertainty \( U_{SM} \) by using benchmark experimental data and, when conditions permit, estimating the sign and magnitude of the modeling error \( \delta_{SM} \) itself. The comparison error \( E \) (which is the difference between the data value \( D \) and simulation value \( S \)) and the validation uncertainty \( U_V \), are used in this process. \( U_V \) is the combination of all uncertainties in the data and the portion of the simulation uncertainties that can be estimated (i.e., all but those due to modeling assumptions). Validation is achieved at the \( U_V \) level for \( |E| < U_V \). If \( |E| > U_V \), then \( E \) can be used to estimate the sign and magnitude of the modeling error. Validation methodology is described in Section 3.3.
3.2. Verification

For many CFD codes, the most important numerical errors and uncertainties are due to iterative solution methods, grid size (and distribution), time step, and other parameters (e.g., artificial dissipation). The errors and uncertainties are highly dependent on the specific application (geometry and conditions) and selection of CFD code inputs such as iteration number \( I \), grid-size \( G \) (or panel size), time-step \( T \), and other parameters \( P \).

The errors due to selection of input parameters are decomposed into error contributions from iteration number \( \delta_i \), grid size \( \delta_g \), time step \( \delta_t \), and other parameters \( \delta_p \), which gives the following expressions for the simulation numerical error and uncertainty

\[
\delta_{SN} = \delta_i + \delta_g + \delta_t + \delta_p = \delta_i + \sum_{j=1}^{n} \delta_j \quad (3)
\]

\[
U^2_{SN} = U^2_i + U^2_g + U^2_t + U^2_p = U^2_i + \sum_{j=1}^{n} U^2_j \quad (4)
\]

Substituting equation (3) into equation (1), one can obtain

\[
S = S_c + \delta_{SN} = S_c + \delta_i + \delta_g + \delta_t + \delta_p = S_c + \delta_i + \sum_{j=1}^{n} \delta_j \quad (5)
\]

where

\[
S_c = T + \delta_{SM} = S - \delta_{SN} = S - (\delta_i + \sum_{j=1}^{n} \delta_j) \quad (6)
\]

is the simulation corrected for numerical errors.

3.2.1. Convergence Studies

Iterative and parameter convergence studies are conducted using multiple \((m)\) solutions and systematic parameter refinement by varying each input parameter while holding all other parameters constant. Equation (5) can be written for the \( m \)th solution for the \( k \)th parameter as

\[
S_k = S_c + \delta_{Sw} + \delta_{St} + \sum_{j=1}^{n} \delta_{Sj} \quad (7)
\]

Iterative convergence must be assessed and \( S_k \) corrected for iterative errors prior to evaluation of parameter convergence since the level of iterative convergence may not be the same for all \( m \) solutions used in the parameter convergence studies. Methods for estimating \( \delta_i \) and/or \( U_i \) are described in Section 3.2.2. With \( \delta_{Sw} \) evaluated, \( S_k \) is corrected for iterative errors as

\[
\hat{S}_k = S_k - \delta_{Sw} = S_c + \delta_k + \sum_{j=1}^{n} \delta_{Sj} \quad (8)
\]

\( \hat{S}_k \) can be calculated for both integral (e.g., resistance coefficients) and point (e.g., surface pressure, wall-shear stress, and velocity) variables. If point variables are not at the same location (e.g., when grid doubling is not used) or time, interpolation to a common location and time is required. Roache (1998) discusses methods for evaluating interpolation errors. \( \hat{S}_k \) can be presented as an absolute quantity (i.e., non-normalized) or normalized with the solution as a percentage change; however, if the solution value is small, a more appropriate normalization may be the range of the solution.

Convergence studies require a minimum of \( m=3 \) solutions to evaluate convergence with respect to input parameter and to estimate error and/or uncertainty. Note that \( m=2 \) is inadequate, as it only indicates sensitivity and not convergence, and that \( m>3 \) may be required. Consider the situation for 3 solutions corresponding to fine \( \hat{S}_k \), medium \( \hat{S}_k \), and coarse \( \hat{S}_k \) values for the \( k \)th input parameter. Solutions changes \( \varepsilon \) for medium-fine and coarse-medium solutions and their ratio \( R_k \) are defined by

\[
\varepsilon_{21} = \hat{S}_{k2} - \hat{S}_{k1}
\]

\[
\varepsilon_{21}/\varepsilon_{23} = \hat{S}_{k2} - \hat{S}_{k3}
\]

Three convergence conditions are possible:

(i) Converging condition: \( 0 < R_k < 1 \)

(ii) Oscillatory condition: \( R_k < 0 \)

(iii) Diverging condition: \( R_k > 1 \)

For the converging condition (i), generalized RE is used to estimate \( U_k \) and/or \( \delta_i \) itself. In the methodology presented here, RE is generalized for \( J \) input parameters and accounting for the effects of the higher-order terms. Methods for estimating errors and/or uncertainties for the converging condition (i) are described in Section 3.2.3.

For oscillatory solutions, the oscillatory condition (ii), may be erroneously identified as condition (i) or (iii). This is apparent if one considers evaluating convergence condition from three points on a sinusoidal curve (Coleman et al., 1999). Depending on where the three points fall on the curve, the condition could be incorrectly diagnosed as either converging or diverging. Methods for estimating uncertainties \( U_k \) for the oscillatory condition (ii) are described in Section 3.2.4.

For the diverging condition (iii), errors and/or uncertainties cannot be estimated. Additional discussion is given in Section 3.2.5.

3.2.2. Iterative Convergence

Iterative convergence must be assessed and simulation results \( S_k \) corrected for iterative errors prior to evaluation of parameter convergence since the level of iterative convergence may not be the same for all \( m \) solutions used in the parameter convergence studies. Methods for estimating \( \delta_i \) and/or \( U_i \) are described in this section. The methods are applicable to both integral and point variables. Further work is needed on assessing iterative errors and their role in parameter convergence studies and for assessing iterative errors and/or uncertainties for unsteady flows.
Typical CFD solution techniques for obtaining steady state solutions involve beginning with an initial guess and performing time marching or iteration until a steady state solution is achieved. For time-accurate calculations using implicit methods, convergence of the solution is required at each time step. Care must be exercised in evaluating iterative convergence based solely on the change in solution from one iteration to the next. Small time steps and/or relaxation parameters can result in small solution changes while iterative error can be large (Ferziger and Peric, 1997). One approach that removes this problem is to use the residual imbalance of the steady state equations as a measure of convergence, since the iterative error satisfies the same equation as this residual imbalance.

The number of order magnitude drop and final level of solution residual can be used to determine stopping criteria for iterative solution techniques. Iterative convergence to machine zero is desirable, but for complex geometry and conditions it is often not possible. Three or four orders of magnitude drop in solution residual to a level of $10^{-4}$ is more likely for these cases. Methods for estimation of iterative errors and/or uncertainties can be based on graphical, as discussed below, or theoretical approaches and are dependent on the type of iterative solution techniques. Iterative convergence to machine zero is often impractical. Alternatively, solutions without interpolation; however, parameter doubling solutions without interpolation; however, parameter doubling also provides sufficient parameter refinement and at least enables prolongation of the coarse-parameter solutions without interpolation; however, parameter doubling is often impractical. Alternatively, $r_k = \sqrt{2}$ is suggested which also provides sufficient parameter refinement and at least enables prolongation of the coarse-parameter solution as an initial guess for the fine-parameter solution.

The terms in the Taylor series expansion provide estimates for $S_{i_k}$, the accuracy of the estimates depends on how many terms are retained in the Taylor series and the magnitude (importance) of the higher-order terms. For sufficiently small $\Delta x_k$, the solutions are in the asymptotic range such that higher–order terms are negligible. However, achieving the asymptotic range for practical geometry and conditions is usually not possible; therefore, methods are needed to account for effects of higher-order terms for practical application of RE.

An increase in the amplitude of the solution envelope as the iteration number increases indicates that the solution is divergent.

Estimates of the iterative error based on theoretical approaches are presented in Ferziger and Peric (1997) and involve estimation of the principal eigenvalue of the iteration matrix. The approach is relatively straightforward when the eigenvalue is real and the solution is convergent. For cases in which the principal eigenvalue is complex and the solution is oscillatory or mixed, the estimation is not as straightforward and additional assumptions are required.

### 3.2.3. Convergent Condition: Generalized Richardson Extrapolation

For the converging condition (i), generalized RE is used to estimate $U_i$ and/or $\delta_{i_k}$. RE is generalized for $J$ input parameters and accounting for the effects of the higher-order terms, as summarized in the following. Appendix B of Stern et al. (1999) provides a more detailed description.

For each input parameter $\Delta x_k$, the $m$ solutions $\hat{S}_{i_k}$ are used to evaluate terms in a Taylor series expansion of $\hat{S}_{i_k}$ about $S_{C_k}$ in powers of input parameter $\Delta x_k$ and with uniform parameter refinement ratio $r_k = \Delta x_2/\Delta x_1 = \Delta x_3/\Delta x_2$. Parameter refinement ratio $r_k = 2$ is ideal as it provides sufficient parameter refinement and enables use of the coarser-parameter solutions as initial guesses for the finer-parameter solutions without interpolation; however, parameter doubling is often impractical. Alternatively, $r_k = \sqrt{2}$ is suggested which also provides sufficient parameter refinement and at least enables prolongation of the coarse-parameter solution as an initial guess for the fine-parameter solution.

The terms in the Taylor series expansion provide estimates for $\delta_{i_k}$. The accuracy of the estimates depends on how many terms are retained in the Taylor series and the magnitude (importance) of the higher-order terms. For sufficiently small $\Delta x_k$, the solutions are in the asymptotic range such that higher–order terms are negligible. However, achieving the asymptotic range for practical geometry and conditions is usually not possible; therefore, methods are needed to account for effects of higher-order terms for practical application of RE.

The number of terms that can be evaluated by RE depends on $m$. Moreover, $m$ must be odd since RE involves solution changes $\varepsilon$ (i.e, subtraction of equations), which eliminates the error terms due to other input parameters ($j \neq k$) and enables solution of the remaining equations for $\delta_{i_k}$ as a function of solution changes $\varepsilon$ and parameter refinement ratio $r$. For $m=3$, only the first-order term can be evaluated. Equations are obtained for $\delta_{i_k}$ and order-of-accuracy $p_k$. 

$$\delta_{i_k} = S - CF_{m}$$

$$U_i = |S - CF_{m}|$$

$$\delta_{i_k} = S - \frac{1}{2}(S_U + S_L)$$

$$U_i = \frac{1}{2}(S_U - S_L)$$

$$\delta_{i_k} = S - \frac{1}{2}(S_U + S_L)$$

$$U_i = \frac{1}{2}(S_U - S_L)$$
\[
\delta_{k_i} = \delta_{RE_{k_i}} = \frac{\varepsilon_{2k_i}}{r_{k_i}^p - 1} \quad (14)
\]

\[
p_k = \frac{\ln(\varepsilon_{2k_i})}{\ln(r_{k_i})} \quad (15)
\]

Estimation of \( n \) terms in the Taylor series expansion for RE requires \( m=2n+1 \) \((n=1,2,3,\ldots)\) solutions. However, \( m>3 \) is undesirable from a resources point of view.

Verification for analytical benchmarks (see Appendix C of Stern et al., 1999) for which the RE error estimates were confirmed through comparisons with the modified equations and analytical solution benchmarks (one-dimensional wave and two-dimensional Laplace equations) show that accurate estimation of order of accuracy \( p_k \) is most important and that a simple correction factor

\[
C_k = \frac{r_{k_i}^p - 1}{r_{k_i}^{p_k} - 1} \quad (16)
\]

can be used where \( p_k \) is the theoretical order of accuracy based on the modified equation. Note that \( C_k < 1 \) or \( C_k > 1 \) indicates that the first-order term overpredicts (higher-order terms net negative) or underpredicts (higher-order terms net positive) the error, respectively. Note that \( C_k \) as given by equation (16) may be fairly universal in that it only implicitly depends on geometry and conditions through its dependence on \( C_k(r_x,p_k) \).

Combining equations (14) and (16) provides an estimate for \( \delta_{k_i} \) accounting for the effects of the higher-order terms

\[
\delta_{k_i} = C_k \delta_{RE_{k_i}} = C_k \left( \frac{\varepsilon_{2k_i}}{r_{k_i}^p - 1} \right) \quad (17)
\]

with \( p_k \) given by equation (15). The estimate includes both sign and magnitude. Equation (17) is used to estimate \( U_{k_i} \) and/or \( \delta_{k_i} \) itself depending on how close the solutions are to the asymptotic range (i.e., how close \( C_k \) is to 1) and one's confidence in equation (17). There are many reasons for lack of confidence, especially for complex three-dimensional flows. Point variables invariably are not uniformly convergent, which is particularly evident near inflection points and zero crossings. Equations (16) and (17) need further testing both for additional analytical benchmarks and practical applications. Note that equation (17) differs significantly from the GCI proposed by Roache (1998) in that herein it is proposed that \( C_k = C_k(\varepsilon,r,p) \), whereas in the GCI \( C_k \) is a constant referred to as a factor of safety \( F_2 \) which equals 1.25 for careful grid studies and 3 for cases for which only two grids are used.

For \( C_k \) sufficiently less than or greater than 1 and lacking confidence, \( U_{k_i} \) is estimated, but not \( \delta_{k_i} \). Based on the analytical benchmark studies (Appendix C of Stern et al., 1999), it appears that equation (17) can be used to estimate the uncertainty by bounding the error by the sum of the absolute value of the corrected estimate from RE and the absolute value of the amount of the correction

\[
U_{k_i} = \left| C_k \delta_{RE_{k_i}} \right| + \left| \frac{1}{C_k} - 1 \right| \delta_{RE_{k_i}} \quad (18)
\]

For \( C_k \) sufficiently close to 1 and having confidence, both \( U_{k_i} \) and \( \delta_{k_i} \) are estimated. Equation (17) is used to estimate the error \( \delta_{k_i} \), which can then also be used in the calculation of \( S_C \) [in equation (6)]. The uncertainty in the error estimate is based on the amount of the correction

\[
U_{k_i} = \left| 1 - C_k \right| \delta_{RE_{k_i}} \quad (19)
\]

Note that in the limit of the asymptotic range, \( C_k = 1 \), \( \delta_{k_i} = \delta_{RE_{k_i}} \), and \( U_{k_i} = 0 \).

3.2.4. Oscillatory Condition

For the oscillatory condition (ii), uncertainties can be estimated, but not the signs and magnitudes of the errors. Uncertainties are estimated based on determination of the upper \( (S_U) \) and lower \( (S_L) \) bounds of solution oscillation, which requires more than \( m=3 \) solutions. The estimate of uncertainty is based on half the solution range

\[
U_{k_i} = \frac{1}{2} (S_U - S_L) \quad (20)
\]

3.2.5. Diverging Condition

For the diverging condition (iii), errors and or uncertainties can not be estimated. The preparation and verification steps must be reconsidered. Improvements in iterative convergence, parameter specification (e.g., grid quality), and/or CFD code may be required to achieve converging or oscillatory conditions.

3.3. Validation

Validation is defined as a process for assessing modeling uncertainty \( U_{SM} \) by using benchmark experimental data and, when conditions permit, estimating the sign and magnitude of the modeling error \( \delta_{SM} \) itself. The verification process described in the previous section provides the estimates for the numerical errors and/or uncertainties. Approaches to estimating experimental uncertainties are presented and discussed by Coleman and Steele (1999).

C&S viewed the comparison process in attempting to validate simulations using experimental data, as illustrated in figure 1. For the data points, uncertainties in both the experimentally-determined \( r \) and the experimentally-determined value of the independent variable \( X \) should be considered, giving an uncertainty "box" around each experimental data point. Additionally, the result from the simulation should not be viewed as an infinitesimally thin \( r \) vs. \( X \) line, but rather as a "fuzzy band" that represents the
simulation plus and minus the uncertainty that should be associated with the simulation.

Figure 1 shows the variables and their uncertainties, but the comparison it shows is deceptive because it is two-dimensional. The independent variable \( X \) must be considered a vector \( (X) \) of \( n \) dimensions -- fluid velocity as a function of position and time, \( V(x, y, z, t) \), for example -- and the "box" around \( X \) will therefore be \( n \)-dimensional. The (total) uncertainty in \( r \) that should be used in a comparison should include the experimental uncertainty in \( r \) and the additional uncertainty in \( r \) arising from experimental uncertainties in the measurements of the \( n \) independent variables.

As discussed in Section 3.1, contributors to simulation errors \( \delta_r(X) \) and/or uncertainties \( U_r(X) \) can be divided into two distinct sources -- numerical and modeling. Techniques for estimating numerical errors and/or uncertainties were discussed in Section 3.2. The modeling errors and uncertainties are considered in the following sections.

Figure 1. Uncertainties in data and predictions to be considered in validations

3.3.1. Coleman and Stern (1997) Validation Methodology

The numerical uncertainty category includes uncertainties due to the numerical solution of the mathematical equations. The modeling uncertainty category includes uncertainties due to assumptions and approximations in the mathematical representation of the physical process (geometry, mathematical equation, coordinate transformation approximations, free-surface boundary conditions, turbulence models, etc) and also uncertainties due to the incorporation of previous experimental data (such as fluid property values) into the model.

Consider the situation shown in figure 2. Using the example mentioned previously, the single-plane representation \( r \) vs. \( X \) might be a mean velocity component \( V \) vs. distance \( (z) \)

normal to a solid surface at a given time and position \((x, y, t)\) on that surface. Define the computed \( r \)-value from the simulation (code) as \( S \) and the experimentally determined \( r \)-value of the \((X, r)\) data point as \( D \). \( S \) and \( D \) differ from the truth \( T \) as

\[
T = S + \delta_s
\]

and

\[
T = D + \delta_d
\]

\( \delta_s \) and \( \delta_d \) are the errors in the simulation and in the data, respectively, and where obviously then

\[
D + \delta_d = S + \delta_s
\]

The simulation error is composed of error from the numerical solution \( \delta_{SN} \), error from the use of previous data \( \delta_{SPD} \), and error from modeling assumptions \( \delta_{SMA} \) (as per equation (1), but with \( \delta_{SM} \) decomposed into the sum of \( \delta_{SPD} \) and \( \delta_{SMA} \))

\[
\delta_s = \delta_{SN} + \delta_{SPD} + \delta_{SMA}
\]

Figure 2. Definition of comparison error

Defining the comparison error, \( E \), as the difference of \( D \) and \( S \) one obtains

\[
E = D - S = \delta_s - \delta_d = \delta_{SN} + \delta_{SPD} + \delta_{SMA} - \delta_d
\]

Thus \( E \) is the resultant of all of the errors associated with the experimental data and the errors associated with the simulation. Here it is assumed that a correction has been made to \( D \) for any error whose value is known and \( S \) is either the uncorrected \( S \) or corrected \( S_c \) simulation result as discussed in Section 3.2. Note that use of \( S_c \) in equation (25) is equivalent to moving the \( \delta_{SN} \) term to the left-hand side such that the right-hand side consists solely of errors from the previous data, modeling assumptions, and benchmark data. Thus, the errors that are the subject of this discussion have unknown sign and
magnitude, and the estimates of these errors are uncertainties. (As will be shown, during the validation process an estimate of the sign and magnitude of $\delta_{\text{SMA}}$ can be made under certain conditions.)

There has been discussion in the verification and validation community that errors such as $\delta_{\text{SN}}$ are deterministic rather than stochastic and thus how they should be treated in an uncertainty analysis is unclear. What does seem clear, however, is that an estimate for $\delta_{\text{SN}}$ must be made in verification and validation efforts -- regardless of how it is combined with other errors or estimated as an uncertainty. In analogy to the treatment of systematic errors in experimental uncertainty analysis, C&S chose to consider any particular $\delta_{\text{SN}}$ as a single realization from some parent population of $\delta_{\text{SN}}$'s and treated the uncertainty $U_{\text{SN}}$ accordingly, and that is the approach used here.

If $X_i, r_i$, and $S$ share no common error sources, then the uncertainty $U_E$ in the comparison error can be expressed as

$$U_E^2 = \left( \frac{\partial E}{\partial D} \right)^2 U_D^2 + \left( \frac{\partial E}{\partial S} \right)^2 U_S^2 = U_D^2 + U_S^2 \quad (26)$$

where subscripts are used in the same manner as for the $\delta$s.

The simulation uncertainty $U_S$ can be represented as (as per equation (2), but with modeling uncertainty decomposed into uncertainties due to use of previous data and uncertainties due to modeling assumptions)

$$U_S^2 = U_{\text{SN}}^2 + U_{\text{SPD}}^2 + U_{\text{SMA}}^2 \quad (27)$$

so that

$$U_E^2 = U_D^2 + U_{\text{SN}}^2 + U_{\text{SPD}}^2 + U_{\text{SMA}}^2 \quad (28)$$

Ideally, we would like to postulate that if the absolute value of $E$ is less than its uncertainty $U_E$, then validation is achieved. In reality, the authors know of no approach that gives an estimate of $U_{\text{SMA}}$, so $U_E$ cannot be estimated. That leaves a more stringent validation test as the practical alternative. If we define the validation uncertainty $U_v$ as the combination of all uncertainties that we know how to estimate (i.e., all but $U_{\text{SMA}}$), then

$$U_v^2 = U_E^2 - U_{\text{SMA}}^2 = U_D^2 + U_{\text{SN}}^2 + U_{\text{SPD}}^2 \quad (29)$$

If $|E|$ is less than the validation uncertainty $U_v$, the combination of all the errors in $D$ and $S$ is smaller than the estimated validation uncertainty and validation has been achieved at the $U_v$ level. This quantity $U_v$ is the key metric in the validation process. $U_v$ is the validation “noise level” imposed by the uncertainties inherent in the data, the numerical solution, and the previous experimental data used in the simulation model. It can be argued that one cannot discriminate once $|E|$ is less than this; that is, as long as $|E|$ is less than this, one cannot evaluate the effectiveness of proposed model “improvements.”

As discussed above, for the data point $(X_i, r_i)$, $U_D$ should include both the experimental uncertainty in $r_i$ and the additional uncertainties in $r_i$ arising from experimental uncertainties in the measurements of the $n$ independent variables $\{X_j\}$ in $X_i$. The expression for $U_D$ that should be used in the $U_v$ calculation is then

$$U_D^2 = U_{X_i}^2 + \sum_{j=1}^{n} \left( \frac{\partial r_i}{\partial X_j} \right)^2 \left( U_{X_j} \right)^2 \quad (30)$$

In some cases, the terms in the summation in equation (30) may be shown to be very small, using an order-of-magnitude analysis, and then neglected. This would occur in situations in which the $U_{X_i}$ values are of “reasonable” magnitude and gradients in $r$ are small. In regions with high gradients (e.g., near a surface in a turbulent flow), these terms may be very significant and the partial derivatives would be estimated using whatever $(X_i, r_i)$ data is available.

There is also a very real possibility that measurements of different variables might share identical bias errors. This is easy to imagine for measurements of $x$, $y$, and $z$. Another possibility is $D$ and $S$ sharing an identical error source, for example if $r$ is drag coefficient and the same density table (curve fit) is used both in data reduction in the experiment and in the simulation. In such cases, additional correlated bias terms must be included in equation (28).

To estimate $U_{\text{SPD}}$ for a case in which the simulation uses previous data $d_i$ in $m$ instances, one would need to evaluate

$$U_{\text{SPD}}^2 = \sum_{i=1}^{m} \left( \frac{\partial S}{\partial d_i} \right)^2 (U_{d_i})^2 \quad (31)$$

where the $U_{d_i}$ are the uncertainties associated with the data.

### 3.3.2. Results for Single CFD Code

Consideration of equation (29) shows that (1) the more uncertain the data, and/or (2) the more inaccurate the code (greater $U_{\text{SN}}$ and $U_{\text{SPD}}$), the easier it is to validate a code, since the greater the uncertainties in the data and the code predictions, the greater the noise level ($U_v$). If this value of $U_v$ is greater than that designated as necessary in a research/design/development program, however, the required level of validation could not be achieved without improvement in the quality of the data, the code, or both. Similarly, if $U_{\text{SN}}$ and $U_{\text{SPD}}$ are not estimated but $|E|$ is less than $U_D$, then validation has been achieved but at an unknown level.

If there is a programmatic validation requirement, denote it as $U_{\text{req}}$ since validation is required at that uncertainty level or below. From a general perspective, if we consider the three
variables \( U_Y, |E|, \) and \( U_{\text{reqd}} \) there are six combinations (assuming none of the three variables are equal):

1. \( |E| < U_Y < U_{\text{reqd}} \)
2. \( |E| < U_{\text{reqd}} < U_Y \)
3. \( U_{\text{reqd}} < |E| < U_Y \)
4. \( U_Y < |E| < U_{\text{reqd}} \) \hspace{1cm} (32)
5. \( U_Y < U_{\text{reqd}} < |E| \)
6. \( U_{\text{reqd}} < U_Y < |E| \)

In cases 1, 2 and 3, \( |E| < U_Y \); validation is achieved at the \( U_Y \) level; and the comparison error is below the noise level, so attempting to decrease the error \( \delta_{\text{SMA}} \) due to the modeling assumptions in the simulation is not feasible from an uncertainty standpoint. In case 1, validation has been achieved at a level below \( U_{\text{reqd}} \), so validation is successful from a programmatic standpoint.

In cases 4, 5 and 6, \( U_Y < |E| \), so the comparison error is above the noise level and using the sign and magnitude of \( E \) to estimate \( \delta_{\text{SMA}} \) is feasible from an uncertainty standpoint. If \( \delta_{\text{SN}} \) is estimated and \( S_C \) is used in equation (25) for \( S \), then \( E \) more likely corresponds to \( \delta_{\text{SMA}} \). If \( U_Y << |E| \), then \( E \) corresponds to \( \delta_{\text{SMA}} \) and the error from the modeling assumptions can be determined unambiguously. In case 4, validation is successful at the \( |E| \) level from a programmatic standpoint.

In general, validation of a code’s predictions of a number (\( N \)) of different variables is desired, and this means that in a particular validation effort there could be \( N \) different \( E, U_Y, U_{\text{reqd}} \) values and (perhaps) some successful and some unsuccessful validations. For each variable, a plot of the simulation prediction versus \( X \) compared with the \((X, r_i)\) data points gives a traditional overview of the validation status, but the interpretation of the comparison is greatly affected by choice of the scale and the size of the symbols. A plot of \( \pm U_Y \) and \( E \) (and \( U_{\text{reqd}} \) if known) versus \( X \) for each variable is particularly useful in drawing conclusions, and the interpretation of the comparison is more insensitive to scale and symbol size choices. This is illustrated in figure 5 of C&S, which compares data and predictions for the three components of velocity versus position in a flow field.

### 3.3.3. Results for Comparison of Multiple Codes and/or Models

When a validation effort involves multiple codes and/or models, the procedure discussed above -- comparison of values of \( E \) and \( U_Y \) (and \( U_{\text{reqd}} \) if known) for the \( N \) variables -- should be performed for each code/model.

Since each code/model may have a different \( U_Y \), some method to compare the different codes’/models’ performance for each variable in the validation is useful. The range within which (95 times out of 100) the true value of \( E \) lies is \( E \pm U_E \). From equation (29), when \( U_{\text{SMA}} \) is zero then \( U_Y = U_E \), so for that ideal condition the maximum absolute magnitude of the 95% confidence interval is given by \( |E| + U_Y \). Comparison of the \((|E| + U_Y)\)'s for the different codes/models then shows which has the smallest range of likely error assuming all \( U_{\text{SMA}} \)'s are zero. This allows appropriate comparisons of \((\text{low} E)/(\text{high} U_Y)\) with \((\text{high} E/\text{low} U_Y)\) codes/models.

### 3.3.4. Results for Predictions of Trends

In some instances, the ability of a code or model to predict the trend of a variable may be the subject of a validation effort. An example would be the difference in drag for two ship configurations tested at the same Froude number. The procedure discussed above -- comparison of \( |E| \) and \( U_Y \) for the drag -- should be performed for each configuration. The difference \( \Delta \) in drag for the two configurations should then be considered as the variable that is the subject of the validation. As discussed in Coleman and Steele (1999), because of correlated bias uncertainty effects in the experimental data the magnitude of the uncertainty in \( \Delta \) may be significantly less than the uncertainty in either of the two experimentally determined drag values. This means that the value of \( U_Y \) for \( \Delta \) may be significantly less than the \( U_Y \)'s for the drag values, allowing for a more stringent validation criterion for the difference than for the absolute magnitudes of the variables.

### 3.3.5. Use of Corrected vs. Uncorrected Simulations

If one has enough confidence in the estimates of the numerical errors \( \delta_i, \delta_g, \delta_r, \) and \( \delta_p \), so that the corrected simulation \( S_C \) is used in place of \( S \) in equation (25), then \( U_{\text{SN}} \) (which is then the overall uncertainty in those error estimates) should be smaller than in the uncorrected case. Referring to equation (28), this means that \( U_Y \) should also be smaller than in the uncorrected case, thus allowing validation at a lower uncertainty level.

If such a validation at a condition is successful, then if one chooses to associate that validation uncertainty level with the simulation’s prediction at a neighboring condition that prediction must also be corrected. That means enough runs are required at the new condition to allow estimation of the numerical errors. If this is not done, then the comparison error \( E \) and validation uncertainty \( U_Y \) corresponding to the use of the uncorrected \( S \) and its associated (larger) \( U_{\text{SN}} \) should be the
ones considered in the validation with which one wants to associate the prediction at a new condition. It should also be noted that when $S_C$ is used and $|E| > U_V$, the sign and magnitude of $\delta_{\text{sma}}$ can be estimated with less uncertainty than when $S$ is used (assuming that the correction leads to lower uncertainty).

4. EXAMPLE FOR RANS CFD CODE

Example results of verification and validation are presented for a single CFD code and for specified objectives, geometry, conditions, and available benchmark information. The CFD code is CFDSHIP-IOWA, which is a general-purpose, multi-block, high performance computing (parallel), unsteady RANS code (Paterson et al., 1998; Wilson et al., 1998) developed for computational ship hydrodynamics. The RANS equations are solved using higher-order upwind finite differences, PISO, $k-\omega$ turbulence model, and exact and approximate treatments, respectively, of the kinematic and dynamic free-surface boundary conditions. The objectives are to demonstrate the usefulness of the proposed verification and validation procedures and methodology and establish the levels of verification and validation of the simulation results for an established benchmark for ship hydrodynamics CFD validation.

4.1. Geometry, Conditions, and Benchmark Data

The geometry is the Series 60 cargo/container ship. The Series 60 was used for two of the three test cases at the last international workshop on validation of ship hydrodynamics CFD codes (CFD Workshop Tokyo, 1994). The conditions for the calculations are Froude number $Fr = 0.316$, Reynolds number $Re = 4.3 \times 10^6$, and zero sinkage and trim. These are the same conditions as the experiments, except the resistance and sinkage and trim tests, as explained next. The variables selected for verification and validation are resistance $C_T$ (integral variable) and wave profile $\zeta$ (point variable).

The benchmark data is provided by Toda et al. (1992), which was also the data used for the Series 60 test cases at the CFD Workshop Tokyo (1994). The data includes resistance and sinkage and trim for a range of $Fr$ for the model free condition (i.e., free to sink and trim); and wave profiles, near-field wave pattern, and mean velocities and pressures at numerous stations from the bow to the stern and near wake, all for $Fr = (0.16, 0.316)$ and the zero sinkage and trim model fixed condition. The data also includes uncertainty estimates, which were recently confirmed/updated by Longo and Stern (1999) closely following standard procedures (Coleman and Steele, 1999).

The resistance is known to be larger for free vs. fixed models. Data for the Series 60 indicates about an 8% increase in $C_T$ for the free vs. fixed condition over a range of $Fr$ including $Fr=0.316$ (Ogiwara and Kajatani, 1994). The Toda et al. (1992) resistance values were calibrated (i.e., reduced by 8%) for effects of sinkage and trim for the present comparisons.

4.2. Verification and Validation of Integral Variable: Resistance

Verification was performed with consideration to iterative and grid convergence studies, i.e., $\delta_{SN} = \delta_f + \delta_g$ and $U_{SN}^2 = U_f^2 + U_G^2$.

![Figure 3. Grids used in verification and validation studies for Series 60: (a) coarse; (b) medium; and (c) fine grid.](image-url)
considerations where the first point from the no-slip surface is placed at a normalized wall distance of \( y^+ < 1.0 \).

The iteration errors and uncertainties were negligible in comparison to the grid errors and uncertainties for all three solutions i.e. \( \delta_t \ll \delta_G \) and \( U_1 \ll U_G \) such that \( \delta_{SN} = \delta_G \) and \( U_{SN} = U_G \).

The results from the grid convergence study for \( C_T \) are summarized in table 2. The solutions for \( C_T \) indicate the converging condition (i) of equation (9) with \( R_G = \varepsilon_{21}/\varepsilon_{12} = 0.42 \). The correction factor \( C_G \) [in equation (16)], order of accuracy \( p_G \) [in equation (15)], and first-order RE estimate \( \delta_{RE_G} \) [in equation (14)] are

\[
C_G = \frac{p_{GS} - 1}{r_{GS} - 1} = \left( \frac{\sqrt{2}}{2} \right)^2 - 1 = 1.29
\]

\[
p_G = \frac{\ln(\varepsilon_{21}/\varepsilon_{12})}{\ln(r_g)} = \frac{\ln(0.81/0.34)}{\ln(\sqrt{2})} = 2.5
\]

\[
\delta_{RE_G} = \left( \frac{\varepsilon_{21}}{r_{GS} - 1} \right) = \left( \frac{0.34 \times 10^{-3}}{\left( \frac{\sqrt{2}}{2} \right)^2 - 1} \right) = 0.26 \times 10^{-3}
\]

where \( p_G = 2 \) was used in equation (33). Uncertainty and/or error estimates are made next both considering \( C_G \) as sufficiently less than or greater than 1 and lacking confidence and \( C_G \) as close to 1 and having confidence, as discussed in Section 3.2.3.

For \( C_G = 1.29 \) considered as sufficiently less than or greater than 1 and lacking confidence, \( U_G \) is estimated and not \( \delta_G \). Equation (18) along with equations (33) - (35) are used to estimate \( U_G \)

\[
U_G = \left| C_G \delta_{RE_G} + \left[ 1 - C_G \right] \delta_{RE_G} \right| = 0.34 \times 10^{-3} + 0.08 \times 10^{-3} = 0.42 \times 10^{-3}
\]

\( U_G \) is 7.8% \( S_{GI} \).

For \( C_G = 1.29 \) considered close to 1 and having confidence, both and \( \delta_G \) and \( U_G \) are estimated. Equation (17) and (19) along with equations (33) - (35) are used to estimate \( \delta_G \) and \( U_G \)

\[
\delta_G = C_G \delta_{RE_G} = 0.34 \times 10^{-3}
\]

\[
U_G = \left| 1 - C_G \right] \left[ 1 - \delta_G \right] = 0.08 \times 10^{-3}
\]

Equation (6) along with equation (37) is used to calculate \( S_C \)

\[
S_C = S_{GI} - \delta_G = 5.07 \times 10^{-3}
\]

\( \delta_G \) and \( U_G \) are 6.7% and 1.6% \( S_{CI} \), respectively.

Validation is performed using both \( S \) and \( S_C \). First using \( S \), the comparison error is calculated from equation (25) as

\[
E = D - S = 5.42 \times 10^{-3} - 5.41 \times 10^{-3} = 0.01 \times 10^{-3} = 0.2\%
\]

The validation uncertainty is calculated from equation (29) as

\[
U_V = \sqrt{U_{SN}^2 + U_D^2} = 0.44 \times 10^{-3} = 8.1\% D
\]

where \( U_{SN} = U_G \) and \( U_D = 2.5\% D \). In conclusion, \( |E| < U_V \) such that the simulation results are validated at the level 8.1% \( D \).

Note that \( U_{SN}/U_D = 3.1 \) such that reduction in \( U_V \) mostly requires reduction in \( U_{SN} \) (e.g., use of finer grids).

Second using \( S_C \), the comparison error is calculated from equation (25) as

\[
E = D - S_C = 5.42 \times 10^{-3} - 5.07 \times 10^{-3} = 0.35 \times 10^{-3} = 6.5\% D
\]

The validation uncertainty is calculated from equation (29) as

\[
U_V = \sqrt{U_{SN}^2 + U_D^2} = 0.16 \times 10^{-3} = 3.0\% D
\]

In conclusion, \( |E| > U_V \) such that the simulation results are not validated. Comparison error \( E \) shows that \( S_C \) underpredicts \( D \). Validation uncertainty \( U_V \) is relatively small and \( U_{SN}<U_D \) suggests that \( E \) is mostly due to modeling errors. Therefore, modeling issues should/can be improved to reduce \( E \) and validate \( C_T \) at the reduced level \( U_V = 3.0\% D \) in comparison to equation (41). However, finer grids and calculations including the effects of sinkage and trim are required to reach definitive conclusions.

<table>
<thead>
<tr>
<th>Grid</th>
<th>Coarse 101x26x16</th>
<th>Medium 144x36x22</th>
<th>Fine 201x51x31</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_T )</td>
<td>6.56</td>
<td>5.75</td>
<td>5.41</td>
<td>5.42</td>
</tr>
<tr>
<td>( C_T )</td>
<td>-12.3%</td>
<td>-17.0%</td>
<td>-4.3%</td>
<td>+0.1%</td>
</tr>
<tr>
<td>( S_T )</td>
<td>4.33</td>
<td>3.90</td>
<td>3.65</td>
<td>3.42</td>
</tr>
<tr>
<td>( S_T )</td>
<td>-9.9%</td>
<td>-6.4%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Grid convergence of ship forces \((x10^3)\) for Series 60.

### 4.3. Verification and Validation of Point Variable: Wave Profile

Verification for the wave profile was conducted as per that described for resistance in Section 4.2. The same 3 grids were used and, here again iteration errors and uncertainties were negligible in comparison to the grid errors and uncertainties for all three solutions i.e. \( \delta_t \ll \delta_G \) and \( U_t \ll U_G \) such that \( \delta_{SN} = \delta_G \) and \( U_{SN} = U_G \).

Determination of the convergence ratio \( R_G = \varepsilon_{21}/\varepsilon_{12} \) for point variables can be problematic since solution changes \( \varepsilon_{21} \) and \( \varepsilon_{12} \) can both go to zero (e.g., in regions where the solution contains an inflection point). In this case, the ratio becomes ill-conditioned. However, the convergence ratio can be used in regions where the solution changes are both non-zero (e.g., local maximums or minimums in solution change). A global measure of convergence ratio \( R \), which overcomes the ill-conditioning problem, is computed from the global solution changes \( \varepsilon_{21} \) and \( \varepsilon_{12} \) (i.e., L2 norm of the profile, \( R = \| \varepsilon_{21} \| / \| \varepsilon_{12} \| \)).

For the wave profile, calculation of the convergence ratio \( R_G \) at local maximums and minimums (i.e., \( x/L = 0.1, 0.4, \) and \( 0.65 \) in figure 4a) shows convergence. In addition, calculation
and was found to be
\[ \langle p_G \rangle = \frac{\ln \| e_{2L_G} \|}{\ln(r_G)} = 1.4 \]  
(44)
where \( \langle \cdot \rangle \) is used to denote a profile-averaged value and \( \| \cdot \| \) denotes the L2 norm of solution change over the region, \( 0 < x/L < 1 \). Correction factor is computed from equation (16) using the global order of accuracy \( p_G \) in equation (44) and \( p_R=2.0 \)
\[ \langle C_G \rangle = \frac{\varepsilon_{2L_G}}{r_G^{p_G} - 1} = \frac{(\sqrt{2})^4 - 1}{(\sqrt{2})^2 - 1} = 0.62 \]  
(45)
The global estimate for order of accuracy and correction factor in equations (44) and (45) were used to estimate grid error and uncertainty for the wave profile at each grid point.

For \( \langle C_G \rangle = 0.62 \) considered as sufficiently less than or greater than 1 and lacking confidence, pointwise values for \( U_G \) are estimated and not \( \delta_G \). Equation (18) is used to estimate \( U_G \)
\[ U_G = \left\{ \langle C_G \rangle \right\} \left\{ \frac{\varepsilon_{2L_G}}{r_G^{p_G} - 1} \right\} \]  
(46)
For \( C_G=0.62 \) considered close to 1 and having confidence, pointwise values for both \( \delta_G \) and \( U_G \) are estimated. Equations (17) and (19) are used to estimate \( \delta_G \) and \( U_G \)
\[ \delta_G = \langle C_G \rangle \left( \frac{\varepsilon_{2L_G}}{r_G^{p_G} - 1} \right) \]  
(47)
\[ U_G = \left\{ 1 - \langle C_G \rangle \right\} \left( \frac{\varepsilon_{2L_G}}{r_G^{p_G} - 1} \right) \]  
(48)
Equation (6) is used to calculate \( S_C \) at each grid point
\[ S_C = S_{G_i} - \delta_G \]  
(49)
As with validation of an integral value, validation of the wave profile is performed using both \( S \) and \( S_C \) in the definition of the comparison error \( E \) and simulation uncertainty \( U_{SN} \). Profile-averaged values for both definitions of the comparison error \( E \), validation uncertainty \( U_V \), and simulation uncertainty \( U_{SN} \) are given in table 3. Values are normalized with the maximum value for the wave profile \( \zeta_{max}=0.014 \) and the uncertainty in data was reported to be 3.7% \( \zeta_{max} \). With the comparison error defined as \( E=D-S \), the average error \( E \) and validation uncertainty \( U_V \) are 7.4% and 5.2% \( \zeta_{max} \), respectively. With the comparison error defined as \( E=D-S_C \), the average error \( E \) and validation uncertainty \( U_V \) are 6.2% and 3.9% \( \zeta_{max} \), respectively. Defining the comparison error as \( E=D-S_C \) reduces both the error \( E \) and validation uncertainty \( U_V \). With both definitions of comparison error, \( |E| > U_V \) such that the simulation results are not validated and modeling issues.

Figure 4. Wave profile for Series 60: (a) grid study, (b) validation using \( S \); and (c) validation using \( S_C \).

The point wise comparison error \( E=D-S \) is compared to validation uncertainty \( U_V \) in figure 4b, while error \( E=D-S_C \) is compared to validation uncertainty \( U_V \) in figure 4c. For the
latter definition of comparison error, the validation uncertainty $U_v$ in figure 4c is due mostly to the uncertainty in the data $U_D$.

<table>
<thead>
<tr>
<th>$E=S-D$</th>
<th>$%E$</th>
<th>$%U_v$</th>
<th>$%U_D$</th>
<th>$%U_{SN}$</th>
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<tbody>
<tr>
<td>7.4</td>
<td>5.2</td>
<td>3.7</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td>$E=S_e-D$</td>
<td>6.2</td>
<td>3.9</td>
<td>3.7</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 3. Profile-averaged values from validation of wave profile for Series 60.

5. CONCLUSIONS AND RECOMMENDATIONS

The verification and validation procedures and methodology presented should have applicability to a fairly broad range of CFD codes, including RANS, Navier-Stokes, Euler, boundary-element methods, and others. More work is needed for other CFD codes (such as large-eddy simulations) and additional error sources and error and uncertainty estimation methods, e.g., single-grid methods and effects of higher-order terms in RE. Clearly more experience is needed through application for different codes and geometry and conditions.

The verification and validation procedures and methodology are recommended for adoption. It will be useful in guiding future developments in CFD through documented, verification, and validation studies and in transition of CFD codes to design through establishment of credibility. Presumably, with a sufficient number of documented, verified, and validated solutions along with selected verification studies a CFD code can be accredited for a certain range of applications. The contribution of the present work is in providing procedures and methodology for the former, which hopefully will help lead to the latter.

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