written to the non-dimensional form

$$A_3K_3=\frac{1}{2}(A_1K_1)^2(A_2K_2)(\omega_1 t)F(\gamma)(2-\gamma)^2. \qquad (\text{VIII}-4)$$

In case of perpendicular waves, non-dimensional coefficient is calculated to be $F(\gamma)(2-\gamma)^2=0.633$.

AppendixIX Analysis of Interaction Equations

1 Construction of Single Equation

We wright down again the interaction equations (3 − 3 − 1) ~ (3 − 3 − 3) such that

$$i\frac{dB_1}{dt} = [T_{11}b_1^2 + T_{12}b_2^2 + T_{13}b_3^2]\ B_1 + T_1 B_1^* B_2 B_3 e^{i\Delta t}, \qquad (IX-1-1)$$

$$i\frac{dB_2}{dt} = [T_{21}b_1^2 + T_{22}b_2^2 + T_{23}b_3^2]\ B_2 + T_2 B_3^* B_1 B_1 e^{-i\Delta t} \qquad (IX-1-2)$$

and

$$i\frac{dB_3}{dt} = [T_{31}b_1^2 + T_{32}b_2^2 + T_{33}b_3^2]\ B_3 + T_3 B_2^* B_1 B_1 e^{-i\Delta t} \qquad (IX-1-3)$$

in which $b_n^2 = B_n B_n^*$ and $\Delta = \omega_1 + \omega_1 - \omega_2 - \omega_3$. Interaction coefficients T_n and symmetric matrix elements $[T_{kl}] = [T_{lk}]$ are real constants to be calculated from wave-numbers. The method of solution adopted here is that used by McGoldrick(1972) for second order non-linear equations in the context of capirally-gravity waves.

Multiplying B_1^* to (IX − 1 − 1) we obtain

$$iB_1^*\frac{dB_1}{dt} = [T_{11}b_1^2 + T_{12}b_2^2 + T_{13}b_3^2]\ b_1^2 + T_1 B_1^* B_1^* B_2 B_3 e^{i\Delta t}.$$

Taking the complex conjugate of this equation such as

$$-iB_1\frac{dB_1^*}{dt} = [T_{11}b_1^2 + T_{12}b_2^2 + T_{13}b_3^2]\ b_1^2 + T_1 B_1 B_1 B_2^* B_3^* e^{-i\Delta t},$$

and subtracting this from the former equation, it reduces to

$$i\frac{db_1^2}{dt} = T_1\ (R - R^*)\ . \qquad (IX-2-1)$$

In this expression, a complex quantity R is introduced such as

$$R = B_1^* B_1^* B_2 B_3 \exp(i \Delta t) \quad .$$

Similar relations are obtained by using (IX－1－2), (IX－1－3) that

$$i \frac{d\, b_2^2}{d\, t} = - T_2 (R - R^*) \qquad (\text{IX}-2-2)$$

$$i \frac{d\, b_3^2}{d\, t} = - T_3 (R - R^*) \qquad (\text{IX}-2-3)$$

From the relations (IX－2－1) ～ (IX－2－3) we have three integrals

$$b_1^2 / T_1 + b_2^2 / T_2 = \text{const}_1 = \bar{b}_1^2 / T_1 + \bar{b}_2^2 / T_2 \qquad (\text{IX}-3-1)$$

$$b_1^2 / T_1 + b_3^2 / T_3 = \text{const}_2 = \bar{b}_1^2 / T_1 + \bar{b}_3^2 / T_3 \qquad (\text{IX}-3-2)$$

$$b_2^2 / T_2 - b_3^2 / T_3 = \text{const}_3 = \bar{b}_2^2 / T_2 - \bar{b}_3^2 / T_3 \qquad (\text{IX}-3-3)$$

where $\bar{b}_n = b_n(0)$, (n=1, 2, 3), the initial value of $b_n(t)$.
By use of these integral properties, a complex function $Z(t)$ is introduced such as

$$Z(t) \equiv (\bar{b}_1^2 - b_1^2) / T_1 = (b_2^2 - \bar{b}_2^2) / T_2 = (b_3^2 - \bar{b}_3^2) / T_3 . \qquad (\text{IX}-4)$$

We can easily calculate that

$$dZ / dt = i (R - R^*) = -2\, \mathrm{Im}(R) \qquad (\text{IX}-5)$$

In order to calculate the real part of R, we differentiate R with respect to t, that is,

$$dR / dt = 2 B_1{}^*{}_t B_1^* B_2 B_3 \exp(i \Delta t) + B_1^{*2} B_{2t} B_3 \exp(i \Delta t)$$
$$+ B_1^{*2} B_2 B_{3t} \exp(i \Delta t) + i \Delta B_1^{*2} B_2 B_3 \exp(i \Delta t) .$$

Substituting (IX－1－1) ～ (IX－1－3) to this expression, it is yielded that

$$dR/dt = i\Delta R + 2i\,[T_{11}b_1^2 + T_{12}b_2^2 + T_{13}b_3^2]\,R + 2iT_1 b_1^2 b_2^2 b_3^2$$
$$-i\,[T_{21}b_1^2 + T_{22}b_2^2 + T_{23}b_3^2]\,R - iT_2 b_1^4 b_3^2$$
$$-i\,[T_{31}b_1^2 + T_{32}b_2^2 + T_{33}b_3^2]\,R - iT_3 b_1^4 b_2^2 \quad .$$

Taking the complex conjugate of this equation and adding them together, the result is expressed by

$$d(R+R^*)/dt = i\Delta\,(R-R^*)$$
$$+2i\,[T_{11}b_1^2 + T_{12}b_2^2 + T_{13}b_3^2]\,(R-R^*)$$
$$-i\,[T_{21}b_1^2 + T_{22}b_2^2 + T_{23}b_3^2]\,(R-R^*)$$
$$-i\,[T_{31}b_1^2 + T_{32}b_2^2 + T_{33}b_3^2]\,(R-R^*)\;.$$

Considering the relation (IX−5), it is transformed to

$$d(R+R^*)/dt = \{\Delta + \hat{T}_1 b_1^2 + \hat{T}_2 b_2^2 + \hat{T}_3 b_3^2\}\,dZ/dt$$

where $\hat{T}_n = 2T_{1n} - T_{2n} - T_{3n}$. Next, b_n^2 (n=1, 2, 3) is eliminated by use of (IX−4), and we have

$$d(R+R^*)/dt = \{\Delta + \hat{T}_1(\hat{b}_1^2 - T_1 Z) + \hat{T}_2(\hat{b}_2^2 + T_2 Z)$$
$$+\hat{T}_3(\hat{b}_3^2 + T_3 Z)\}\,dZ/dt\;.$$

In this formula, direct integration is possible such that

$$2\mathrm{Re}(R) = R + R^* = H + \{\Delta + \hat{T}_1\hat{b}_1^2 + \hat{T}_2\hat{b}_2^2 + \hat{T}_3\hat{b}_3^2\}\,Z$$
$$-\frac{1}{2}\{\hat{T}_1 T_1 - \hat{T}_2 T_2 - \hat{T}_3 T_3\}\,Z^2 \qquad \text{(IX−7)}$$

where H is a real constant determined by initial conditions.
In order to fulfil the apparent equality that

$$|R|^2 = \{\mathrm{Re}(R)\}^2 + \{\mathrm{Im}(R)\}^2\;,$$

The relations (IX − 5) and (IX − 7) are connected to

$$4(\acute{b}_1^2 - T_1 Z)^2 (\acute{b}_2^2 + T_2 Z)(\acute{b}_3^2 + T_3 Z)$$

$$= (H + \xi Z + \eta Z^2)^2 + (dZ/dt)^2 \qquad \text{(IX − 8)}$$

where ξ and η are the coefficients determined in (IX − 7).

2 Analysis of Resonant Growth

In the case that tertiary wave component does not exist initially, we can set the constant $H = 0$ and $\acute{b}_3^2 = 0$ in (IX − 8) so that we investigate the equation of the form

$$(dZ/dt)^2 = f(Z) \qquad \text{(IX − 9)}$$

where f is a quaritic function of Z such as

$$f(Z) = 4(\acute{b}_1^2 - T_1 Z)^2 (\acute{b}_2^2 + T_2 Z) T_3 Z$$

$$- [\{\Delta + \hat{T}_1 \acute{b}_1^2 + \hat{T}_2 \acute{b}_2^2\} - \tfrac{1}{2}\{\hat{T}_1 T_1 - \hat{T}_2 T_2 - \hat{T}_3 T_3\} Z]^2 Z^2 .$$

$$\text{(IX − 10)}$$

In general, real solution Z exists and can be solved by means of a integration

$$\int_0^t dt = \int_0^Z \frac{dx}{\sqrt{f(x)}} \qquad \text{(IX − 11)}$$

if $f(x)$ is positive at $0 < x \leqq Z$.

In order to obtain a formal solution, we must rearrange the polynomial $f(x)$ in its standard form such as (see Jeffreys & Jeffreys (1972)),

$$f(x) = a_0 x^4 + a_1 x^3 + a_2 x^2 + a_3 x = \phi(x)$$

and it is resolved to the factors such that

$$\phi(x) = \phi_1(x)\,\phi_2(x)$$

where

$$\phi_1(x) = ax^2 + bx + c \quad \text{and} \quad \phi_2(x) = x^2 + \beta x \ .$$

A bilinear transformation of the variable is performed by

$$x = (Ay + B) / y + 1 \ , \qquad (\text{IX}-12)$$

in which A and B are real roots of the following equation,

$$(b - a\beta)\zeta^2 + 2c\zeta + c\beta = 0 \ . \qquad (\text{IX}-13)$$

In this procedure, integrant of (IX−11) is transformed as

$$\frac{dx}{\sqrt{\phi(x)}} = \frac{(A-B)\,dy}{\sqrt{P\{y^2+M\}\{y^2+N\}}} \ . \qquad (\text{IX}-14)$$

There are several cases according to the signs of $P = \phi(A)$, $M = \phi_1(B) / \phi_1(A)$ and $N = \phi_2(B) / \phi_2(A)$.

Case I ; $P > 0$, $M = \mu^2 > 0$, $N = -\nu^2 < 0$
In this case, (IX−14) is rewritten by

$$F(y)\,dy = \frac{(A-B)\,dy}{\sqrt{P\{y^2+\mu^2\}\{y^2-\nu^2\}}} \ . \qquad (\text{IX}-15)$$

Transformation $y^2 = \nu^2 / (1 - u^2)$ is adopted and

$$F(y)\,dy = \frac{(A-B)\,du}{\sqrt{P\{\mu^2+\nu^2\}(1-u^2)(1-k^2u^2)}}$$

$$(\text{IX}-16)$$

results in the form of elliptic integral of first kind after some manipulation. In this formula, $k^2 = \mu^2 / \{\mu^2 + \nu^2\}$ is called the generatrix of the integral.
Defining $\Omega = \sqrt{P\{\mu^2+\nu^2\}} / (A - B)$, integral (IX−11) reduces to

$$\Omega t = \int_{u_0}^{u} \frac{dv}{\sqrt{(1-v^2)(1-k^2v^2)}} \qquad (\text{IX}-17)$$

and

$$u^2 = 1 - \nu^2 (A - Z)^2 / (B - Z)^2 \ . \qquad (\text{IX}-18)$$

From (IX－17), we obtain

$$u = sn(\Omega t + \theta ; k^2)$$

and from (IX－18),

$$(A-Z)/(B-Z) = \nu^{-1} cn(\Omega t + \theta ; k^2) \qquad \text{(IX－19)}$$

in which sn and cn are the Jacobi's elliptic functions.
Thus, the formal solution of (IX－9) is expressed by

$$Z = \frac{A - \mathrm{sig}(B)\, B \nu^{-1} cn(\Omega t + \theta ; k^2)}{1 - \mathrm{sig}(B)\, \nu^{-1} cn(\Omega t + \theta ; k^2)}, \qquad \text{(IX－20)}$$

where sig (B) means the signum of B.
To satisfy the initial condition that $Z=0$ at $t=0$, constant θ is determined by

$$A - \mathrm{sig}(B)\, B \nu^{-1} cn(\theta ; k^2) = 0 . \qquad \text{(IX－21)}$$

An example of this solution is shown in Fig－A－1. In this Figure, the variation of resonant wave amplitude A_3 is described under the conditions that A_1=4cm and A_2=5cm initially with γ=1.80. The solid line is the solution obtained by the method discussed here. The symbol ○ is the numerical solution obtained in Chapter 3 (Fig－3－3 (c)). Both results which are obtained independently, coincide appreciably. Precision of the numerical procedure adopted in Chapter 3 is confirmed to be sufficient.

Case II ; $P>0$, $M=-\mu^2<0$, $N=-\nu^2<0$
In this case, (IX－14) is rewritten by

$$G(y)\,dy = \frac{(A-B)\,dy}{\sqrt{P\{y^2-\mu^2\}\{y^2-\nu^2\}}} . \qquad \text{(IX－22)}$$

Transformation $y^2=\nu^2/u^2$ is adopted this time and

$$G(y)\,dy = \frac{(B-A)\,du}{\sqrt{P\nu^2(1-u^2)(1-k^2u^2)}}$$

(IX－23)

results also in the form of elliptic integral and $k^2 = \mu^2 / \nu^2$.
By the same procedure as in Case I, with $\Omega = \sqrt{P \nu^2} / (B - A)$ we have

$$Z = \frac{A + B \nu^{-1} \mathrm{sn}\,(\Omega t + \theta\ ;\ k^2)}{1 + \nu^{-1} \mathrm{sn}\,(\Omega t + \theta\ ;\ k^2)} \quad . \qquad (\text{IX}-24)$$

To satisfy the initial condition that $Z = 0$ at $t = 0$, constant θ is determined by

$$A + B \nu^{-1} \mathrm{sn}\,(\theta\ ;\ k^2) = 0 \ . \qquad (\text{IX}-25)$$

The transition from Case I to Case II occures under the condition of maximum growth of tertiary resonant wave which is clearly shown also by the numerical solution discussed in Ch 3 of this paper.

3 Non-Periodic Solution

If we change the initial condition $\acute{b}_1^2$ or $\acute{b}_2^2$, two types of solution appear as interpreted in the previous section. Although both types of solution are periodic, there exist an aperiodic solution just at the critical region between Case I and Case II.

Returning to (IX−10), if the relation

$$\{\Delta + \hat{T}_1 \acute{b}_1^2 + \hat{T}_2 \acute{b}_2^2\} - \tfrac{1}{2} \{\hat{T}_1 T_1 - \hat{T}_2 T_2 - \hat{T}_3 T_3\} \acute{b}_1^2 / T_1 = 0 \qquad (\text{IX}-26)$$

is assumed to be realized, that is, the parameter $\acute{b}_1^2$, say, is sought so as to satisfy the following equation to the fixed $\acute{b}_2^2$, T_n, $\hat{T}_n$ (n=1, 2, 3) and Δ

$$\Delta = - \hat{T}_2 \acute{b}_2^2 - \tfrac{1}{2} \{\hat{T}_1 T_1 + \hat{T}_2 T_2 + \hat{T}_3 T_3\} \acute{b}_1^2 / T_1 \ , \qquad (\text{IX}-27)$$

the equation $f(Z) = 0$ has a double root at $Z = \acute{b}_1^2 / T_1 = \beta$ and $f(Z)$ is represented by

$$f(Z) = - a Z (Z - \beta)^2 (Z - \gamma) \qquad (\text{IX}-28)$$

where

$$a = -4T_1^2 T_2 T_3 + \frac{1}{4}\{\hat{T}_1 T_1 + \hat{T}_2 T_2 + \hat{T}_3 T_3\}^2 > 0,$$

$$\beta = \hat{b}_1^2 / T_1 > 0$$

and

$$\gamma = 4T_1^2 T_3 \hat{b}_2^2 / a > 0$$

are the positive constants in this situation with $\beta < \gamma$.

In this special case, (IX−9) is easily solved and the non-periodic solution is obtained as follows,

$$Z = \frac{\beta\gamma \tanh^2 \lambda t}{(\gamma - \beta) + \beta \tanh^2 \lambda t} \qquad \text{(IX−29)}$$

where $\lambda = \{a\beta(\gamma - \beta)\}^{1/2} / 2$.

It is remarkable that Z approaches a constant β when t goes to infinity and all the energy initially contained in the first primary wave is transferred monotonically to the other components. Note that maximum amplitude realized by tertiary resonant wave a_3 is determined only by the initial value of the first primary wave amplitude a_1 and is independent of a_2 as discussed in Ch3. The condition (IX−27) is fulfiled even $\Delta = 0$ (exact resonance $\gamma = 1.736$). In this condition, the ratio of amplitudes of two primary waves is determined $a_2 / a_1 = 3.1655\cdots\cdots$. To the values computed numerically in Ch3, it corresponds that $a_1 = 1.5795\cdots\cdots$cm and the asymptotic growth of tertiary wave would be $a_3 = 1.332\cdots\cdots$cm which are consistent with the numerical results.

For the case of wave instability problem, we can apply this theory by the following manner. This time b_1^2 is a primary wave and $b_2^2 = b_3^2 = b_s^2$ are two side band components recognized as small perturbations. To the leading order, $T_1 = T_2 = T_3 = T = k_1^3 / 4\pi^2$, $\hat{T}_1 = \hat{T}_2 = \hat{T}_3 = 0$ and $\Delta = 0$ so that (IX−9) and (IX−10) are reduced to

$$(dZ/dt)^2 = 4T^4 (Z - \beta)^2 (Z + \gamma)^2 . \qquad \text{(IX−30)}$$

where $\beta = \hat{b}_1^2 / T$, $\gamma = \hat{b}_s^2 / T$ and $\beta \gg \gamma$. This equation is easily solved as

$$Z = \gamma\{\exp(2\beta T^2 t) - 1\}$$

and evolution of the amplitude of side band components is expressed in terms of the steepness of primary wave such that

$$a_s(t) = a_{s0} \exp\{\tfrac{1}{2}(a_1 k_1)^2 \omega_1 t\} . \qquad (IX-31)$$

The growth rate of side band components $\frac{1}{2}(a_1 k_1)^2 \omega_1$ obtained in this theory is in accordance with the Benjamin-Feir(1967) theory.

REFERENCES

Barrick, D. E. & Weber, B. L. (1977): On the nonlinear theory for gravity waves on the ocean's surface. Part2. J. Phys. Oceanogr., Vol. 7., pp11～21.

Bendat, J. S. & Piersol, A. G. (1967): Measurement and Analysis of Random Data, John Wiley & Snos

Benney, D. J. (1962): Non-linear gravity wave interactions. J. Fluid Mech., Vol. 14, pp577～584.

Benjamin, T. B. & Feir, J. E. (1967): The disintegration of wave trains on deep water. J. Fluid Mech., Vol. 27, pp417～430.

Bogoliubov, N. N. & Mitropolskii, Y. A. (1965): Nonlinear Oscillation, Kyouritu (Japanese translation)

Chereskin, T. K. & Mollo-Christensen, E. (1985): Modulational development of nonlinear gravity-wave groups. J. Fluid Mech., Vol. 151, pp337～365.

Crawford, D. R., Lake, B. M., Saffman, P. G. & Yuen, H. C. (1981): Stability of weakly nonlinear deep-water waves in two and three dimension. J. Fluid Mech., Vol. 105, pp177～191.

Crawford, D. R., Lake, B. M., Saffman, P. G. & Yuen, H. C. (1981): Effects of nonlinearity and spectral bandwidth on the dispersion relation and component phase speed of surface gravity waves. J. Fluid Mech., Vol. 112, pp1～32.

Donelan, M. A. (1987): The affect of swell on the growth of wind waves. The JHU Appl. Phys. Lab. Tech. Dig., Vol. 8, pp18～23.

Dysthe, K. B. (1979): Note on a modification to the nonlinear Schroedinger equation for application to deep water waves. Proc. Roy. Soc. London, Ser. A369, pp105～114.

Fox, M. J. H. (1976): On the nonlinear transfer of energy in the peak of a gravity-wave spectrum. Proc. R. Soc. Lond. A. 348, pp467～483.

Gerstner, F. J. v. (1809): Theorie der Wellen. Ann. der Physik, Vol. 32, pp412～440.

Hamada, T. (1965): The secondary interactions of surface waves. Rept. Port and Harbour Tech. Res. Inst., Vol. 4, pp1～28 (in Japanese).

Hasselmann, K. (1962): On the nonlinear energy transfer in a gravity wave spectrum. Part1. J. Fluid Mech., Vol. 12, pp481～500.

Hasselmann, K. (1963a): On the nonlinear energy transfer in a gravity wave spectrum. Part2. J. Fluid Mech., Vol. 15, pp273～281.

Hasselmann, K. (1963b): On the nonlinear energy transfer in a gravity wave spectrum. Part3. J. Fluid Mech., Vol. 15, pp385～398.

Huang, N. & Tung, C. C. (1976): The dispersion relation for a nonlinear random gravity wave field. J. Fluid Mech., Vol. 75, pp337～345.

Ishida, S. & Watanabe, I. (1980): On the properties of oblique waves generated with Snake motion wave generator. J. Western Naval Architects, Vol. 69, pp135～141 (in Japanese).

Jeffreys, H. & Jeffreys, B. S. (1972): Methods of Mathematical Physics, 3-rd ed., Cambridge University Press.

Kinsman, B. (1965): Wind Waves, Prentice Hall

Leibovich, S. & Seebass, A. R. (1974): Non-Linear Waves, Cornell Univ. Press

Li, J. C., Hui, W. H. & Donelan, M. A. (1987): Effects of velocity shear on the stability of surface deep water wave train. Proc. IUTAM Sympo. Nonlinear Water Waves., Springer-Verlag, pp213～220.

Longuet-Higgins, M. S. (1962): Resonant interactions between two trains of gravity waves. J. Fluid Mech., Vol. 12, pp321～332.

Longuet-Higgins, M. S. (1976): On the nonlinear transfer of energy in the peak of a gravity-wave spectrum:a simplified model. Proc. R. Soc. Lond. A. 347, pp311～328.

Longuet-Higgins, M. S. & Phillips O. M. (1962): Phase velocity effects in tertiary wave interactions. J. Fluid Mech., Vol. 12, pp333～336.

Longuet-Higgins, M. S. & Smith N. D. (1966): An experiment on third order resonant interactions. J. Fluid Mech., Vol. 25, pp417～435.

Mase, H. & Iwagaki, Y. (1985): Analysis of the wave-group characteristics of the field data based upon the modulation-instability theory. Papers 32-th Coast. Eng., pp184～188 (in Japanese).

Masuda, A., Kuo, Y. Y. & Mitsuyasu, H. (1979): On the dispersion relation of random gravity waves. Part1. J. Fluid Mech., Vol. 92, pp717～730.

McGoldrick, L. F. (1972): On the rippling of small waves:a harmonic nonlinear nearly resonant interaction. J. Fluid Mech., Vol. 52, pp725～751.

McGoldrick, L. F., Phillips, O. M., Huang, N. E. & Hodgson, T. H. (1966): Measurements of third-order resonant wave interactions. J. Fluid Mech., Vol. 25, pp437～456.

McLean, J. W. (1982): Instabilities of finite-amplitude water waves. J. Fluid Mech., Vol. 114, pp315～330.

Milder, D. M. (1977): A note regarding "On Hamilton's principle for surface waves". J. Fluid Mech., Vol. 83, pp159～161.

Miles, J. W. (1977): On Hamilton's principle for surface waves. J. Fluid Mech., Vol. 83, pp153～158.

Mitsuyasu, H., Kuo, Y. Y. & Masuda, A. (1979): On the dispersion relation of random gravity waves. Part2. J. Fluid Mech., Vol. 92, pp731～749.

Mollo-Christensen, E. (1981): Modulational stability of short-crested free surface waves. Phys. Fluids, Vol. 24, pp775～776.

Mollo-Christensen, E. & Ramamonjiarisoa, A. (1978): Modeling the presence of wave groups in a random wave field. J.G.R., Vol. 83, pp4117～4122.

Mollo-Christensen, E. & Ramamonjiarisoa, A. (1982): Subharmonic transitions and group formation in a wind wave field. J.G.R., Vol. 87, pp5699～5717.

Nagata, Y. (1970): Ocean waves. Lectures in Oceanography 3, 1. Tokai Univ. Press, pp1～107 (in Japanese).

Nayfeh, A.H. (1973): Perturbation Methods, John Wiley & Sons

Newman, J. (1977): Marine Hydrodynamics, The MIT Press

Okamura, M. (1984): Instabilities of weakly non-linear standing gravity waves. J. Phys. Soc. Japan, Vol. 53, pp3788～3796.

Okamura, M. (1985): On the instability of weakly non-linear three-dimensional standing waves. J. Phys. Soc. Japan, Vol. 54, pp3313～3320.

Peregrine, D.H. (1983): Water waves, nonlinear Schroedinger equations and their solutions. J. Austral. Math. Soc., Ser. B25, pp16～43.

Phillips, O.M. (1960): On the Dynamics of unsteady gravity waves of finite amplitude Part1. J. Fluid Mech., Vol. 9, pp193～217.

Phillips, O.M. (1977): The Dynamics of the Upper Ocean, 2-nd ed., Cambridge

Pierson, W.J., Jr. (1952): A unified mathematical theory for the analysis, propagation and refraction of storm generated ocean surface waves Part1 and 2. N.Y.U., Coll. of Eng. Res. Div., Dept. of Meteorol. and Oceanogr. Prepared for the Beach Erosion Board, Dept. of the Army, and Office of Naval Res., Dept. of the Navy., pp461.

Pierson, W.J., Jr., Neumann, G. & James, R.W. (1955): Practical methods for observing and forecasting ocean waves by means of wave spectra and statistics. U.S. Navy Hydrogr. Office Pub., No. 603.

Sand, S.E. (1988): Discussion of Report Ⅰ.1 (Environmental Conditions) in 10th ISSC (Copenhagen).

Shiba, H. (1961): On the Mitaka Ship Experimental Basin. Rep. Inst. Trans., Vol. 11, pp625～647 (in Japanese).

Snodgrass, F.E., Groves, G.W., Hasselmann, K.F. Miller, G.R., Munk, W.H. & Powers, W.H. (1966): Propagation of ocean swell across the pacific. Phil. Trans. Roy. Soc., Vol. 259, pp431～497.

Sobey, R.J. & Colman, E.J. (1982): Natural wave trains and scattering transform. J. Waterway Port Coastal and Ocean Div. ASCE, Vol. 108, pp272～290.

Sobey, R. J. & Colman, E. J. (1983): Scattering analysis and synthesis of wave trains. J. Australian Math. Soc. Ser. B25, pp44～63.

Stiassnie, M. & Kroszynski, U. I. (1982): Long-time evolution of an unstable water-wave train. J. Fluid Mech., Vol. 116, pp207～225.

Stiassnie, M. & Shemer, L. (1984): On modification of the Zakharov equation for surface gravity waves. J. Fluid Mech., Vol. 143, pp47～67.

Stokes, G. G. (1847): On the theory of oscillatory waves. in Math. Phys. Papers, Vol. 1, pp197～229.

Strizhkin, I. I. & Ralentnev, V. I. (1986): The experimental study of three and four-waves resonance interactions of the surface sea waves. Izv. Phys. Atomos. Ocean, Tom22, pp412～417 (in Russian).

Su, M. Y. (1982): Three-dimensional deep-water waves. Part1. J. Fluid Mech., Vol. 124, pp73～108.

Su, M. Y., Bergin, M., Marler, P. & Myrick, R. (1982): Experiments on nonlinear instabilities and evolution of steep gravity-wave train. J. Fluid Mech., Vol. 124, pp45～72.

Taira, K. (1975): Dynamics of Ocean Surface. Lectures in Oceanography 3, 2. the Univ. Tokyo Press, pp47～83 (in Japanese).

Takaishi, T., Sugai, K. & Ogawa, A. (1973a): Recent Development of the Model Ship Test in Rectangular Wave Basin(Part1). J. Naval Architects, Vol. 525, pp16～25 (in Japanese).

Takaishi, T., Sugai, K. & Ogawa, A. (1973b): Recent Development of the Model Ship Test in Rectangular Wave Basin(Part2). J. Naval Architects, Vol. 526, pp14～23 (in Japanese).

Tani, I., Kobasi, Y. & Sato, H. (1977): Experimental Methods in Fluid Dynamics, Iwanami (in Japanese)

Tick, L. J. (1959): A non-linear random model of gravity waves 1. J. Math. Mech., Vol. 5, pp643～651.

Tomita, H. (1985a): On a mutual effect of large amplitude waves. Proc. 46-th Lec. Meeting S. R. I., pp143～146 (in Japanese).

Tomita, H. (1985b): On non-linear water wave groups and the induced mean flow. Proc. The Ocean Surface Sympo., D. Reidel Pub., pp59～64.

Tomita, H. (1986): On non-linear sea waves and the induced mean flow. J. Oceanogr. Soc. Japan, Vol. 42, pp153～160.

Tomita, H. (1987): Etude numerique sur l'interaction resonante entre des vagues d'amplitude finie. La mer, Tome25, pp53～61.

Tomita, H. (1988a): Ocean Waves and Ship motions. KOKAI, Vol. 96, pp8～16 (in Japanese).

Tomita, H. (1988b): Wind Wave Characteristics in North-Western Pacific Ocean(Part1). Rept. Ship Res. Inst. (in Japanese).

Tomita, H. (1988c): Wind Wave Characteristics in North-Western Pacific Ocean(Part2). Rept. Ship Res. Inst. (in Japanese) .

Tomita, H. & Sawada, H. (1987): An experimental investigation into non-linear resonant wave interactions in the ship model basin. Proc. IUTAM Sympo. Nonlinear Water Waves., Springer-Verlag, pp341～348.

Waters, J. & Ford, J. (1966): Method of solution for resonant nonlinear coupled oscillator systems. J. Math. Phys., Vol. 7, pp399～403.

Weber, B. L. & Barrick, D. E. (1977): On the nonlinear theory for gravity waves on the ocean's surface. Part1. J. Phys. Oceanogr., Vol. 7, pp3～10.

West, B. J. (1981): Deep Water Gravity Waves, Springer-Verlag

Whitham, G. B. (1974): Linear and Nonlinear Waves, John Wiley & Sons

Yue, D. K. P. & Mei, C. C. (1980): Foreward diffraction of Stokes waves by a thin wedge. J. Fluid Mech., Vol. 99, pp33～52.

Yuen, H. C. & Lake, B. M. (1982): Nonlinear dynamics of deep-water gravity waves. Adv. Appl. Mech., Vol. 22, pp67～229.

Zakharov, V. E. (1968): Stability of periodic waves of finite amplitude on the surface of a deep fluid. J. Appl. Mech. Tech. Phys., Vol. 9, pp190～194.

Zakharov, V. E. & Shabat, A. B. (1972): Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. Soviet Phys. JETP, Vol. 34, pp62～69.

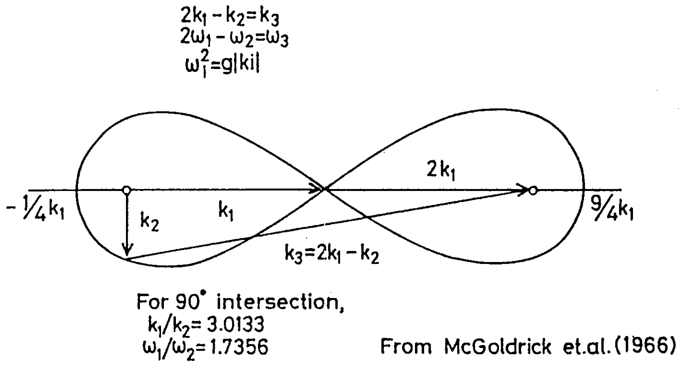


Fig. -1-1
Resonance curve;
Solutions to the resonance conditions.
K_1 : first-primary wave
K_2 : second-primary wave
K_3 : tertiary resonant wave

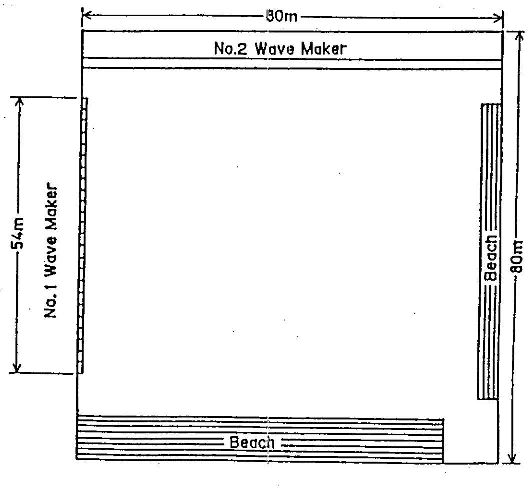


Fig. -2-1
Plan of the basin.
80m (length)×80m (width)×4.5m (depth)

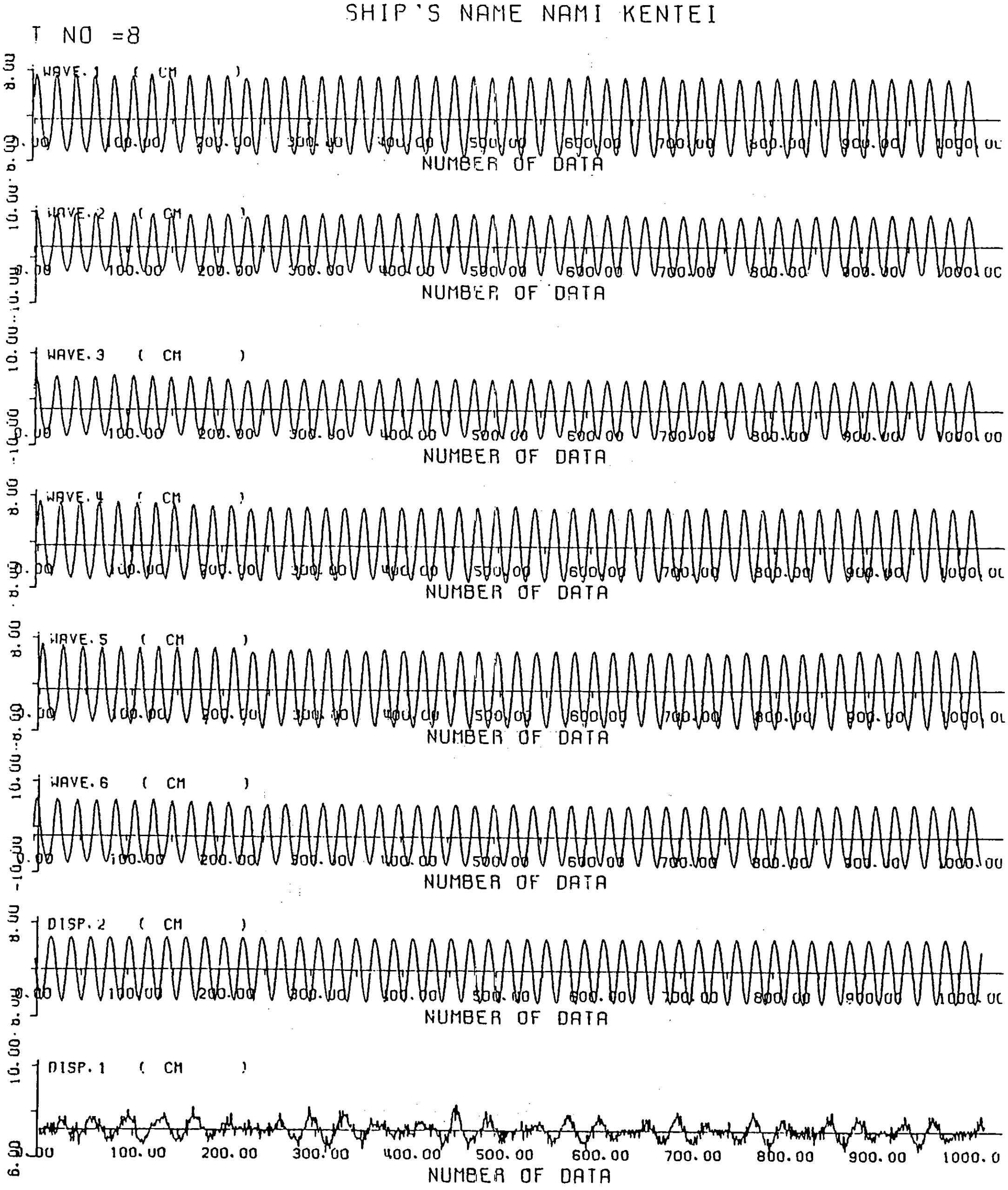


Fig. -2-2 (a)
Examples of the mesurement.
first primary wave is generated.
Upper six rows are wave records.
Lower two rows are records of stroke
of wave-makers.

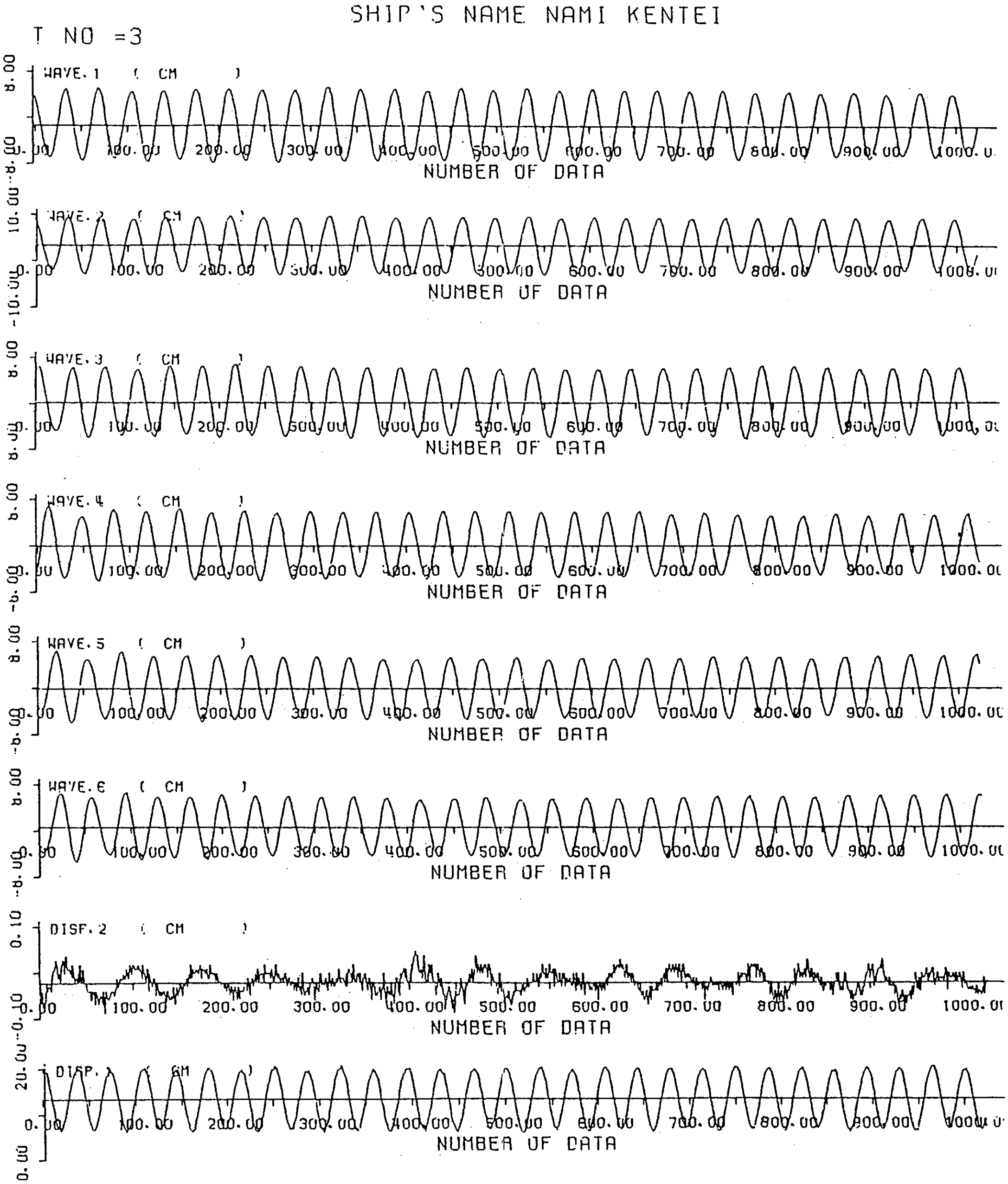


Fig. -2-2 (b)
Examples of the mesurement.
second primary wave is generated.
Upper six rows are wave record.
Lower two rows are records of stroke
of wave-makers.

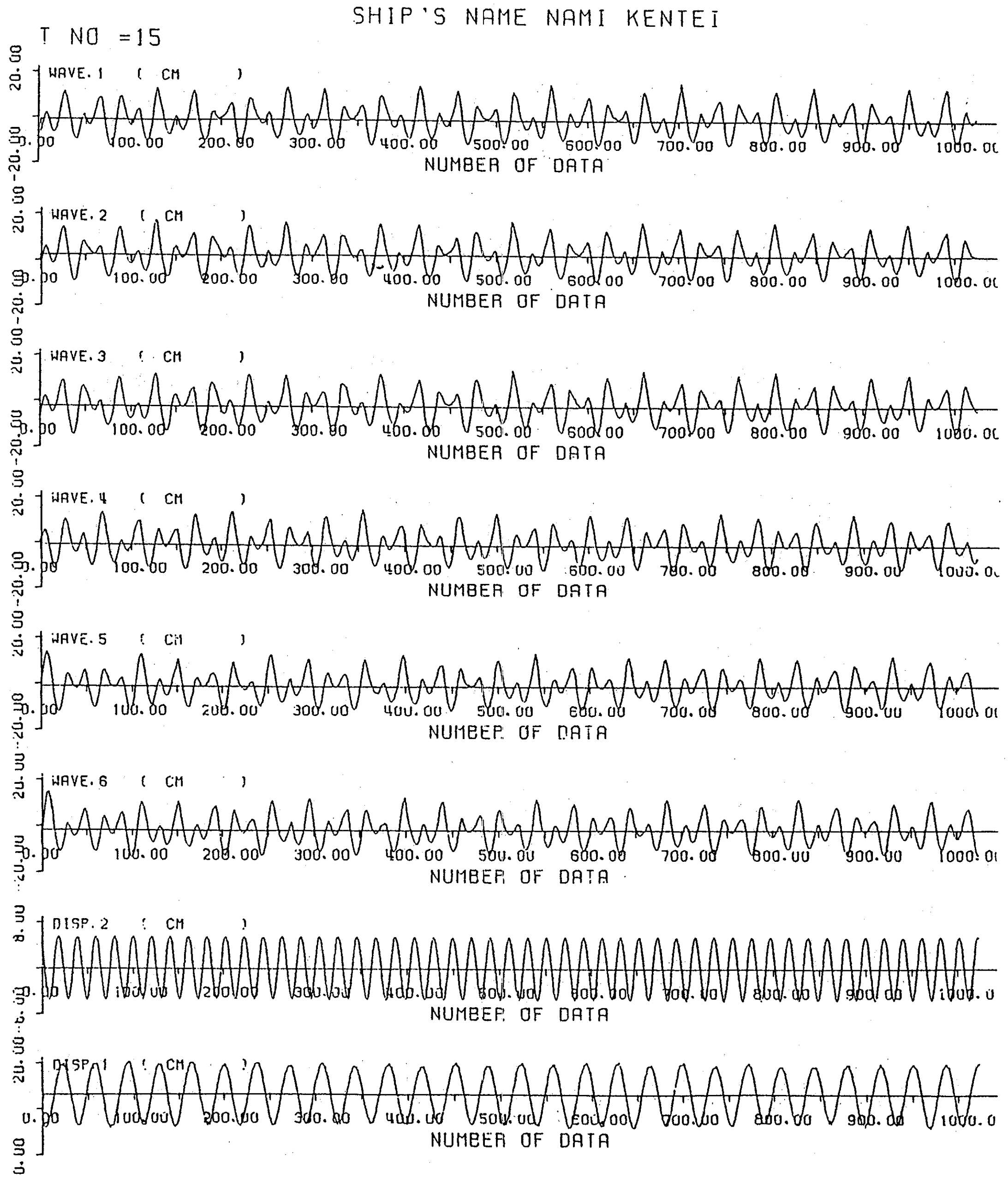


Fig. -2-2 (c)
Examples of the measurement.
both primary waves are generated.
Upper six rows are wave records.
Lower two rows are records of stroke
of wave-makers.

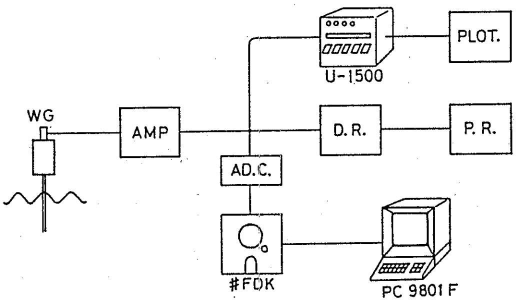


Fig. -2-3
Data collection system.
WG (wave gauge), AMP (amplifier), AD. C (AD converter)
D. R. (data recorder), P.R. (printer), PLOT(plotter)
FDK (disquet)

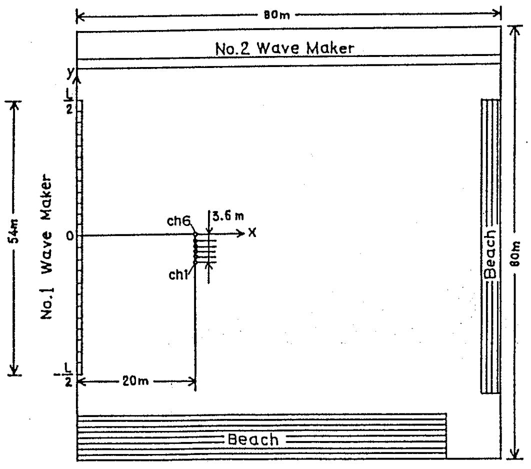


Fig. -2-4
Arrangement of the wave gauges (Case I)
For analysing the short term growth and the direction of the resonant waves.

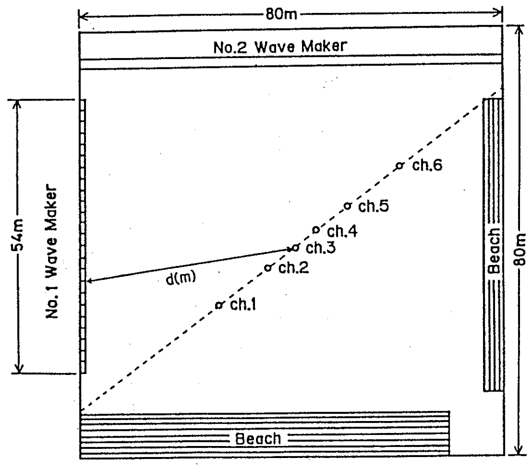


Fig. -2-5
Arrangement of the wave gauges (Case II).
For analysing the long term growth of the resonant waves.

Table-2-1 Elements of Mechanically Generated Waves

1-ST PRIMARY WAVE		2-ND PRIMARY WAVE		
PERIOD	WAVE HEIGHT	PERIOD	WAVE HEIGHT	γ
0. 93	3～13	1. 77	2. 5～10	1.897
0. 96				1.845
0. 99				1.793
1. 02				1.724
1. 10	3～13	2. 09	～5	1.898
1. 15				1.816
1. 19				1.755

PERIOD(sec), WAVE HEIGHT(cm), $\gamma = \omega_1 / \omega_2$

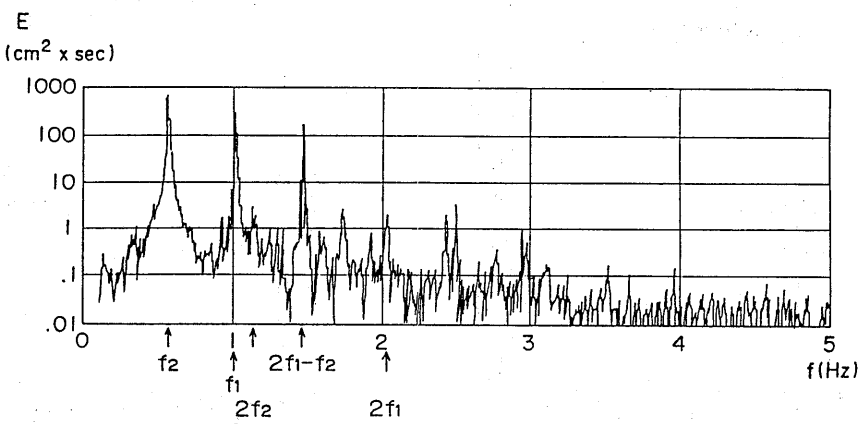


Fig. -2-6
An example of power spectrum. γ=1.793, d=45m
f_1 : 1-st primary wave, $2f_1$: 2-nd harmonics
f_2 : 2-nd primary wave, $2f_2$: 2-nd harmonics
$2f_1$-f_2 : tertiary resonant wave

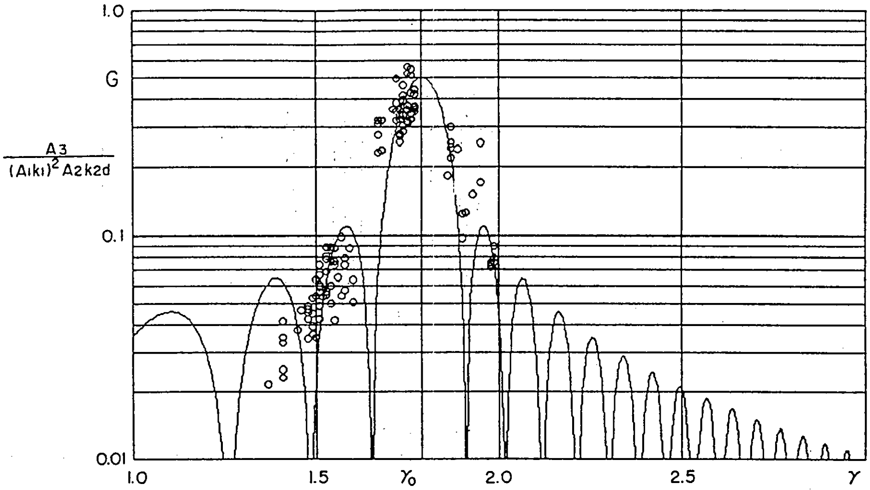


Fig. -2-7
Growth rate of the tertiary resonant waves.
G : the growth rate
γ_θ : γ of the most strong resonance
The solid curves are due to detuning effect.

Table-2-2 Observations of Initial Growth Rate

	G	γ	d
Longuet-Higgins(1962) theoretical value	0.442	1.736	
MacGoldrick et. al. experiment(1966)	0.57	1.78	15 m *
Tomita et.al. experiment(1986)	0.50	1.79	20.25 m

#The distance is converted to the size of our experiment.

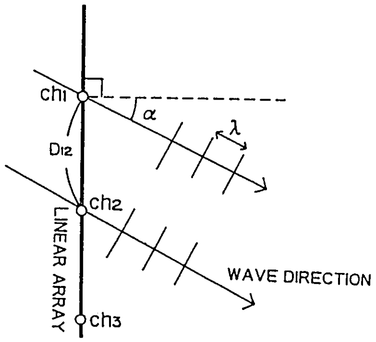


Fig. -2-8
The principle of wave direction measurement.
Ch 1 ~ Ch 3 on the array in a obliquely incident wave

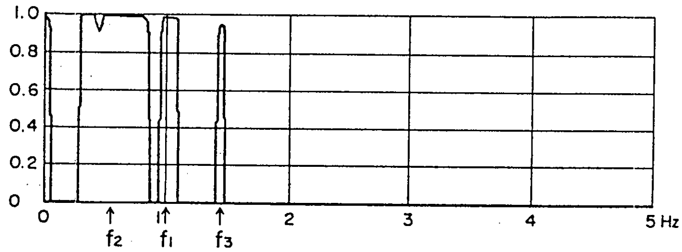


Fig. -2-9
Coherence between wave data at the locations 1 and 3 .
f_1 : 1-st primary wave
f_2 : 2-nd primary wave
f_3 : tertiary resonant wave

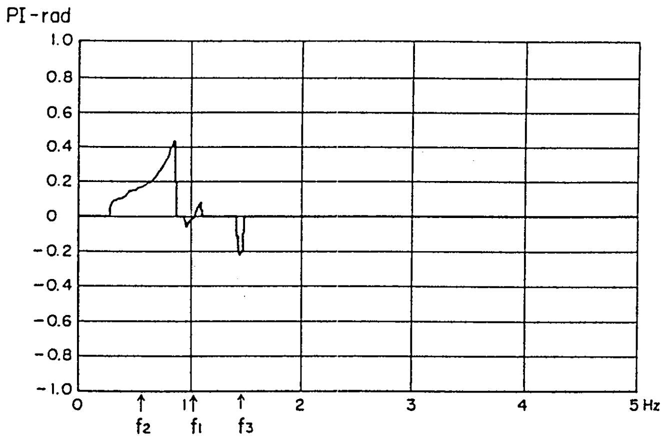
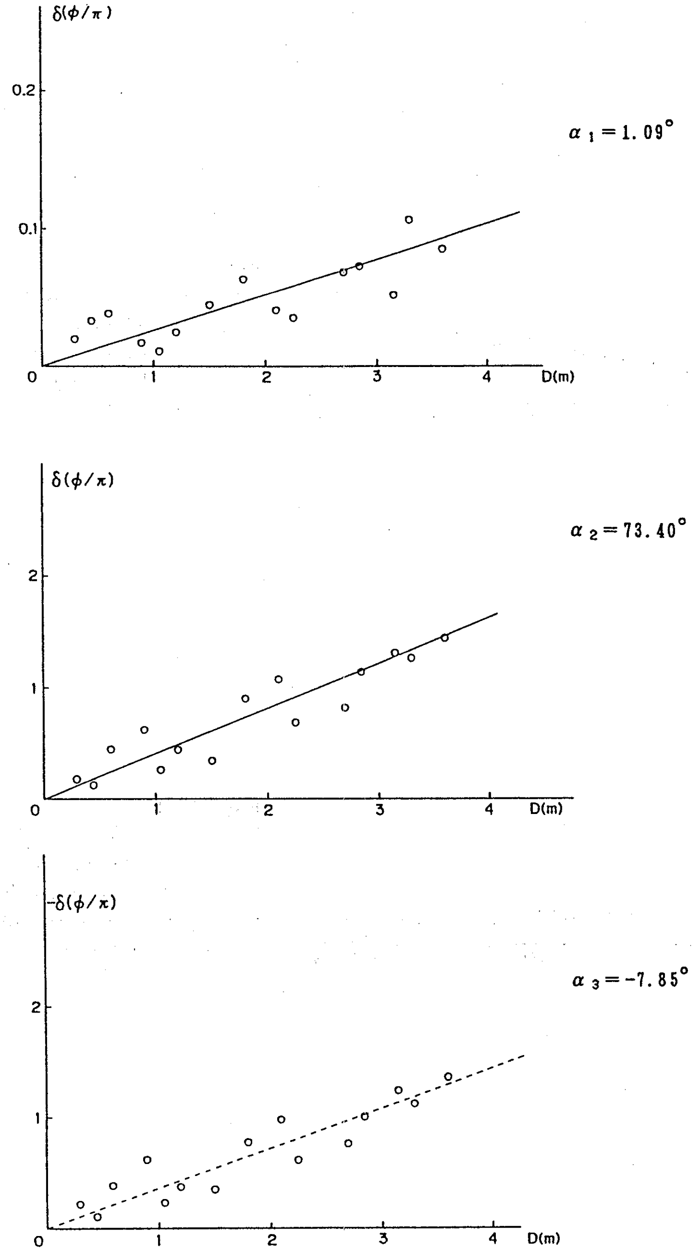


Fig. -2-10
Phase spectrum between wave data at the locations 1 and 3.
f_1 : 1-st primary wave
f_2 : 2-nd primary wave
f_3 : tertiary resonant wave



$\alpha = \alpha_3 - \alpha_1 = -8.94°$

Fig. -2-11
Phase differences along the linear array
(a) 1-st primary wave
(b) 2-nd primary wave
(c) tertiary resonant wave

α : angle between the resonant wave and 1-st wave

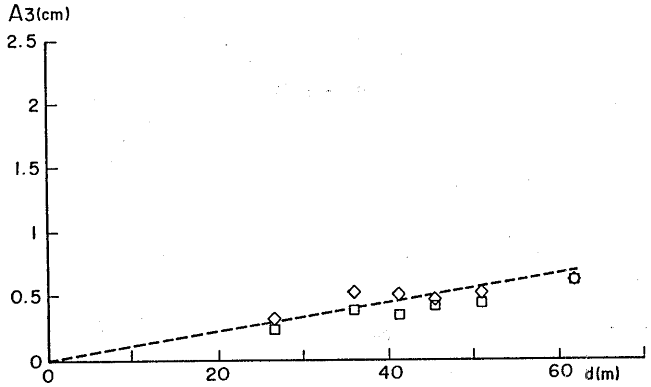


Fig. -2-12
Long term variation of A_3 ($\gamma=1.72$)
------- : Theory (Longuet-Higgins)
□ : Experiment (cm) $A_1=2.29$, $A_2=2.51$
◇ : Experiment (cm) $A_1=2.84$, $A_2=2.50$
Examples of linear growth of resonant waves

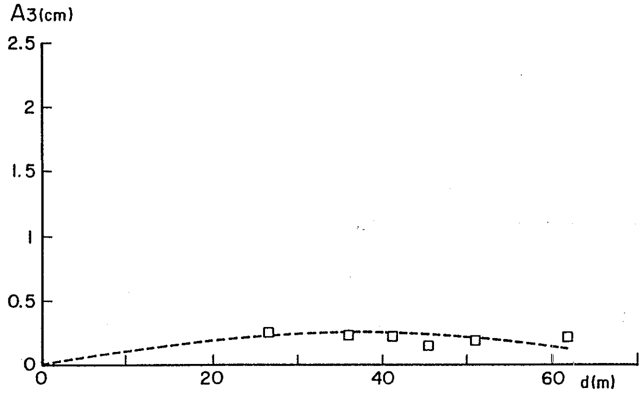


Fig. -2-13
Long term variation of A_3 ($\gamma=1.72$)
------- : Theory (Zakharov)
□ : Experiment (cm) $A_1=4.06$, $A_2=2.51$
Example of weak resonance at the exact (linear) resonance condition

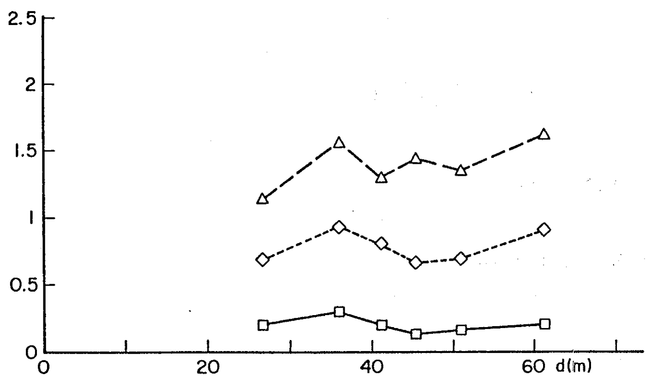


Fig. -2-14
Long term variation of A_3 ($\gamma=1.79$)
□ : Experiment (cm) $A_1=1.80$, $A_2=5.29$
◇ : Experiment (cm) $A_1=2.49$, $A_2=5.03$
△ : Experiment (cm) $A_1=2.84$, $A_2=5.12$
Large resonant wave appears at the off resonance condition.

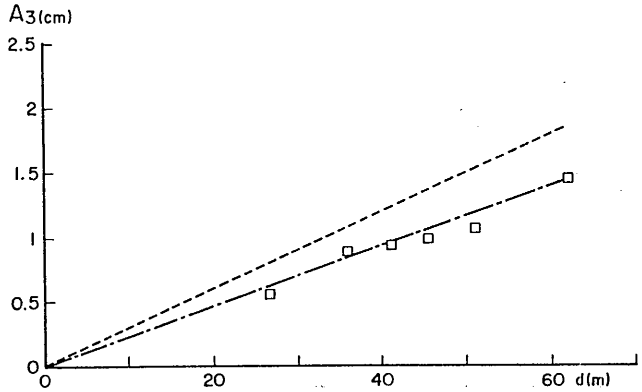


Fig. -2-15
Long term variation of $A_3(\gamma=1.79)$
------- : Theory (Longuet-Higgins)
·········· : Least square fitting
□ : Experiment (cm) $A_1=3.36$, $A_2=2.61$
An example of non-linear resonance

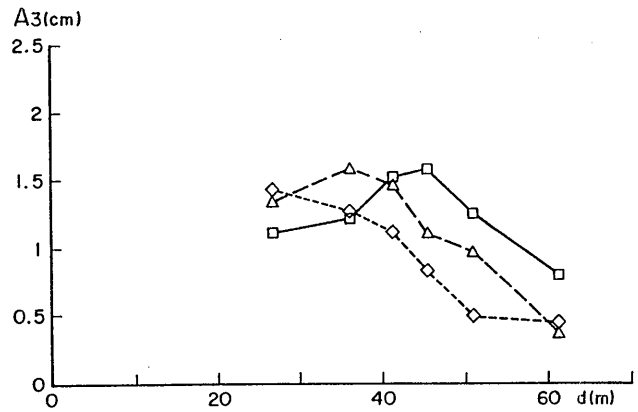


Fig. -2-16
Long term variation of A_3 $(\gamma=1.79)$
□ : Experiment (cm) $A_1=2.91$, $A_2=5.07$
◇ : Experiment (cm) $A_1=3.24$, $A_2=5.28$
△ : Experiment (cm) $A_1=3.44$, $A_2=5.14$
Decreasing of resonant wave amplitudes with fetch

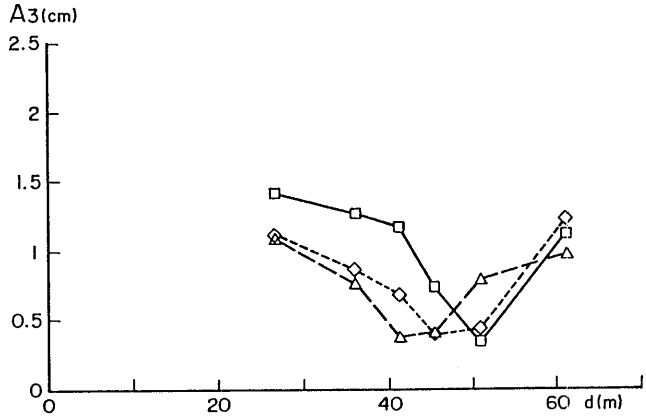


Fig. -2-17
Long term variation of A_3 $(\gamma=1.82)$
□ : Experiment (cm) $A_1=3.63$, $A_2=5.38$
◇ : Experiment (cm) $A_1=3.78$, $A_2=5.40$
△ : Experiment (cm) $A_1=4.15$, $A_2=5.41$
Evidences of recurrence phenomena

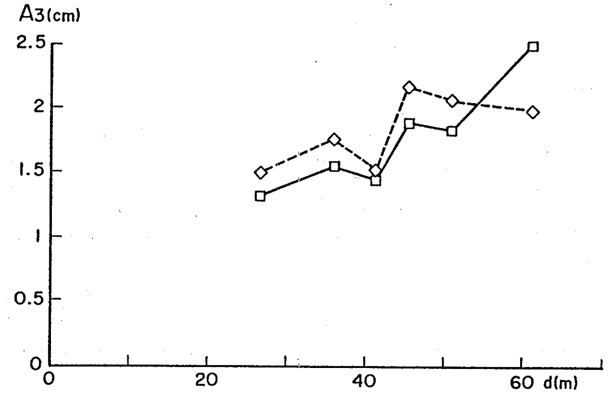


Fig. -2-18
Long term variation of A_3 ($\gamma=1.82$)
□ : Experiment (cm) $A_1=4.76$, $A_2=5.29$
◇ : Experiment (cm) $A_1=5.47$, $A_2=5.35$
The largest amplitudes of resonant waves observed in the experiment.

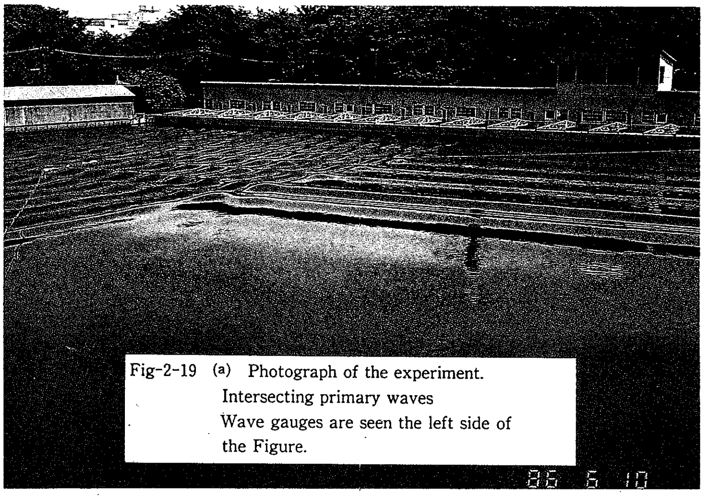


Fig-2-19 (a) Photograph of the experiment.
Intersecting primary waves
Wave gauges are seen the left side of the Figure.

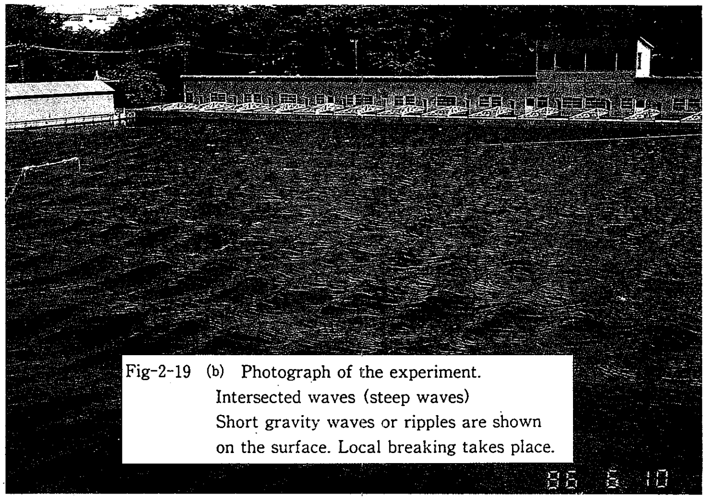


Fig-2-19 (b) Photograph of the experiment.
Intersected waves (steep waves)
Short gravity waves or ripples are shown on the surface. Local breaking takes place.

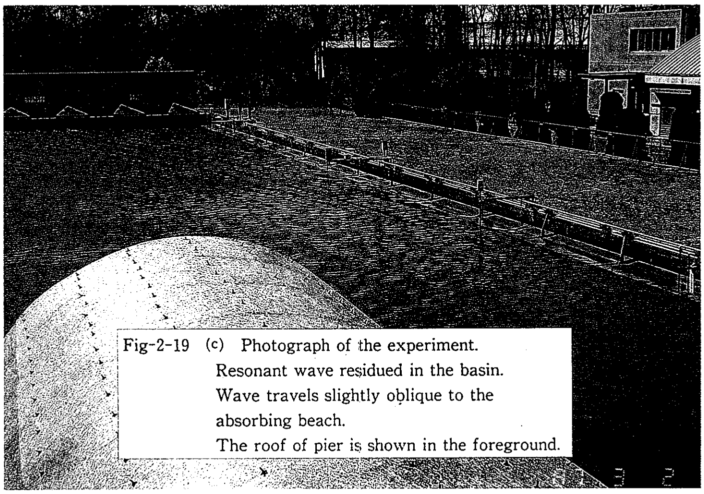


Fig-2-19 (c) Photograph of the experiment.
Resonant wave residued in the basin.
Wave travels slightly oblique to the absorbing beach.
The roof of pier is shown in the foreground.

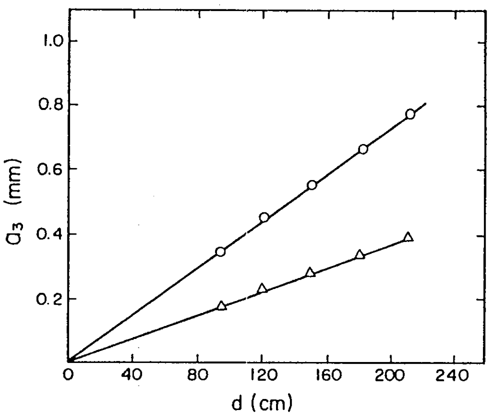


Fig. -3-1
Comparison of the Zakharov theory with the experiments by McGoldrick et. al. (1966) at the short fetch.
○, △ : Experiments (a_2 is one half in the latter)
—— : Theory

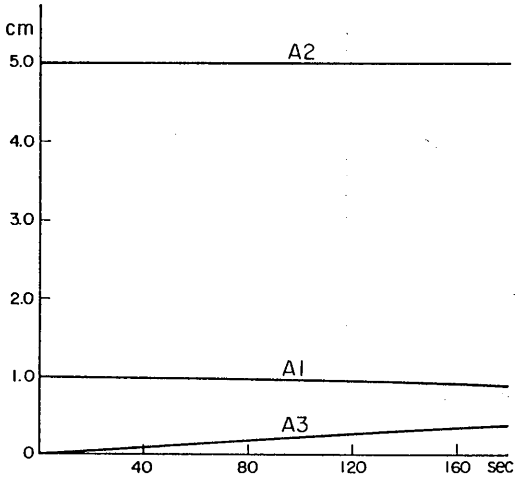


Fig. -3-2 (a)
Solution of the Zakharov equation (γ=1.735)
Initial values : A_1=1.0cm
A_2=5.0cm
A_3=0.0cm
Growth of resonant wave is nearly straight.

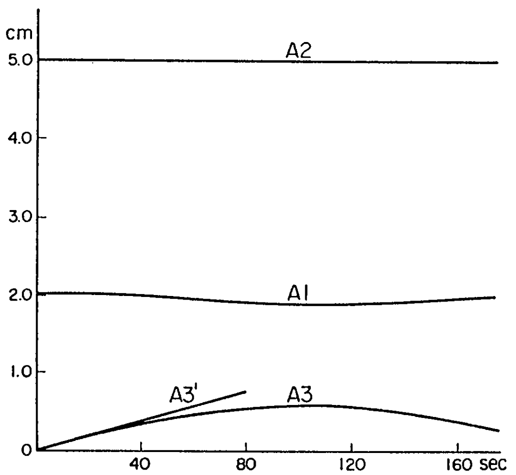


Fig. -3-2 (b)
Solution of the Zakharov equation (γ=1.735)
Initial values : A_1=2.0cm
A_2=5.0cm
A_3=0.0cm
Growth of resonant wave ceases at around 100 sec.
Initial growth rate coinsides with classical one.

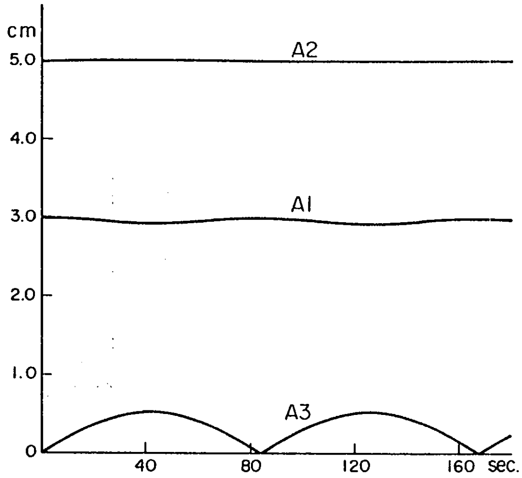


Fig. -3-2 (c)
Solution of the Zakharov equation (γ=1.735)
Initial values : A_1=3.0cm
A_2=5.0cm
A_3=0.0cm
Recurrence phenomena appear.

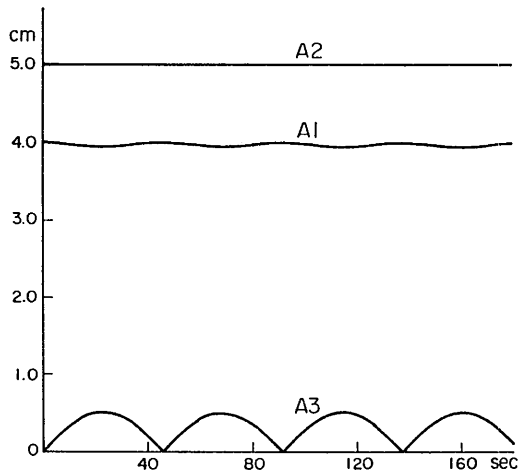


Fig. -3-2 (d)
Solution of the Zakharov equation (γ=1.735)
Initial values : A_1=4.0cm
A_2=5.0cm
A_3=0.0cm
Resonant wave amplitude does not increase proportional to the primary waves.