DNS of Drag-reducing Turbulent Channel Flow with Coexisting Newtonian and Non-Newtonian Fluid

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Abstract

In the present study, we numerically investigated drag-reducing turbulent channel flows by surfactant additives. Surfactant additives were assumed to be uniformly distributed in the entire flow region by turbulent convection and diffusion etc., but it was assumed that the shear-induced structure (SIS) (network of rod-like micelles) could form either in the region next to the walls or in the center region of the channel, making the fluid viscoelastic; while in other regions surfactant additives were assumed to be incapable of building a network structure, and to exist in form of molecules or micelles which do not affect the Newtonian properties of the fluid. With these assumptions, we studied the drag-reducing phenomenon with coexisting Newtonian and non-Newtonian fluids. From the study we identified the effectiveness of the network structures at different flow regions, and showed that the phenomenon of drag-reduction (DR) by surfactant additives is not only closely associated with the reduction of Reynolds shear stress but also related to the induced viscoelastic shear stress.

1. Introduction

It is well known that surfactant additives are good drag-reducers with long-term action, and can be applied to district heating and cooling systems. The mechanism of drag-reduction by surfactant additives is still not well understood, but it is generally accepted that drag-reduction is associated with the network structures in the surfactant solution. These network structures show elasticity, prevent the generation of turbulence and thus reduce frictional drag. Recently we carried out a direct numerical simulation (DNS) study to quantitatively study the mechanism of drag-reduction. By using viscoelastic Giesekus constitutive equations to model the interaction between the network structures and solvent (water), we successfully reproduced many features observed in the experiments such as a wider buffer layer and decrease of Reynolds shear stress (Yu et al., 2003). However, our previous DNS study had certain limitations: (1) the network structures were assumed to exist in the entire flow region and the fluid was non-Newtonian in the whole computational domain. However, experiments show that the network structures are sensitive to shear stress and temperature; for example, high temperature and high shear rate can destroy the network structures. The recent experiment of our research group (Li et al., 2004) shows the profile of the Reynolds shear stress displays a multi-layer structure as shown in Fig. 1, which can be explained by the different states of surfactant additives due to the local shear stress and supports that the network structure region and non-network structure region may coexist. (2) Since we assumed the fluid was non-Newtonian in the whole computational domain, the effectiveness of the network structures in reducing frictional drag at different layers was not isolated.

In the present study, we investigated turbulent channel flow with coexisting Newtonian and non-Newtonian fluids in order to identify the effectiveness of the network structures at different flow layers. The prediction of limiting condition of network structure is extremely important in practice, such as designing a flow system using the DR additives. Once the function of the network structures is clearly understood, some methods can be proposed to control the turbulence by controlling the network structures through adjusting the concentration, temperature, and shear rate distribution in the surfactant solution. Another purpose of this study was to better reproduce some phenomena observed in practical problems which were not found in our previous DNS study.
2. Modeling of Flow

The flow to be studied was a fully-developed channel flow. Two types of fluid motion, Flow A and Flow B shown in Fig. 2, were studied, where Newtonian and non-Newtonian fluids separately flow at different layers with the interface between them being parallel to the walls. In Flow A, the network structures exist at the center region of the channel, whereas in Flow B they exist at the near-wall region. By moving the interface position, it is possible to study how the network structures reduce frictional drag at different flow layers. For Flow A, we did four calculations with non-Newtonian fluid thicknesses of $h_0$, $h_0^2$, $h_0^3$, and $h_0^4$. For flow B, we carried out three computations with non-Newtonian fluid thicknesses of $h_0$, $h_0^2$, and $h_0^3$. The first case of Flow A is Newtonian fluid and the last case of Flow B is non-Newtonian fluid. For all the other cases, Newtonian and non-Newtonian fluids coexist.

3. Governing equations

We employed viscoelastic Giesekus constitutive equations to model the interaction between the network structures and solvent. The governing equations for fully developed turbulent channel flow can be written as:

\[
\frac{\partial u_i}{\partial x_i} = 0
\]

\[
\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \eta_s \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_j}{\partial x_j} \right) + \frac{\partial \tau_{ij}}{\partial x_j}
\]

\[
\tau_{ij} + \lambda \left( \frac{\partial \tau_{ij}}{\partial t} + \frac{\partial u_m \tau_{ij}}{\partial x_m} - \frac{\partial u_i \tau_{mj}}{\partial x_m} - \frac{\partial u_j \tau_{mi}}{\partial x_m} + \frac{\alpha}{\eta_s} \tau_{ij} \tau_{mj} \right) = \eta_a \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

where $\lambda$ and $\alpha$ are, respectively, relaxation time and mobility factor. $\eta_s$ and $\eta_a$ are the contribution of solvent and additive to the zero-shear-rate solution viscosity. $\tau_{ij}$ is the extra stress related to the network structures. Note that in the Newtonian fluid region, $\tau_{ij}$ is zero.

By introducing the following non-dimensional variables:

\[
x^* = \frac{x}{h}, \quad t^* = \frac{t}{h/U_r}, \quad u_i^* = \frac{u_i}{U_r}, \quad p^* = \frac{p - p_i}{\rho U_r^2}, \quad c_{ij}^* = \frac{\tau_{ij}}{\eta_{ij}} = \frac{\tau_{ij}}{\eta_s} + \delta_{ij}
\]

the governing equations can be written in dimensionless form as follows:
\[
\frac{\partial u^+}{\partial x_j} = 0
\]
(5)
\[
\frac{\partial u^+}{\partial t} + u^+_j \frac{\partial u^+}{\partial x_j} = - \frac{\partial p^+}{\partial x^+_j} + \frac{1}{\text{Re}_r} \frac{\partial}{\partial x^+_j} \left( \frac{\partial u^+}{\partial x^+_j} \right) + \frac{\beta}{\text{We}_r} \frac{\partial c^+_y}{\partial x^+_j} + \delta_{ij}
\]
(6)
\[
\frac{\partial c^+_y}{\partial t} + \frac{\partial u^+_m c^+_y}{\partial x^+_m} - \frac{\partial u^+_m c^-_m}{\partial x^+_m} + \frac{\partial u^+_m c^-_m}{\partial x^+_m} + \frac{\text{Re}_L}{\text{We}_r} \left[ c^+_y - \delta_{ij} + \alpha \left( c^+_m - \delta_{im} \right) \left( c^+_m - \delta_{mj} \right) \right] = 0
\]
(7)

where \( p^+ \) is excess hydrostatic pressure, \( \delta_{ij} \) is mean pressure gradient and \( c^+_y \) is conformation tensor associated with the deformation of the network structures. There are four important non-dimensional parameters in the governing equations: \( \text{Re}_r, \text{We}_r, \alpha \) and \( \beta \). \( \text{Re}_r \) is the frictional Reynolds number based on the frictional velocity and half of the channel height. \( \text{We}_r = \rho \lambda U^+ / \eta_s \) is Weissenberg number, which is a non-dimensional relaxation time. Mobility factor \( \alpha \) indicates the extensibility of the network structures in the surfactant solution. \( \beta \) is a ratio defined as \( \beta = \eta_s / \eta_s \), where \( \eta_s \) and \( \eta_v \) are surfactant contribution and solvent contribution to the zero-shear rate viscosity of the solution (\( \eta_0 = \eta_s + \eta_v \)). For Newtonian fluid, Navier-Stokes’ equation is recovered by setting \( \beta \) to zero.

Calculations were performed with parameters \( \text{Re}_r = 125, \text{We}_r = 25, \alpha = 0.001 \) and \( \beta = 0.1 \) in the non-Newtonian fluid region and \( \text{Re}_r = 125 \) in the Newtonian fluid region. A computational box \( 3h \times 2h \times 2h \) in the \( x, y \) and \( z \) directions was chosen for simulation and the computational domain in wall units (\( \eta_s, U_r \) and \( \rho \)) was \( 375 \times 250 \times 250 \) \( (x \times y \times z) \). A grid system of \( 64 \times 78 \times 64 \) \((x, y \) and \( z) \) meshes was adopted. Non-uniform grids in the wall-normal direction were used with grids clustered in the near-wall region and at the interface region. Grid-spacing \( \Delta y^+ \) ranged from around 0.3 next to the wall to 6 in the center. Uniform grids were used in the \( x \) and \( z \) directions and the corresponding grid-spacings were \( \Delta x^+ = 5.86 \) and \( \Delta z^+ = 3.9 \) respectively. The periodic boundary conditions were imposed in both the streamwise (\( x^- \)) and spanwise (\( z^- \)) directions, while the nonslip condition was adopted for the top and bottom walls. The numerical method used here was a fractional-step method. The Adams-Bashforth scheme was used for time-advancement to ensure second-order accuracy in time. A second-order faithful finite difference scheme of Yu et al. (2004) was used to enhance the numerical stability. To avoid a zigzag pressure field, staggered grid was employed in which pressure and conformation components were stored at the cell center while velocity components were located at the cell faces. Calculations started from an instantaneous velocity field of Newtonian fluid at Reynolds number \( \text{Re}_t=150 \) in our previous work (Yu et al., 2004). The initial fields for pressure and conformation tensor were simply set as a zero field. At the interface of the Newtonian and non-Newtonian fluid regions, the following shear stress and normal stress balance equations were satisfied:

\[
\frac{\partial u^+}{\partial y^+} \bigg|_N = \left( \frac{\partial u^+}{\partial y^+} \right)_{NN} + \frac{\text{Re}_r c^+_x}{\text{We}_r}
\]
(8)
\[
\frac{\partial w^+}{\partial y^+} \bigg|_N = \left( \frac{\partial w^+}{\partial y^+} \right)_{NN} + \frac{\text{Re}_r c^+_z}{\text{We}_r}
\]
(9)
\[
\frac{\partial v^+}{\partial y^+} \bigg|_N = \left( \frac{\partial v^+}{\partial y^+} \right)_{NN} + \beta \frac{\text{Re}_r (c^+_y - 1)}{\text{We}_r}
\]
(10)
4. Results

Ensemble averaging in the periodic direction and time was carried out to obtain statistical steady turbulent quantities:

$$\langle q \rangle = \frac{1}{N_x N_y N_z} \sum_{x=0}^{N_x} \sum_{y=0}^{N_y} \sum_{z=0}^{N_z} q(x,y,z,t)$$  \hspace{1cm} (11)

In the present study, after the flow reached the steady state, calculations were carried out for a further 6000 $\nu/U_r^2$ time units for statistical average processing. One hundred statistically independent fields were saved at equal interval to make the averages.

<table>
<thead>
<tr>
<th>Flow</th>
<th>$h$</th>
<th>$Re_b$</th>
<th>$C_f$</th>
<th>DR %</th>
<th>DR / DR max</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(1)</td>
<td>$2 \times 0h$</td>
<td>3653</td>
<td>0.00936</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td>A(2)</td>
<td>$2 \times 0.4h$</td>
<td>3651</td>
<td>0.00937</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td>A(3)</td>
<td>$2 \times 0.6h$</td>
<td>3726</td>
<td>0.00940</td>
<td>3.63%</td>
<td>0.15</td>
</tr>
<tr>
<td>A(4)</td>
<td>$2 \times 0.9h$</td>
<td>4071</td>
<td>0.00754</td>
<td>17.5%</td>
<td>0.73</td>
</tr>
<tr>
<td>B(1)</td>
<td>$2 \times 0.2h$</td>
<td>3848</td>
<td>0.00844</td>
<td>8.94%</td>
<td>0.37</td>
</tr>
<tr>
<td>B(2)</td>
<td>$2 \times 0.4h$</td>
<td>4175</td>
<td>0.00717</td>
<td>21.0%</td>
<td>0.88</td>
</tr>
<tr>
<td>B(3)</td>
<td>$2 \times h$</td>
<td>4263</td>
<td>0.00688</td>
<td>23.9%</td>
<td>1.00</td>
</tr>
</tbody>
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Table 1 Reynolds number, friction factor and DR rate

![Figure 3](image-url)  
(a) based on results of Flow A  
(b) based on results of Flow B

Figure 3 local fractional contribution to drag-reduction rate

Our calculations were made at a constant frictional Reynolds number (constant pressure loss); an enhanced flow rate as compared to that of Newtonian fluid means the occurrence of drag-reduction. Table 1 lists the mean bulk Reynolds numbers obtained for different cases. It is seen that the flow rate is largest for uniform non-Newtonian fluid (Flow B(3)) and smallest for Newtonian fluid (Flow A(1)). In order quantitatively to show the drag-reducing ability of each case, the Fanning friction factor was adopted for the evaluation of frictional drag:

$$C_f = \frac{\tau_w}{\rho U_m^2 / 2} \hspace{1cm} (12)$$

Drag-reduction rate was defined as the reduction of friction factor with respect to Newtonian fluid at equal mean Reynolds number $Re_b$, i.e.

$$DR\% = \frac{C_f^D - C_f}{C_f^D} \times 100\% \hspace{1cm} (13)$$

$C_f^D$ was evaluated by Dean’s equation. Table 1 shows the calculated frictional factor $C_f$, evaluated frictional factor $C_f^D$ and drag-reduction rate. It is seen that for Newtonian fluid, the calculated frictional factor agrees quite well with Dean’s correlation. It is also seen that there is no drag-reduction for Flow A(2) and the drag-reduction rate is largest for Flow B(3). For
Flow A(3), a small drag-reduction rate of around 4 percent is obtained. Flow A(2) and Flow A(3) show the network structures are not effective in reducing frictional drag in the bulk flow region, especially in the center of the channel. In Flow B(1), the non-Newtonian fluid region is 20% of the entire flow region, so a drag-reduction rate of about 9 percent is obtained. In Flow B(2) the non-Newtonian fluid region is 40% of the entire flow region and the drag-reduction rate approaches that in Flow B(3). Flow B(1) and Flow B(2) indicate the network structures are most effective at the region near the wall. By comparing Flow A(4) and Flow B(3), it is seen in Flow A(4) that the network structures cover most of the fluid region except the viscous sublayer (0 ≤ y’ ≤ 5) and a small part of the buffer layer (5 < y’ ≤ 12.5), but its drag-reduction ability is 27% less than that of Flow B(3). In addition, the non-Newtonian fluid region of A(4) is 2.25 times as large as that of Flow B(2), but it has a smaller drag-reduction rate. All these findings support the hypothesis of Lumley (1969, 1973) that the primary action of additives takes place in the buffer layer. The local fractional contribution to DR in Figure 3 shows more clearly that additives are important in the near wall region. There are somewhat discrepancies in the contributions in Flow A and Flow B, which must come from the nonlinear effects.

Figure 4 shows the mean velocity profiles. For both Flow A and Flow B, as the drag-reduction rate increases, the velocity profile upshifts in the logarithmic region and the buffer layer becomes larger. The velocity profile of Flow A(2) almost collapses to that of Flow A(1) in the viscous sublayer and buffer layer, but differs from Flow A(1) in the center region of the channel. This indicates that though the network structures in the bulk flow region are not effective in reducing frictional drag, they do interact with the solvent. The velocity profile of Flow B(2) is almost the same as that of Flow B(3) up to y’ = 40. The network structures in Flow B(2) exist up to y’ = 50, but in region next to the interface from y’ = 40 to y’ = 50, the velocity values are smaller than those of Flow B(3). This means the effect of Newtonian fluid penetrates to the non-Newtonian region.

The root-mean-square velocity fluctuations are compared in Figs. 5–7. Generally, the peak value positions of $u_{rms}$ shift to the bulk flow region for drag-reduction cases. For
drag-reduction cases, $u_{\text{rms}}$ is larger than that of solvent except Flow B(1). For Flow A(2) and A(3), $v_{\text{rms}}$ and $w_{\text{rms}}$ decrease in the center region. For Flow B(1) and B(2), $v_{\text{rms}}$ and $w_{\text{rms}}$ decrease in the near-wall region. Flow B(1) and B(2) have larger drag-reduction rate, which means that larger drag-reduction is associated with the suppression of cross flow in the near-wall region. In general, the smaller $v_{\text{rms}}$ and $w_{\text{rms}}$ in the near-wall region, the larger the drag-reduction rate. In Flow B(3), $v_{\text{rms}}$ and $w_{\text{rms}}$ are appreciably suppressed in both the near-wall region and the center region and the largest drag-reduction rate was obtained.

The total shear stress can be used as an indicator of whether the calculation has reached a statistically steady state. When a steady state is reached, the following balance equation is satisfied:

$$\tau_{\text{total}} = 1 - \frac{y^+}{\text{Re}_r} = -u^+ v^+ + \frac{\partial U^+}{\partial y^+} + \beta \frac{C_{xy}^+}{\text{We}_r} \tag{14}$$

The last term on the right-hand side of Eq. (14) is the mean viscoelastic stress due to the elasticity of the network structures. Statistically steady states have been confirmed for all the calculations as seen in Fig. 8. The jump of viscous shear stress at the interface of the Newtonian fluid and non-Newtonian fluid regions is clearly seen especially in Flow A(4) and Flow B(1), which is due to the induced viscoelastic shear stress in the non-Newtonian fluid region.

![Figure 8 Budget of shear stress](#)
The Reynolds shear stresses are compared in Fig. 9. Drag-reduction is often explained by the decrease of Reynolds shear stress. It is seen that in our calculations, the Reynolds shear stress decreases for all the drag-reduction cases. However, Flow A(4) has a larger Reynolds shear stress than Flow B(1) but it also has a larger drag-reduction rate. This is because in Flow B(1), though the Reynolds shear stress is smaller, the viscoelastic shear stress is large in the near-wall region, which increases frictional drag. The comparison of Reynolds shear stress indicates that drag-reduction rate is associated with, but not proportional to, the decrease of Reynolds shear stress. The decrease of Reynolds shear stress is only one important factor for achieving a large drag-reduction rate.

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<tbody>
<tr>
<td>DR %</td>
<td>0%</td>
<td>0%</td>
<td>3.63%</td>
<td>17.5%</td>
<td>8.94%</td>
<td>21.0%</td>
<td>23.9%</td>
</tr>
<tr>
<td>$V^c$ (%)</td>
<td>34.5%</td>
<td>34.7%</td>
<td>35.2%</td>
<td>37.7%</td>
<td>36.6%</td>
<td>38.4%</td>
<td>39.9%</td>
</tr>
<tr>
<td>$T^c$ (%)</td>
<td>65.5%</td>
<td>64.8%</td>
<td>63.2%</td>
<td>57.2%</td>
<td>57.8%</td>
<td>57.2%</td>
<td>54.1%</td>
</tr>
<tr>
<td>$E^c$ (%)</td>
<td>0%</td>
<td>0.453%</td>
<td>1.56%</td>
<td>5.14%</td>
<td>5.62%</td>
<td>4.39%</td>
<td>5.97%</td>
</tr>
</tbody>
</table>

Eq. (14) shows that frictional drag derives from three components: viscous shear stress, Reynolds shear stress and viscoelastic shear stress. An integration equation for friction factor can be obtained by applying two-fold integration $\int_0^1 \int_0^y dy^* dy$ to Eq. (14), transforming multiple integration to single integration as follows:

$$C_f = \frac{12}{Re_b} + 6 \int_0^1 \left( -\frac{u'^* v'^*}{U_b^2} \right) (1- y^*) dy^* + 6 \int_0^1 \frac{C_w^* \beta}{We_e U_b^2} (1- y^*) dy^* \tag{15}$$

This integration was first proposed by Fukagata et al. (2002) (FIK integration) to evaluate active turbulent control and we extend it to drag-reducing flow by additives (Yu et al., 2003). The above equation shows the friction factor is decomposed into viscous contribution, which is identical to the laminar solution, the turbulence contribution and viscoelastic contribution. The turbulence contribution and viscoelastic contribution are linear weighted integrations with the weighted factor larger at the near-wall region. This indicates that the decrease of Reynolds shear stress and increase of viscoelastic shear stress at the near-wall region have the largest effect on frictional factor. The smaller Reynolds shear stress and smaller viscoelastic shear stress at the near-wall region can result in a smaller friction factor. The network
structures give rise to viscoelastic shear stress which increases frictional drag, whereas a decrease in Reynolds shear stress reduces frictional drag. When the decrease effect is larger than the increase effect, drag-reduction occurs. Table 2 shows the relative contributions of the three components in Eq. (15) to frictional drag. In Flow A(2), the viscoelastic contribution is negligible, which is due to the small values of viscoelastic shear stress and the small weighted factor, and the viscous and turbulence contributions are almost the same as those of Newtonian fluids. It is seen clearly that with the increase of drag-reduction rate, the viscous contribution becomes larger, indicating the flow is further laminarized. For all the drag-reduction cases, the turbulence contribution becomes smaller as compared to the Newtonian fluid case and Flow B(3) has the smallest turbulence contribution, but drag-reduction rate is not proportional to the decrease of turbulence contribution. The turbulence contributions of Flow A(4), Flow B(1) and Flow B(2) are almost the same, but their drag-reduction rates differ from each other. This is due to different viscoelastic contributions, for example, Flow B(1) has the largest viscoelastic contribution to increase frictional drag, thus its drag-reducing ability is smallest. The comparisons show that a large drag-reduction rate depends on not only the decrease of Reynolds shear stress but also the viscoelastic shear stress.

The integrated balance equations of mean kinetic energy, turbulent kinetic energy and elastic energy can be derived as follows:

\[ \int_{-1}^{1} U^+ \frac{d \bar{u}^+}{dy} \, dy = \int_{-1}^{1} -u^+ v^+ \frac{\partial U^+}{\partial y} \, dy + \int_{-1}^{1} \frac{d U^+}{dy} \, dy + \int_{-1}^{1} \frac{\beta C_{xy}^+}{\text{We}_z} \, dy \]

energy provided by Reynolds gradient shear stress

\[ \int_{-1}^{1} \left( \frac{\partial U^+}{\partial x} \right) \left( \frac{\partial U^+}{\partial x} \right) - \int_{-1}^{1} \frac{\beta C_{ik}^+ \frac{\partial U^+}{\partial x}}{\text{We}_z} \right) \]

turbulent dissipation work by fluctuating viscoelastic shear stress

\[ 0 = \int_{-1}^{1} \frac{\beta}{\text{We}_z} \frac{\partial U^+}{\partial y} C_{xy}^- - \int_{-1}^{1} \frac{\beta}{\text{We}_z} \frac{\partial U^+}{\partial x} C_{ij}^+ + \int_{-1}^{1} \frac{\beta}{2 \text{We}_z^2} [3(C_{ij}^- - \alpha(C_{im}^- - \delta_{im}^0)] \]

elastic dissipation

It is seen that the interaction of network structures with solvent alters the energy transportation process. In the budget equations of mean kinetic energy and turbulent kinetic energy, there are additional terms due to interaction of the network structures and solvent. The elastic energy equation shows that the stretching of the network structures absorbs energy from the mean kinetic energy and turbulent kinetic energy and then the relaxation of the network structures releases the energy to elastic dissipation. Table 3 lists elastic dissipations. It is seen that generally the larger the elastic dissipation, the larger the drag-reduction rate. But Flow B(1) is an exception; it has a larger elastic dissipation than Flow A(4) and Flow B(2), but a smaller drag-reduction rate. This is because in Flow B(1), most of the energy of the elastic dissipation comes from mean kinetic energy and only a small amount of energy comes from turbulent kinetic energy. Thus the turbulence has not been effectively suppressed.

Finally, we use our present DNS results to explain the drag-reduction and post-drag-reduction phenomena. Fig. 10 is a typical diagram of drag-reduction rate versus mean velocity (Gasljevic et al., 2001). The figure is characterized by two regions, DR region and post-DR region. In the DR region, the drag-reduction rate increases with the increase of mean velocity, whereas in the post-DR region, drag-reduction rate decreases. The increase and then
decrease of DR rate can be explained as follows. It is generally accepted that the network structures are shear stress dependent. Above a critical shear stress, some rod-like micelles in the surfactant solution begin to connect to form network structures. With the increase of shear stress, more rod-like micelles connect to form network structures. At the DR region, the local effective shear stress at the wall region (viscous shear stress plus Reynolds shear stress) is above the critical shear stress and at the center region is below the critical value, so the network structures can form in the near-wall region; this flow motion is similar to Flow B. However, as the shear stress increases larger than a second critical value, the network structures begin to destroy. In the post-DR region, the effective shear stress at the near-wall region becomes relatively large, and the network structures there begin to be destroyed. This flow motion can be modeled as Flow A. By modeling the flow in the DR region as Flow B and flow in the post-DR region as Flow A, we can see drag-reduction rate increase and then decrease phenomena have been qualitatively reproduced as shown in Table 1. In addition, we have some turbulence statistics to support the drag-reducing flow at lower mean velocity, and the network structures exist in the near-wall region such as Flow B(1). Experiments show that at lower mean velocity, there is a lower velocity profile at the buffer layer as compared to Newtonian fluid (Eschenbacher, 2002). As shown in Fig. 4, the velocity profile of Flow B(1) is lower than that of Newtonian fluid, in agreement with the experiments. Experiments also show that at a low mean velocity, $u_{rms}$ decreases as compared to Newtonian fluid (Schmidt, 1997 and Yu et al. 2003). The $u_{rms}$ of Flow B(1) decreases, which also agrees with the experiments as shown in Fig. 5.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{DR rate versus mean velocity}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{DR% / DR%$_{\text{max}}$ versus Re$_b$/Re$_{b,\text{max}}$}
\end{figure}

In the above paragraph we have explained that the viscous shear stress and Reynolds shear stress (effective shear stress) can help to form or destroy network structures dependent on their magnitude. When the effective shear stress is larger than the second critical value, which is equal to the viscous shear stress at the wall of the maximum DR rate flow (Flow B(3)), the network structures begin to destroy. However in the present DNSs, the interface of Newtonian and Non-Newtonian fluids was assumed changeable at a fixed frictional Reynolds number. The effective shear stresses (dimensional) at the interface in Flows A(2), A(3) and A(4) are less than the wall shear stress of Flow B(3) with Flows A(2) smallest, but the network structures between the wall and the interface were assumed to destroy. Therefore we can assume there are three Flows, Flow A’(2), Flow A’(3) and Flow A’(4), which have the same DR rate and shear stress budget as Flow A(2), A(3) and A(4) respectively, but the sum of the dimensional viscous shear stress and Reynolds shear stress at the interface of the three flows are the same as that of the dimensional viscous wall shear stress of Flow B(3). Based on this assumption we convert the interface positions to corresponding higher Reynolds numbers of Flow A’(2), Flow A’(3), Flow A’(4), which are 1.23, 1.70 and 2.55 times as large as Flow B(3). The relationships between DR% / DR%$_{\text{max}}$ and Re$_b$/Re$_{b,\text{max}}$ (Re$_{b,\text{max}}$ is the Reynolds number of the maximum DR rate) of the numerical prediction and experiment (Li et al., 2004) are compared in Fig.11. It
is seen that the agreement is quantitatively good. The estimation based on the simple criterion ($\tau_{\text{eff}} > \tau_{\text{critical}}$) is surprisingly good. This fact suggests that the fracture of micellar network structure is strongly related to the local effective shear stress instead of shear rate estimated by the mean velocity gradient. Further investigation on this point may be interesting in the scope of searching the fracture limit in the development of micellar destruction devices (Li et al, 2001).

5. Conclusion

We studied drag-reducing flow with coexisting Newtonian and non-Newtonian fluids. Numerical results show network structures are most effective in reducing frictional drag in the buffer layer. The drag-reduction rate is not only closely associated with the reduction of Reynolds shear stress but also the induced viscoelastic shear stress. The Reynolds number dependency of skin friction in DR and post-DR regions of surfactant solutions are quantitatively explained by a simple two-layer model.

6. Acknowledgements

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