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Nonlinear Response of Moored Floating Structures in Random Waves and its Stochastic Analysis
Part 1. Theory and Model Experiments

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ABSTRACT

This paper deals with stochastic analysis of slow drift responses(forces and motions) of floating structures moored in random seas and their statistical predictions. First the study review for slow drift forces causing the slow drift motion is described, and four problems which must be solved in future are discussed. Second it is shown that nonlinear responses (total second order responses) including the slow drift responses can be represented by a two term Volterra functional series. Physical meanings of kernel functions in the functional series are investigated from a viewpoint of transfer functions(or frequency response functions). It is shown that the kernel functions can be estimated not only from bispectral analyses of experimental data but also by numerical calculations based on the potential theory. Furthermore on the basis of the mathematical fact that the second term in the Volterra functional series can be expressed by an equivalent linear process of instantaneous wave power in stochastic sense, new functional model is developed. This is based on the Wiener filter theory. This model is used to solve the problems excluded in the investigations obtained up to now. The problems are as folows: a) Hydrodynamic forces of slow drift motion in still water are modified in waves; b) Newman- Pinkster's approximation for slowly varying drift forces does not satisfy the condition of physical causality. Comparisons between simulated results and experimental ones have been conducted in both frequency and time domains. Main results are as follows: 1) Viscous drift force exists in addition to the drift force driven from the potential theory and it becomes significant compared with the potential drift force for large wave height. It is shown that the approximate method which takes into account the viscous drift force; 2) The hydrodynamic forces of slow drift motions are modified in waves and this phenomenon is caused not only by the wave drift damping (speed dependence of added resistance in waves) but also by increase of viscous damping force in waves. The ratio between the damping force in waves and that in still water was not more than 1.6 in the experiments which we carried out during this research project. But the problem why the hydrodynamic forces in still water are modified in waves remains completely unsolved; 3) It has been confirmed that the experimental and simulated results are in good agreement with each other provided we know how much the added mass and the damping forces in still water are modified in waves.

Finally a theory of probability density functions (p.d.f.'s) is developed for an instantaneous total second order response and its maxima, in order to predict 1/n th highest mean amplitudes and extreme statistics of total second order responses. New formula for the total second order p.d.f.'s which include not only quadratic but also linear responses are derived. These p.d.f.'s can be represented by the generalized Laguerre polynomials of which the first term is a Gamma p.d.f. consisting of three parameters. Assuming that the response and its time derivative processes are mutually independent, the 1/n th highest mean amplitude can be evaluated numerically from the derivative of the instantaneous response p.d.f.. This method is first applied to the sway motions of moored floating semi-circular and rectangular two dimensional cylinders, and the applicability of the method is studied by comparisons with Naess' exact solution. The variation of the 1/n th highest mean amplitude of the total second order response is then investigated following increases in damping and restoring forces. And comparisons between the experimental results and the estimated ones obtained from the present theory are carried out. The applicability of the present theory has been confirmed. The results are as follows: 1) It is confirmed through comparisons with Naess' exact solution that the present method is an accurate approximation for pure second order responses(slow drift responses); 2) The p.d.f. of the total second order response differs from that of the pure second order response. In fact it becomes a widely-banded distribution with an increase in the damping coefficient. Additionally it significantly deviates from Gaussian p.d.f.; 3) It is confirmed that the usual prediction method based on the Longuet-Higgns' method significantly underestimates the measured results while the present method estimates them very well. And it is shown that the extreme response of the total second order response is greater than that based on the assumption of the pure second order response.

Chapter 1

Introduction

A floating city and a floating airport interest people more than before, and floating drilling rigs are forced to operate under severe environmental conditions. The accurate estimation of motions and wave forces acting on these structures is important for economical and safety design of these structures[1),2),3),4)]. For instance, an accurate motion prediction is required to evaluate the workability of the structure and the predictions of mooring forces and horizontal excursions are needed to design the safety mooring system. Since all of these responses are random variables, these evaluations must be conducted by extreme values of the responses. In order to obtain the extreme statistics of these random responses, instantaneous and peak probability density functions are required. If the probability density functions (p.d.f.'s) are obtained, short term and long term predictions of the responses of the structure become possible[5)].

There are two methods for obtaining peak p.d.f.'s[6)]. The one is deterministic, and the other is nondeterministic. For both methods, frequency response functions of exciting forces to incident waves and hydrodynamic forces (i.e. added mass and damping force coefficients) are required. In the case of ships, many studies have already been reported for these hydrodynamic problems. For example, there is the strip method[7),8)] as a popular calculation method, and detail investigations[9),10)] for the accuracy of the strip method have sufficiently been made. There is the three dimensional source distribution method[11)] to get an exact solution for ideal fluid flow. (But the validation is still required)

The deterministic manner is summarized as follows:

The wave force time history in random seas is obtained from the wave force frequency response function and the random wave time history and the motion time series is numerically calculated by solving a motion equation in time domain. By the statistical analysis of the motion time history, a histogram corresponding to a peak p.d.f. is obtained. The merits of this manner are that the motion time history can be obtained even if the motion equation is nonlinear and that the peak p.d.f. can be found out without calculating the motion frequency response function. However since the statistical values obtained in this way are nothing but one sample in statistical sense, numerous calculations are required to get stable statistical values. Namely, in order to reduce a scatter of statistical values, it is necessary to get ensemble of sample statistical values obtained from many calculations of motion time histories. Even if the Ergodicity is assumed, statistical analysis of a motion time history with an infinite duration is required to get ensemble statistical values. Thus it is remarkably difficult to get ensemble statistical values by using the deterministic manner.

Another problem in the deterministic manner is a frequency dependency of hydrodynamic coefficients in the motion equation. Since the hydrodynamic coefficients of the motion equation is a function of wave frequencies in general, exactly speaking, the motion equation becomes a differential-integral equation. It is very difficult to solve the equation in time domain since the hydrodynamic coefficient in an infinite frequency is required.

While, on the nondeterministic manner, Cartwright and Longuet-Higgins[12)] have shown that the peak p.d.f. for linear responses can be represented by a response variance and a band width parameter. Thus in the case of linear responses, the peak p.d.f. can be obtained analytically if a frequency response function is found out.

In the field of ship and ocean engineering, most responses can be regarded as linear, but some can not, of which nonlinear components become significant. Nonlinearities of wave excitations or a motion response functions to external forces should be considered. For floating offshore structures, it is usual that both nonlinear phenomena happen. As an example of the former phenomenon, wave drift forces in regular waves and slowly varying drift forces in irregular waves must be considered, and as an example of the latter viscous damping forces and mooring forces must be considered.

It has been considered that the slowly varying drift forces in irregular waves occur by the following reason:

Because of the nonlinearity of the wave drift force, the existence of two waves of different frequencies always implies the existence of wave excitations at the sum and difference frequencies. The latter frequency may occur near the resonance frequency of the floating structure moored in horizontal motions. And if the damping is low(as it is usually in such motions), a highly tuned resonance motion must be expected even though the low-frequency force is generally small in magnitude. Accordingly, the motion of a floating structure moored in irregular waves consists of sum of a slowly varying component and a component oscillating at wave frequencies. The spectrum of this time history has two peaks. The one peak occurs within the wave frequency range and the other occurs below the lowest frequency (close to resonance frequency) at which there is any significant energy in the incident waves. For the combined responses with a low and wave frequency components (i.e. total second order response), in general, maxima and minima are not equal, so the probabilities of them are different. Thus in order to analyze statistically such nonlinear responses, new approach is required.

The application of probability theory to this problem was accomplished by Neal[13]. He assumed that a nonlinear response could be represented by a two term Volterra functional series, and he provided a closed form for a characteristic function(c.f.) of the response by using the Kac and Siegert method[14] (K-S method). According to K-S method, the problem of obtaining a c.f. for a random variable, which is represented by the sum of linear and quadratic forms of Gaussian random variables with mutual independence, can be reduced to a problem of solving eigenvalues and eigenfunctions of an integral equation. Since a probability density function (p.d.f.) of the response corresponds to an inverse Fourier transform of the c.f., Neal's method gives important information to estimate the p.d.f. of the nonlinear response to second order. This p.d.f., however, cannot be generally expressed in a closed form.

Naess[15],[16],[17] introduced a slow drift approximation and a pure quadratic response approximation to obtain the second order response p.d.f., and showed that the resulting eigenvalue problem generated a set of equal double eigenvalues. The p.d.f. of the response can be obtained by his approximations except when the equal double eigenvalues exist. Equal double eigenvalues may occur because a large number of eigenvalues are needed to describe a highly tuned response as shown by the authors et al[18]. Included are many nearly zero eigenvalues, thus caution is required. The Naess' method requires a pure quadratic response.

Vinje[19] assumed that the considered nonlinear response is weakly nonlinear and the p.d.f. is close to a Gaussian p.d.f., and he provided the series form of the p.d.f. from Taylor expansions of cumulants. His method is a kind of the approximate method called the Gram-Charlier expansion(or Edgeworth expansion).

Naess' method can be applied to obtain the instantaneous p.d.f. of the nonlinear response, but cannot be applied to get the peak p.d.f. while the Vinje's method can. In order to get the peak p.d.f. of the nonlinear response, a joint p.d.f. of response acceleration, velocity, and displacement is needed. But it is very difficult to exactly obtain this p.d.f. and some approximation is required. Hineno[20] applied the Vinje's method to the peak p.d.f. of nonlinear responses and obtained a series form(Hermite polynomial series). Recently Naess[21] developed the SRSS (Square Root form of Sum of Squares) method, which is the method that the extreme statistical values can be represented by the square root form of sum of squares of stochastic variables, and applied it to get the extreme response of the nonlinear response. But theoretical background is not clear.

Besides these studies, Yamanouchi[22] studied on nonlinear roll spectrum, and he investigated the relationship between the degree of nonlinearity and roll spectrum. And Roberts[23],[24] obtained the approximate steady p.d.f.'s by means of Fokker-Planck equation method (or stochastic differential equation method). This is a promising method in future, but has the defects that the external force is limited only to white noise and it is difficult to solve numerically because of infinite boundary conditions.

In this way, the consistent method to get the statistics of the nonlinear response has not yet been developed. As things are, the deterministic manner is the main current[25),26)]. As indicated earlier, this deterministic manner has many demerits, thus a new probability method may be required.

The objective of the present study is to develop a simulation model for total second order response of floating structures moored in random seas and its stochastic analysis method.

In Chapter 2, the study review for slow drift force(second order force) is described, and four problems which must be solved infuture are discussed. As the most important problem in them, the following problems are treated in this paper.

a) Hydrodynamic forces of slow drift motion in still water are modified in waves.

b) The Newman-Pinkster's approximation[25)] for the slowly varying drift force does not satisfy the condition of physical causality.

In Chapter 3, it is shown that the total second order force including slow drift forces can be represented by a two term Volterra functional series. Physical meanings of the kernel functions in the functional series are investigated from a viewpoint of frequency response functions (or transfer functions) and a method estimating the kernel ones from experimental data is also studied, which is the method using the bispectrum (a kind of higher order spectra). Furthermore a new functional model such that the second term of the Volterra functional series can be represented by the equivalent linear process of instantaneous wave power is developed. The new model is based on the Wiener filter theory[27)].

Several kinds of experiments have been carried out. Relation between the kernel function and the frequency response function of the slow drift force is investigated through comparisons between the experimetal results and numerical calculations. And the applicability of the present functional model is studied by comparing between the experimental data and numerical simulations. Furthermore the unsolved problems a)(i.e. how much the hydrodynamic forces in still water are modified in waves) and b) are investigated by using the new functional model.

The main results obtained in this Chapter are as follows:

(1) The kernel functions in the Volterra series correspond to the linear and quadratic transfer functions in frequency domain. The quadratic transfer function expresses a frequency characteristic of slowly varying drift force. The quadratic transfer function estimated by using the bispectral analysis from the experimental results does not agree with the numerical result based on the potential theory, viscous drift force exists in addition to the drift force due to the potential theory and it becomes more significant

than the potential drift force when the wave amplitude has a finite amplitude. If the viscous drift force is taken into account to the quadratic transfer function obtained from the numerical calculations even though it is approximately evaluated, the corrected numerical result is in good agreement with the experimental one.

(2) The hydrodynamic forces of slow drift motions are modified in waves. This phenomenon is caused not only by the wave drift damping proposed by Wichers et al.[28] and but also by increase of viscous damping force in waves newly suggested by the authors[29]. Recently other causes have also been found (e.g. one of the authors and Takaiwa[30]).

When the slow drift motion is dominant compared with the linear motion, the damping force at the slow drift motion increases by 1.6 times as large as one in still water whereas the added mass force becomes smaller than that in still water within limit of our experiment. However the problem how much and why the hydrodynamic forces in still water are modified in waves remains completely unsolved.

(3) Comparison between time domain simulations taking into account the viscous drift force in addition to the potential drift force and measured data is conducted. It has been confirmed that both results are in good agreement if we know how much the added mass and the damping forces in still water are modified in waves.

In Chapter 4, on the basis of the results obtained in Chapter 3 a theory of probability density functions(p.d.f.'s) is developed for an instantaneous total second order response and its maxima, in order to predict $1/n$ th highest mean amplitudes and extreme responses. New formulas for the total second order p.d.f.'s which include not only quadratic but also linear responses are derived. These new p.d.f.'s can be represented by the generalized Laguerre polynomials of which the first term is a Gamma p.d.f. consisting of three parameters. Assuming that the response and its time derivative processes are mutually independent, the $1/n$ th highest mean amplitude can be evaluated numerically from the derivative of the instantaneous response p.d.f.. This method is first applied to the sway motion of moored floating semi-circular and rectangular two dimensional cylinders, and the applicability of the method is studied by comparisons with Naess' exact solution. The variation of the $1/n$ th highest mean amplitude of the total second order response is then investigated following increases in damping and restoring forces. And comparisons between the experimental results in Chapter 3 and the calculated ones obtained from the present theory are carried out. The applicability of the present theory is confirmed.

The main results obtained in this Chapter are as follows:

(1) In the case of pure second order response (slow drift response) the instantaneous p.d.f. and the extreme responses estimated from the present method are in good agreement with the exact results shown by Naess.

(2) Using the present method, an investigation to determine the statistical interference between the first and second order responses was conducted for a system with a linear damping and a linear restoring forces. The p.d.f. of the total second order response differs from that of the pure second order response. In fact it becomes a widely-banded distribution with an increase in the damping coefficient. Additionally it significantly deviates from Gaussian p.d.f..

(3) As to the extreme response, comparison between the result obtained from the present method and one from the model test during long duration has been carried out. It is confirmed that the usual prediction method based on the Longuet-Higgins' method significantly underestimates the measured results while the present method estimates them very well. And it is shown that the extreme response of the total second order response is greater than that based on the assumption of the pure second order response.

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Chapter 2

Review and some Problems of Second order forces

In this section, first we shall describe the state of the art of studies of the slowly varying second order forces. Second, we discuss some problems. The coordinates used in this section are shown in Fig.A.1 in Appendix A.

If the static restoring force by mooring lines is very small, a highly tuned resonance generally occurs at a very low natural frequency in horizontal plane for a moored floating structure. It is important in practice to predict the magnitude of the low frequency horizontal excursions of a platform and to ensure that they are kept within acceptable bounds. This is one of the most important hydrodynamic problems that must be solved in designing ocean platforms. This phenomenon was first reported by Verhagen and Sluijs[1]. They explained the phenomenon as:

Because of the nonlinearity of the free-surface conditions, the existence of two waves with different frequencies always implies the existence of waves at the sum and difference (beat) frequencies. The latter may occur near the resonance frequency of the moored platform in horizontal motion, i.e. sway, surge, or yaw motions. If the incident wave system consists of a continuous spectrum of waves, one is assured that there is always some disturbance at any very low frequency, and, if the damping is small (as it usually is in such motions), a highly tuned resonant motion must be expected.

On a basis of a physical investigation, Hsu and Blenkarn[2] suggested an estimation method of slowly varying nonlinear forces causing the slow drift motion as follows:

In any small time interval, consider the incident waves approximately as if they were simple sinusoidal waves, that is, fit the time history over a very short time interval by a sinusoidal curve with a specific amplitude and the period, and compute the steady force as if these sinusoidal waves existed for all time. At a slightly later time, the waves will have changed, and so the process is repeated with a new sinusoidal wave of different amplitude and period, to which there corresponds a new value of the "*steady force*". And so on. In this way, a slowly varying second order force is predicted, its amplitude varying roughly with the square of the wave envelope.

This argument seems reasonable if the incident waves constitute a narrow band process.

Marthinsen[3] has recently provided a mathematical ground for the method of Hsu and Blenkarn. First, note that, if the incident wave system consists of a single frequency wave, the steady drift force can be expressed:

$$\overline{F}^{(2)} = F_d(\omega)a_1^2 \tag{2.1}$$

whereF_d is a transfer function that depends on the frequency of the primary wave(but not on its amplitude). and a_1 is a primary wave amplitude.

Now suppose that the incident waves contain many frequency components:

$$\zeta_1(x,t) = \sum a_i \cos(\omega_i t - \kappa_i x + \delta_i) \tag{2.2}$$

where a_i is the real amplitude and δ_i is an arbitrary phase constant.

He rewrites (2.2) in the following way:

$$\zeta_1(x,t) = \Re\{A(x,t)\exp[i(\omega_p t - \kappa_p x)]\} \tag{2.3}$$

where

$$A(x,t) = \sum a_i \exp\{i[(\omega_i - \omega_p)t - (\kappa_i - \kappa_p)x + \delta_i]\} \tag{2.4}$$

and ω_p is some frequency at or near the peak of the wave spectrum, with κ_p the corresponding wave number. The quantity $A(x,t)$ is clearly slowly varying in both space and time, if the wave spectrum is narrow banded. So by using a slowly varying function $a(x,t)$ can be represented as:

$$\zeta_1(x,t) = a(x,t)\cos(\omega_p t - \kappa_p x + \psi(x,t)) \tag{2.5}$$

where $a(x,t)$ is the slowly varying amplitude and ψ the slowly varying phase function. They are represented as:

$$\begin{aligned} a(x,t) \;&= [(\sum a_i \cos\{(\omega_i - \omega_p)t - (\kappa_i - \kappa_p)x + \delta_i\})^2 \\ &+(\sum a_i \sin\{(\omega_i - \omega_p)t - (\kappa_i - \kappa_p)x + \delta_i\})^2]^{1/2} \end{aligned}$$

$$\psi(x,t) = \tan^{-1}[\frac{\sum a_i \sin\{(\omega_i - \omega_p)t - (\kappa_i - \kappa_p)x + \delta_i\}}{\sum a_i \cos\{(\omega_i - \omega_p)t - (\kappa_i - \kappa_p)x + \delta_i\}}]$$

If the **local frequency** ω_L and **local wave number** κ_L are introduced as

$$\omega_L = \omega_p + \frac{\partial\psi}{\partial t}$$
$$\kappa_L = \kappa_p - \frac{\partial\psi}{\partial t} \tag{2.6}$$

finally the following representation can be obtained.

$$F^{(2)}(t) = \overline{F}^{(2)} + \tilde{F}^{(2)}(t) = F_d(\omega_L)a^2(x_0, t) \tag{2.7}$$

where x_0 is a fixed point, which is typically the location of the centre of gravity of the body, or possibly just the origin of the coordinates used for analyzing the body motion. And $\tilde{F}^{(2)}$ is a slowly varying drift force.

Equation (2.7) represents essentially the method prescribed by Hsu and Blenkarn. This method is justified only if the wave spectrum is narrow banded. Because if the wave spectrum is of wide band, the concept of local frequency can no longer be used.

Robert[4] developed a formula like (2.7). His formula is $\omega_L = \omega_p$, i.e. $\psi_t = 0$. Marthinsen shows that this method gives valid results if $\frac{dF_d}{d\omega} \ll 1$, that is, the transfer function of steady drift force is flat for wave frequencies, and invalid results if $\frac{dF_d}{d\omega} \gg 1$.

Newman[5] followed a different formulation but derived a similar result. His approach has been used by many subsequent investigators. His argument is as follows:

Let the wave elevation at some x be represented by

$$\zeta_1(t) = \Re\{\sum a_i \exp(i\omega_i t)\} \tag{2.8}$$

where a_i is the complex wave amplitude of frequency ω_i. The first order force caused by these waves can be represented as:

$$F^{(1)}(t) = \Re\{\sum f_{1i} a_i \exp(i\omega_i t)\} \tag{2.9}$$

where $f_{1i} = f_1(\omega_i)$ is a first order transfer function relating force amplitude and phase to the wave amplitude and phase.

We expect that the second order force components will depend on the square of the wave amplitude. Thus, noting that the products of two wave components can be written:

$$\Re\{a_i \exp(i\omega_i t)\} \times \Re\{a_j \exp(i\omega_j t)\} =$$

$$\frac{1}{2}\Re\{a_i a_j \exp[i(\omega_i + \omega_j)t] + a_i a_j^* \exp[i(\omega_i - \omega_j)t]\}$$

where the asterisk denotes a complex conjugate.

The second order force components should then take the form:

$$F^{(2)}(t) = \Re\{\sum_i \sum_j f_{2ij}^{(+)} a_i a_j \exp[i(\omega_i + \omega_j)t]\}$$
$$+\Re\{\sum_i \sum_j f_{2ij}^{(-)} a_i a_j^* \exp[i(\omega_i - \omega_j)t]\} \tag{2.10}$$

The $(+)$ and $(-)$ notations distinguish between the second order transfer functions relating to the sum frequency and the difference frequency, respectively. We are interested only in the second order zero- and beat-frequency force, and so we neglect the first sum and delete the subscript 2 and $(-)$ notation. So we have simply:

$$F^{(2)}(t) = \Re\{\sum_i \sum_j f_{ij} a_i a_j^* \exp[i(\omega_i - \omega_j)t]\} \tag{2.11}$$

Note that $f_{ij} = f_2(\omega_i, \omega_j)$. The time average of $\overline{F}^{(2)}$ is

$$\overline{F}^{(2)} = \Re\{\sum_i f_{ij} a_i a_j^*\} \tag{2.12}$$

Since $\overline{F}$ and the products $a_i a_j^*$ are real, $\Im f_{ii}$ is of no interest. Thus,

$$f_{ii} = F_d(\omega_i)$$

The coefficients, f_{ij} with $i = j$, are generally complex. Since the force expression in (2.11) does not depend on the choice of i and j (which are arbitrary), we require that

$$f_{ij} = f_{ji}^*$$

that is, the second order force matrix must be a Hermitian matrix. f_{ij} can be viewed as a surface in a three dimensional space with coordinates ω_i, ω_j, and f_{ij}. For each pair of frequencies, which define a point with coordinates (ω_i, ω_j) in the $\omega_i - \omega_j$ plane, the height of the surface is given by $\Re f_{ij}$ or by $\Im f_{ij}$. The height of the surface is known along the 45° line in the base plane: The real part is just f_{ii}, and the imaginary part is zero. Newman assumes that these surfaces are smooth and that their tangent planes make small angles with the base plane.

If this assumption is valid, then

$$f_{ij} = f_{ii} + O(\omega_i - \omega_j) \tag{2.13}$$

for frequency pairs lying near the 45° line. Then the off diagonal terms in f_{ij} can be approximated, the following formula is given for the slowly varying force

$$F^{(2)}(t) = \Re\{\sum_i \sum_j f_{ii} a_i a_j \exp[i(\omega_i - \omega_j)t]\} \times\{1 + O(\omega_i - \omega_j)\} \quad as \quad \omega_i - \omega_j \to 0 \tag{2.14}$$

Triantafylou[6] has pointed out that the Newman's approximation might be valid unless the second order waves could be considered as shallow water waves.

Pinkster[7] has developed the same formula as Newman's. He indicated that if F_d can be represented by a linear function, f_{ij} in (2.11) can be approximated as:

$$f_{ij} = f_{\frac{i+j}{2}, \frac{i+j}{2}}$$

And he gave a slow drift force spectrum in the following form:

$$S_F(\omega) = 2\rho^2 g^2 \int S_\zeta(\omega') S_\zeta(\omega' + \omega) F_d^2(\omega + \frac{\omega'}{2})\, d\omega' \tag{2.15}$$

where S_ζ is a incident wave spectrum. All of these analyses are approximate solutions.

Recently Pinkster[8),9] and Ogilvie[10] have shown the exact expressions for the second order forces and moments within the potential theory, those expressions were obtained based on the method of direct integration of fluid pressure acting on the instantaneous wetted surface of a body.

Their expressions of the second order force can be represented as the sum of the following components (see Appendix A):

(1) : Component caused by fluid pressure between mean and instantaneous wave surfaces:

$$\vec{F}_1^{(2)} = -\frac{\rho g}{2} \oint_C \vec{n}(\zeta_1 - \xi_{31} - y\xi_{41} + x\xi_{51})^2\, ds \tag{2.16}$$

(2) : Component due to quadratic pressure term in Bernoulli equation :

$$\vec{F}_2^{(2)} = \frac{\rho}{2} \iint_{S_m} \vec{n} \mid \nabla\varphi_1 \mid^2\, dS \tag{2.17}$$

(3) : Component caused by variation of the acting point of fluid pressure:

$$\vec{F}_3^{(2)} = \rho \iint_{S_m} \vec{n}\{(\vec{\xi}^{(1)} + \vec{\alpha}^{(1)} \times \vec{x}) \cdot \nabla\varphi_{1t}\}\, dS \tag{2.18}$$

(4) : Component caused by variation of direction of first order wave force with respect to rotation of a body:

$$\vec{F}_4^{(2)} = \vec{\alpha}^{(1)} \times \vec{F}^{(1)} \tag{2.19}$$

(5) : Component due to second order potentials:

$$\vec{F}_5^{(2)} = \rho \iint_{S_m} \vec{n}(\varphi_{2t}^I + \varphi_{2t}^d)\, dS \tag{2.20}$$

where ζ_1 is the first order surface elevation, and $\vec{\alpha}^{(1)} = (\xi_{41}, \xi_{51}, \xi_{61})$, which is the first order rotational motion vector, and ξ_{31} is the first order heave motion In addition, there exists a term $\rho g A_{wp}(x_f\xi_{41}+y_f\xi_{51})$ in the vertical second order forces, which term is caused by the product of first order rotational motions.

If the instantaneous wave surface elevation is expressed by (2.8), the second order forces can be represented like (2.10). The components (1) through (4) are caused by the product of first order quantities, the component (5) is only caused by second order potentials, that is, it can be obtained only by solving the boundary value problem of second order potentials.

Although the Pinkster-Ogilvie theory has become popular, there are several problems that can not be determined by the theory. They are as follows:

[1] Contribution of second order potentials to the drift force

The component (5) depends on the second order velocity potential φ_2 , which is very difficult to compute. There are several different ways in which to approach this problem. For example, Lighthill[11)] has shown that the second order force can be expressed wholly in terms of first order quantities by use of reciprocal relationships. His expressions, however, require the evaluation of an integral of second order pressures over the entire free surface, which is called a free surface integral. The amount of numerical work required to achieve this is likely to become vast unless some approximation will be found to represent an asymptotic behavior of second order potential away from the body. The purpose of evaluating this term is not to obtain an accurate prediction of slowly varying drift force but rather than to find out if the term is important.

Pinkster[9)], Standing et al.[12)], and Matsui[13] have obtained the following conclusion by evaluating the total slowly varying drift force without calculating the free surface integral.

The contribution of term (5) to the total slowly varying drift force can be negligible at high wave frequencies, at which the first order diffraction effect is significant, but it can be of great importance at low frequencies.

Faltinsen and Løken[14)] formulated the problem precisely to second order, expressed the drift force in terms of first and second order potentials, and then used Green's theorem to eliminate the explicit dependence on the second order potentials. They obtained the same conclusion. They treated only the two dimensional problem, but it certainly is possible that the same conclusion will be obtained in the three dimensional case.

However there is one case in which the second order potential may be important. It is the case that the second order waves may be shallow even though the first order waves are still deep water waves. It is possible for this phenomenon to occur, because the second order waves have the very low frequency component.

[2] Necessity of singular perturbation

One defect of the usual perturbation analysis (regular perturbation analysis) is that it is based on the assumption that motions of the structure are small compared with the dimensions of the structure itself. Since it is well known that the low frequency resonance response of a moored floating structure often involves very large horizontal excursions, then it is clear that this usual perturbation approach becomes invalid.

Triantafyllou[6)] developed a mathematical model that involves only linear hydrodynamic problems, even while it permits large excursions of the platform in the horizontal plane. He used a kind of multi-scale expansion theory, and assumed that the motion response consists of two motions:

1) The one is the usual motion response to the incident waves; the amplitude, velocity, and acceleration are small, and considered to be $O(\varepsilon)$, where ε is the usual perturbation parameter.

2) The other is the low frequency motion having the velocity that is $O(\varepsilon)$, whereas its amplitude is $O(1)$.

If t is the time variable that is normally used, he used a new time variable, $t = \varepsilon t$, for analyzing the low frequency motion. Thus his method is a kind of derivative perturbation analysis, i.e. singular perturbation.

[3] Effect of wave drift damping etc.

It is observed that the damping force on a moored floating structure during low frequency motion in waves becomes greater than the one in still water. Wichers et al.[15),16)] explained that this phenomenon is caused by a kind of added damping force due to the drifting of a structure in waves. They called it **wave drift damping** in order to distinguish it from the linear radiation wave damping. The authors et al.[17] and Standing et al.[18)] examined a simple method , which is called "*drift force gradient method* ", for approximating the additional damping due to the presence of waves, based on drift forces in regular waves at zero forward speed, using the analytical relationship between forward speed and wave encounter frequency together with wave frequency gradients of the drift forces. Wichers et al.[19)] proposed a different way using added resistance gradient and Hearn et al.[20)] computed by so called "*added resistance method* " that models the low frequency motion by steady forward speed of the structure in the fluid structure interaction analysis. Hearn et al. concluded that the drift force gradient method seems to predict the correct trend but not the correct magnitude, and that the subject of theoretically predicting wave drift damping is not fully resolved and more research is required.

On the other hand , the authors[21] showed an increase of the decay of low frequency motion coupled with the wave induced motion due to the nonlinear viscous damping force (**nonlinear coupled viscous damping**). The explanation of this phenomenon is indicated in Appendix D.

Saito and Takagi[22] demonstrated from comparisons between simulations and model experiments that drift forces based on potential theory as well as the nonlinear coupled viscous damping have an influence on the increase of low frequency damping in sway motion. One of the authors and Takaiwa[23] showed that the increase of the viscous damping force due to waves is sometimes much larger than the nonlinear coupled viscous damping even taking off the wave drift damping for a semisubmersible and it depends on the ratio between wave particle velocity and motion velocity. They called it as **drag coefficient change due to waves.** Furthermore they indicated that the low frequency added mass in still water is also modified in waves. But theoretical backgrounds of these phenomena are still not clear yet.

[4] Physical causality of Newman approximation

In general, the slowly varying drift force can be represented by a Volterra system function, which will be stated in the next sections.

$$F^{(2)}(t) = \int_{\tau_1}\int_{\tau_2} g_2^f(\tau_1,\tau_2)\zeta(t-\tau_1)\zeta(t-\tau_2)\,d\tau_1\,d\tau_2 \tag{2.21}$$

where

$$g_2^f(\tau_1,\tau_2) = \frac{1}{4\pi^2}\int_{\omega_1}\int_{\omega_2} G_f^2(\omega_1,\omega_2)\exp\{i(\omega_1\tau_1+\omega_2\tau_2)\}\,d\omega_1\,d\omega_2 \tag{2.22}$$

As stated earlier, Newman[24] introduced the approximation for the quadratic transfer function G_2^f. His approximation is that $G_2^f(\omega_1,\omega_2)$ is estimated by its values along the diagonal $\omega_2=-\omega_1$ as follows:

$$G_2^f(\omega_1,\omega_2) = \begin{cases} G_2^f(\omega_1,-\omega_1) & \text{as } \omega_1\cdot\omega_2 \le 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.23}$$

then Eq.(2.22) becomes

$$g_2^f(\tau_1,\tau_2) = h_2^f(\tau_1)\delta(\tau_2) \tag{2.24}$$

where $\delta(\tau)$ is the Dirac's delta function and

$$h_2^f = \begin{cases} \frac{1}{2\pi}\int_\omega G_2^f(\omega,-\omega)\exp(i\omega\tau)\,d\omega \\ \frac{2}{\pi}\int_0^\infty \Re\{G_2^f(\omega,-\omega)\}\cos(\omega\tau)\,d\omega \\ \frac{2}{\pi}\int_0^\infty -\Im\{G_2^f(\omega,-\omega)\}\sin(\omega\tau)\,d\omega \quad \text{for } \tau\ge 0 \end{cases} \tag{2.25}$$

Substituting (2.24) into (2.21) yields:

$$F^{(2)}(t) = \zeta(t) \cdot \int_{\tau} h_2^f(\tau)\zeta(t-\tau)d\tau \tag{2.26}$$

and since $G_2^f(\omega, -\omega)$ represents the steady drift force, it must be real; i.e. $\Im\{G_2^f(\omega, -\omega)\} \equiv 0$. Eq.(2.26) means that $F^{(2)}(t)$ is approximately written as the product of two Gaussian random processes (which are not statistically independent). However, from (2.25) we must note that $h_2^f(t)$ does not satisfy the physical causality unless $G_2^f(\omega, -\omega)$ takes a constant value. It is physically inconsistent that $G_2^f(\omega, -\omega)$ is constant, i.e. the steady drift force does not depend on wave frequencies. This inconsistency is caused by the lack of the phase information of $G_2^f(\omega, -\omega)$.

In this way, there exist many problems which must be solved on the slow drift phenomenon. This paper treats the slowly varying drift force from a viewpoint of system functional theory in order to solve the problem of physical causality of Newman's approximation. Then there will be a discussion along this approach on the third problem, i.e. how the wave drift damping affects the slow drift motion can also be investigated by this approach. But we will not discuss on the first and second problems in this paper since they require lots of additional research.

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