

Regression Formulae of the Parameters Estimating Lateral Ship Motion -GM and KG-

by

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Abstract

Predicting ship motion in waves is important for evaluating the safety and fuel efficiency of ships in operation. While it is well known that the strip method can predict the ship motion with practical accuracy, an accurate prediction of lateral motion such as roll motion requires the height of center of gravity of a ship as well as sectional data in ship longitudinal direction. In general, accurate data of the height of center of gravity can be obtained from the detailed hull form data, however, such data is available only for shipyards. Taking into account that parties other than shipyards are willing to predict ship motion, simplified estimation of the height of center of gravity should be established. This paper proposes the simplified estimation of the height of center of gravity based on ship principal particulars and reports the application of the simplified method to the prediction of roll motion in waves.

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Nomenclature

A_{44}	; added inertia with respect to the longitudinal axis
B	; ship breadth
B_{44}	; equivalent roll damping coefficient
C_{44}	; rolling restoring coefficient
D	; spreading function
D_S	; ship depth
E	; wave directional spectrum
d	; ship draught at midship
GM	; transverse metacenter height of the ship
H	; significant wave height
k	; circular wavenumber
KG	; height of the center of gravity from the keel
L_{PP}	; ship length between perpendiculars
L_{WL}	; ship length above waterline
M_ϕ	; rolling moment acting on a ship
$RMSE_{GM}$	; root mean square error of GM
$RMSE_{KG}$	; root mean square error of KG
S	; frequency spectrum
T	; mean wave period of short crested irregular waves
T_ϕ	; natural period of roll
α	; wave direction of regular waves
ε_ϕ	; roll phase where positive means a delay of phase
Δ_{Full}	; ship displacement at design full condition
Δ_{Bal}	; ship displacement at ballasted condition
ϕ_a	; dimensional roll amplitude
λ	; wavelength
λ_E	; wavelength corresponding to the encountering wave period that matches the natural roll period
θ	; primary direction of short crested irregular waves
$\sigma_{\phi_{cw}}$	; roll variance in short crested irregular waves
ω	; circular frequency of regular waves

ζ_a ; amplitude of regular waves

1. Introduction

Predicting ship motion in waves is important for evaluating ship safety and fuel efficiency in operation. Recalling that there are several methods for predicting ship motion, the strip method is used as one of the practical methods with accuracy. One of the reasons the strip method is used is that it requires limited data such as longitudinal cross-sectional data of the ship.

While longitudinal motion such as heave and pitch motion can be predicted with sufficient accuracy, lateral motion such as roll motion has difficulties in the prediction. Especially, since roll motion is greatly affected by the viscous effects of the fluid and the ship's forward speed, the roll damping force is approximately taken into account in the strip method. The roll damping force can be determined by a free-rolling test using a model ship, or it can be estimated by Ikeda's method ^{1),2)}.

In addition to the roll damping force, the height of center of gravity of a ship, which should be obtained from the detailed form data including the general arrangement, is important for predicting roll motion. Generally, such detailed form data is available only for shipyards and the other parties such as ship owners or operators rarely can use them. They usually can contact the limited data such as ship principal particulars, therefore, the simplified method for estimating the height of center of gravity available for the parties is expected. In this regard, Kuroda et al. ³⁾ compared the estimated roll motion with full-scale tests data and indicated that the use of the estimated transverse metacenter height overestimates roll motion.

To achieve above, this paper addresses the establishment of the simplified method of the height of center of gravity of a ship. Using the data of the height of center of gravity of actual ships, the author proposes a simplified method which requires only ship principal particulars. This paper also reports that the application of the simplified method to the roll prediction in waves and presents that the simplified method has practical accuracy for predicting roll motion in actual seas.

2. Estimation Method of The Height of Center of Gravity

2.1 Ship data

This study incorporates the data of the height of center of gravity calculated based on the detailed hull form data of existing ships. The ships used in this study are summarized in Table 1. Based on the data of the ships above, simplified estimation method of the height of center of gravity is discussed.

Table 1 Ships used in this study

Ship type	number of ships	range of L_{PP} [m]	range of B [m]
Container ship	22	Abt. 110-320	Abt. 18-46
Vehicle carrier	8	Abt. 100-190	Abt. 20-33
Bulk carrier	22	Abt. 127-285	Abt. 21-50
Tanker	27	Abt. 110-322	Abt. 28-60

2.2 Proposed Method

(1) Transverse metacenter height (GM)

A previous study dealing with simple estimation of the height of the center of gravity conducted by Inoue et al. ⁴⁾. They organized the data for general cargo ship, bulk carrier, vehicle carrier, and container ship, and presented regression equations

for transverse metacenter height (GM) during departure and arrival in full load condition, based on the principal particulars of the ships.

The author carried out surveys ⁵⁾ on the draft at departure and arrival in order to determine the representative draft in a voyage and concluded that the use of the draft at departure is appropriate. Taking this into account, the author referred to the equations applicable for the departure of bulk carrier, vehicle carrier, and container ship. Since Inoue's formula does not cover tanker, this study applies the equations for the departure of bulk carrier to tanker. A minor revision to the regression equations was made by accounting for the size of recent ships. The revised regression equations are expressed in Equation (1).

$$\frac{GM}{B} = a_1 \cdot L_{pp} + a_2 \quad (1)$$

where the coefficients a_1 and a_2 are given as shown in Table 2 including a_2 in the revised equation and Inoue's formula. Table 2 shows that while a_2 increases for container ship, bulk carrier, and tanker comparing with a_2 in original Inoue's formula, a_2 decreases for vehicle carrier. The causes of this result may be the increase in ship size and changes in ship design

Table 2 Coefficients a_1 and a_2 in revised Inoue's formula

Ship type	a_1	a_2 (Revised Inoue's formula)	a_2 (Original Inoue's formula)
Container ship	-2.5×10^{-4}	0.1055	0.0821
Vehicle carrier	-2.4×10^{-4}	0.0847	0.1053
Bulk carrier	1.7×10^{-4}	0.1268	0.0753
Tanker	1.7×10^{-4}	0.1501	0.0753

Although Inoue et al. use data from over 100 ships, the formulation is limited to full load conditions. Since the draft condition usually varies with the amount of cargo, a formulation that can be applied to other draft condition is expected to be established.

This study addressed the establishment of simplified method for estimating GM which can be applied not only to design full condition but also to ballasted condition, using GM obtained from the detailed hull form data of container ship, vehicle carrier, bulk carrier, and tanker. To improve the accuracy of Inoue's formula, ship draft is added to the formula and ship length between perpendiculars is eliminated since GM increases with an increase in the ratio (B/d).

For example, the relationship between (GM/B) and (B/d) at container ship is obtained as shown in Figure 1, showing that (GM/B) can be approximately linear to (B/d).

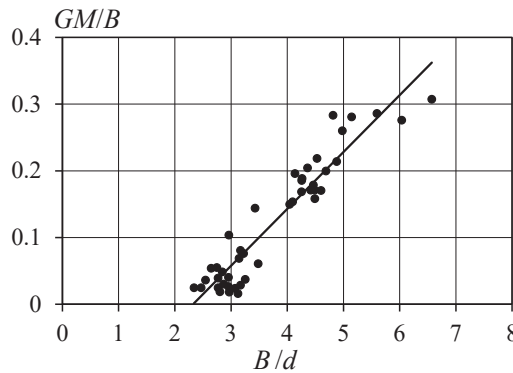


Figure 1 Relationship between (GM/B) and (B/d) for container ship

The simplified estimation of GM is proposed as follows where the coefficients C_1 and C_2 are listed in Table 3. Taking into account that Figure 1 indicates that the obtained (GM/B) is negative where (B/d) is less than 2.5, GM is given 0.15 m in accordance with International Code on Intact Stability ⁶⁾ in case that the obtained GM by Equation (2) is negative.

$$\frac{GM}{B} = C_1 \cdot \frac{B}{d} + C_2 \quad (2)$$

Table 3 Coefficients C_1 and C_2

Ship type	C_1	C_2
Container ship	0.0853	-0.1984
Vehicle carrier	0.0521	-0.1035
Bulk carrier	0.0599	-0.0590
Tanker	0.0639	-0.0631

The comparison between actual and estimated GM is shown in Figure 2. While the revised Inoue's formula tends to underestimate GM, the proposed method shows good agreement with the Actual GM.

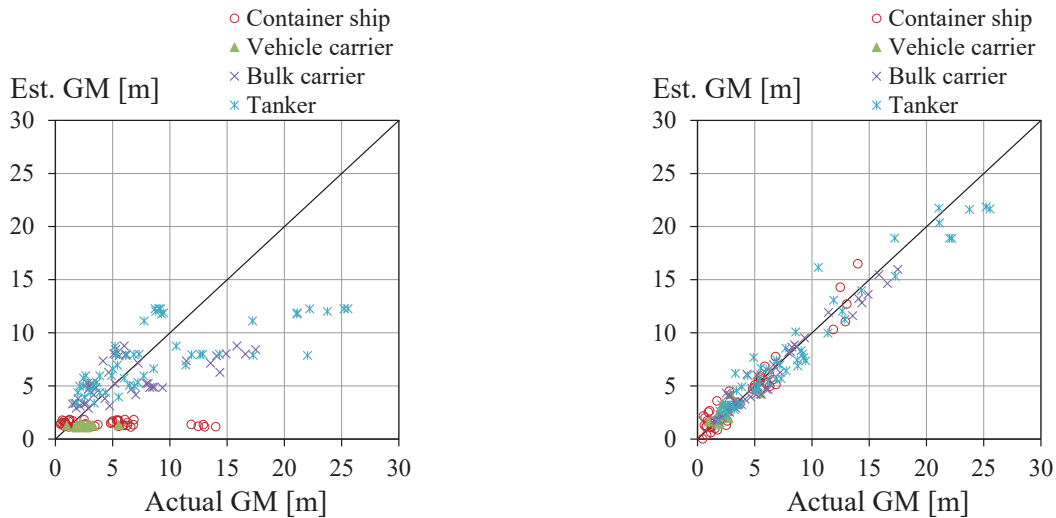


Figure 2 Comparison on actual and estimated GM.

(left: Estimated by Revised Inoue's formula , right: Estimated by the proposed method)

(2) Height of the center of gravity from the keel (KG)

A method for estimating KG is proposed as expressed in Equation (3) ⁶⁾.

$$KG = 4.5884 D_S \cdot L_{PP}^{0.371} \quad (3)$$

In this study, the author constructs a new estimation method to replace Equation (3) since the equation is based on old ships and does not deal with ships in other draft conditions. KG in design full condition and in ballasted condition are calculated as following equations where the coefficients C_3 , C_4 and C_5 are listed in Table 4.

$$\left. \frac{KG}{d} \right|_{Full} = C_3 \cdot L_{PP} + C_4 \quad (4)$$

$$\left. \frac{KG}{d} \right|_{Ballast} = \alpha \times \left. \frac{KG}{d} \right|_{Full} \quad (5)$$

$$\alpha = \begin{cases} 1 + C_5(s - 1) & 0 \leq s \leq 1.3 \\ 1 + 0.3 \cdot C_5 & 1.3 < s \end{cases} \quad (6)$$

$$s = \frac{\Delta_{Bal}}{\Delta_{Full}} \quad (7)$$

Table 4 Coefficients C_3 , C_4 and C_5

Ship type	C_3	C_4	C_5
Container ship	0.0014	0.9128	-0.26
Vehicle carrier	0.0005	1.3782	-0.44
Bulk carrier	-0.0004	0.8884	-1.53
Tanker	0	0.8126	-1.41

Equation (6) can be used with the range from $s = 0.0$ to $s = 1.3$. If s exceeds 1.3, α is given at the value at $s = 1.3$. Figure 3 shows the comparison between actual and estimated KG and demonstrates that applying the proposed method brings more accurate estimation. While Equation (3) does not include draft d , the proposed method can take the draft into account, which increases the accuracy for estimating KG.

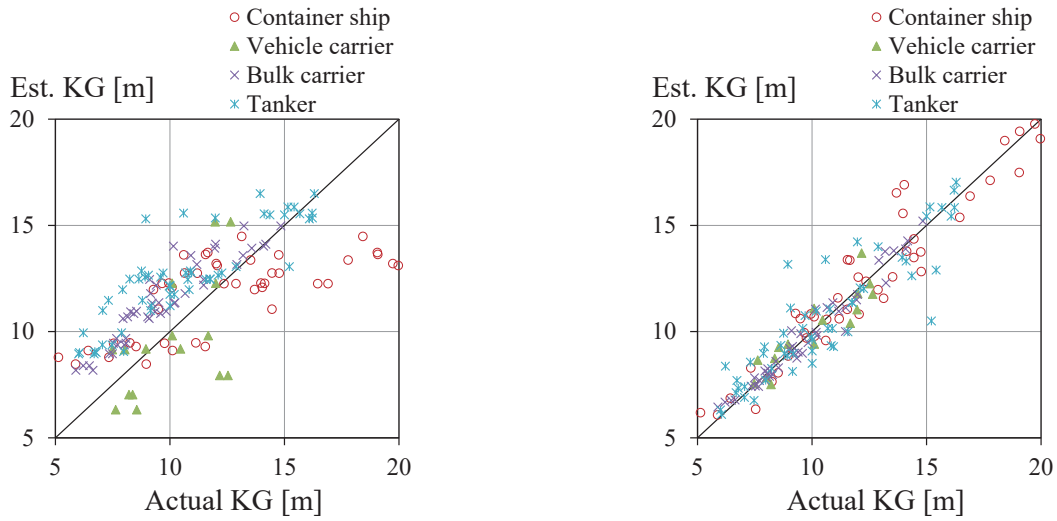


Figure 3 Comparison on actual and estimated KG.

(left: Estimated by Revised Inoue's formula , right: Estimated by the proposed method)

The root mean square error of GM and KG, which are expressed as $RMSE_{GM}$ and $RMSE_{KG}$, are calculated and indicated in Figure 4. It can be observed from the figures that the proposed method for estimating GM and KG is more accurate than the previous method defined by Equation (1) or Equation (3). Especially, $RMSE_{GM}$ by the proposed method is less than half of that by the previous method. The average of $RMSE_{GM}$ and $RMSE_{KG}$ for all ship types are 1.2m and 1.0m, respectively.

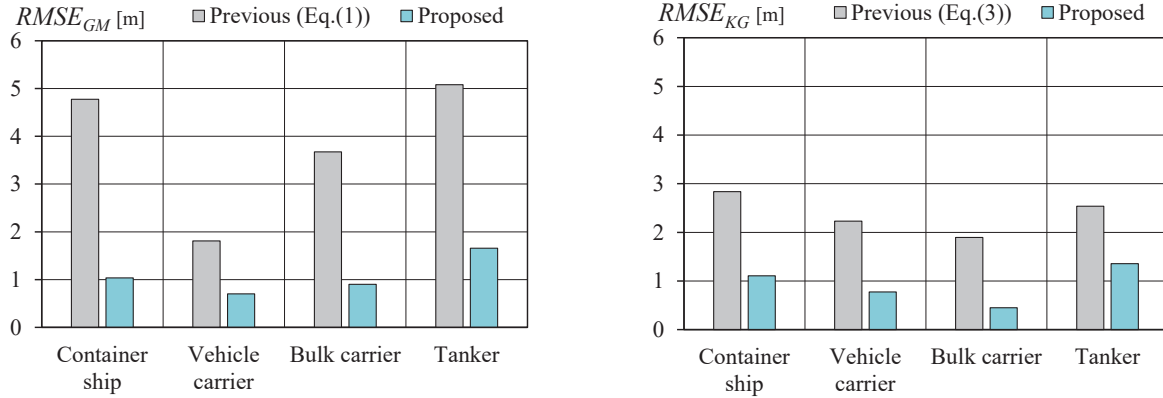


Figure 4 Root mean square error (left: GM ,right: KG)

3. Roll prediction by the proposed method

In this chapter, the roll prediction with the estimated GM and KG by the proposed method is conducted. Through the comparison between the predicted roll motion with the actual GM and KG and that with the estimated GM and KG, the effectiveness of the proposed method is investigated. Ships used for the roll prediction are summarized in Table 5. These ships are selected from the ships shown in Table 1 taking into account that the difference of GM and KG between actual and estimated value for the selected ships is close to the root mean square error. Containers and vehicle carriers are usually operated in laden conditions, therefore their ballast condition is not subject to the roll prediction in terms of practicality in this study.

Table 5 Ships used for roll prediction

Ship type	Container	Vehicle carrier	Bulk carrier		Tanker	
Condition	Full	Full	Full	Ballast	Full	Ballast
ID	CS23-full	VC6-full	BC16-full	BC16-bal	TK25-full	TK25-bal
Length between perpendiculars [m]	320.0	166.0	230.0		320.0	
Breadth [m]	46.0	32.0	43.0		60.0	
Draft at mid [m]	14.0	7.1	12.6	7.8	19.0	9.0
Froude number	0.21	0.24	0.15		0.14	

3.1 Roll motion in regular waves

Roll motion in regular waves is predicted by the New Strip Method. It is well known that the New Strip Method requires the distribution of draft, half beam, and sectional area in the longitudinal direction. Such distribution are obtained by the program ‘EAGLE-OCT.’^{7),8)} since only principal particulars of the ships are available.

The prediction of roll motion in regular waves requires the natural period of roll motion of the ship and the roll damping coefficient as well as GM and KG. The natural period of roll motion is calculated by Equation (8)⁵⁾ which is based on the formula described in IS code. Although the NK guidelines⁹⁾ states that Equation (8) should be applied for ships built recently, the author treats the equation as the base equation for establishing the revised equation.

$$T_{\phi} = \frac{2B}{\sqrt{GM}} \left(C_0 + 0.023 \frac{B}{d} - 0.043 \frac{L_{WL}}{100} \right) \quad (8)$$

C_0 is given 0.433 for container ship, bulk carrier, and tanker while given 0.379 for vehicle carrier for providing the reliable roll period. The natural roll period is calculated using GM estimated by Equation (2) and used both for the prediction of roll motion with actual GM and KG and that with estimated GM and KG. This aims to investigate the sensitivity of GM and KG to the roll prediction.

The roll prediction in regular waves is conducted in accordance with the roll motion equation expressed by Equation (9) in which the nonlinear term on the roll damping coefficient is expressed as the linear term using the equivalent roll damping coefficient B_{44} .

$$(I_{xx} + A_{44})\ddot{\phi} + B_{44}\dot{\phi} + C_{44}\phi = M_{\phi} \quad (9)$$

where I_{xx} and A_{44} is the inertia and added inertia with respect to the longitudinal axis, respectively, C_{44} is the rolling restoring coefficient, and M_{ϕ} is the rolling moment acting on a ship. B_{44} is calculated in accordance with Ikeda Method.

The frequency response of roll motion in regular waves whose direction is given 30deg, 60deg, 90deg, 120deg, and 150deg is calculated as shown in Figure 5 to Figure 10 where 0deg means head waves. ‘Actual’ and ‘Estimated’ denote the roll motion with the actual and estimated GM and KG, respectively. These figures include the actual and estimated value of GM and KG for validating the effectiveness of the proposed method and also indicates λ_E which is the wavelength corresponding to the encountering wave period that matches the natural roll period.

From these figures, in most cases the predicted frequency response of roll amplitude and phase with estimated GM and KG have good agreement with that with actual estimated GM and KG. In the case of CS23-full, the predicted frequency response shows less accuracy where the wave direction is 90deg. Whether such less accuracy is acceptable or not is discussed based on the calculated roll motion in short crested irregular waves, which is dealt with in the next section.

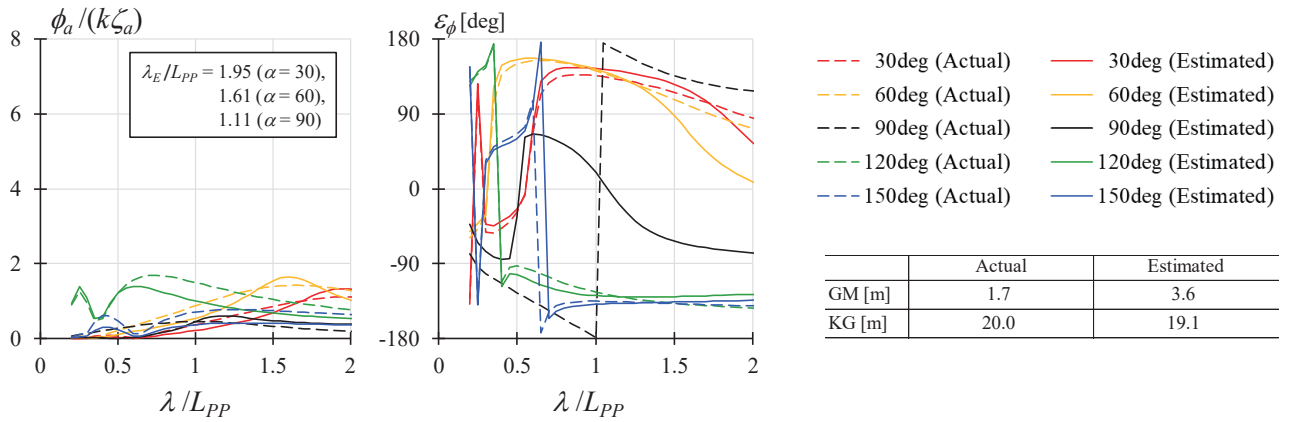


Figure 5 Frequency response of roll motion in regular waves. (CS23-full)

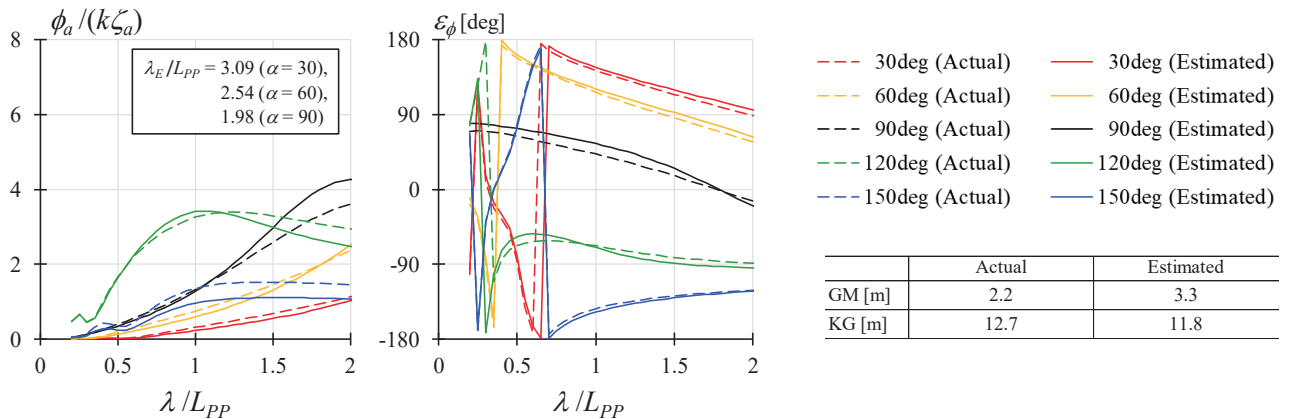


Figure 6 Frequency response of roll motion in regular waves. (VC6-full)

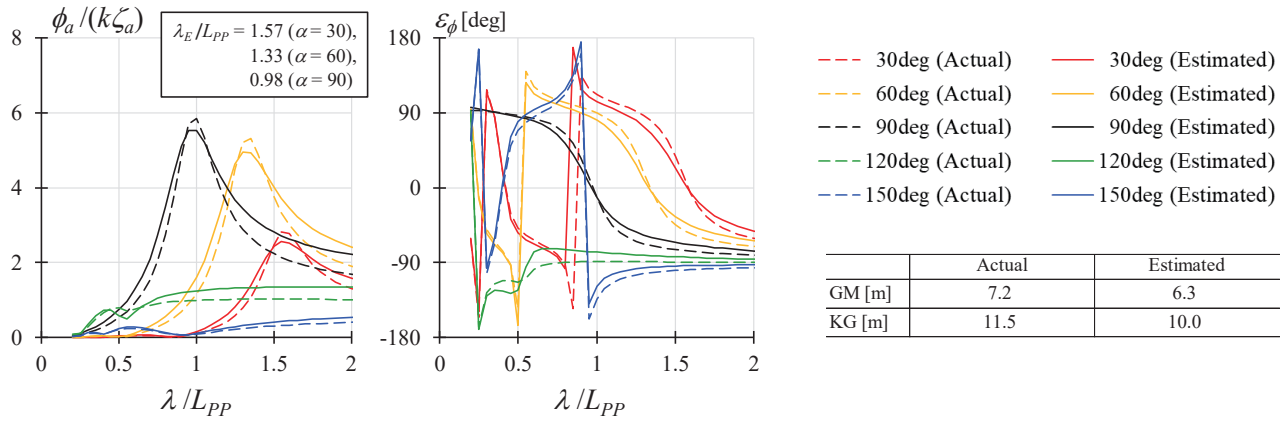


Figure 7 Frequency response of roll motion in regular waves. (BC16-full)

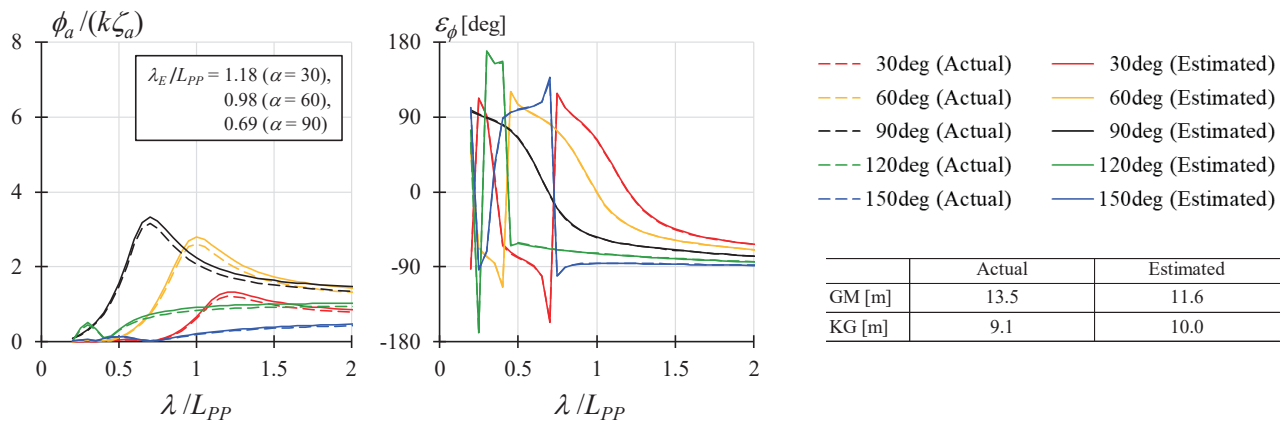


Figure 8 Frequency response of roll motion in regular waves. (BC16-bal)

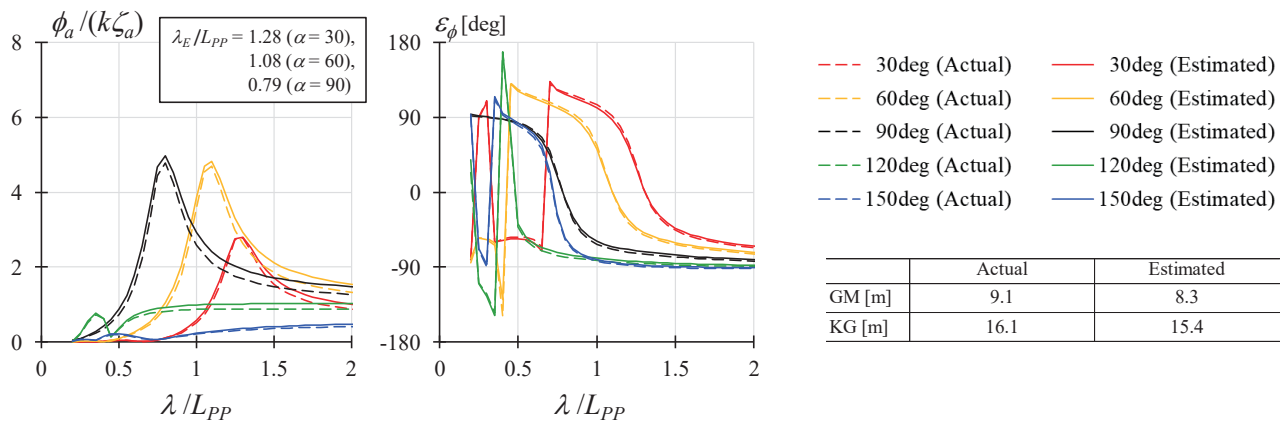


Figure 9 Frequency response of roll motion in regular waves. (TK25-full)

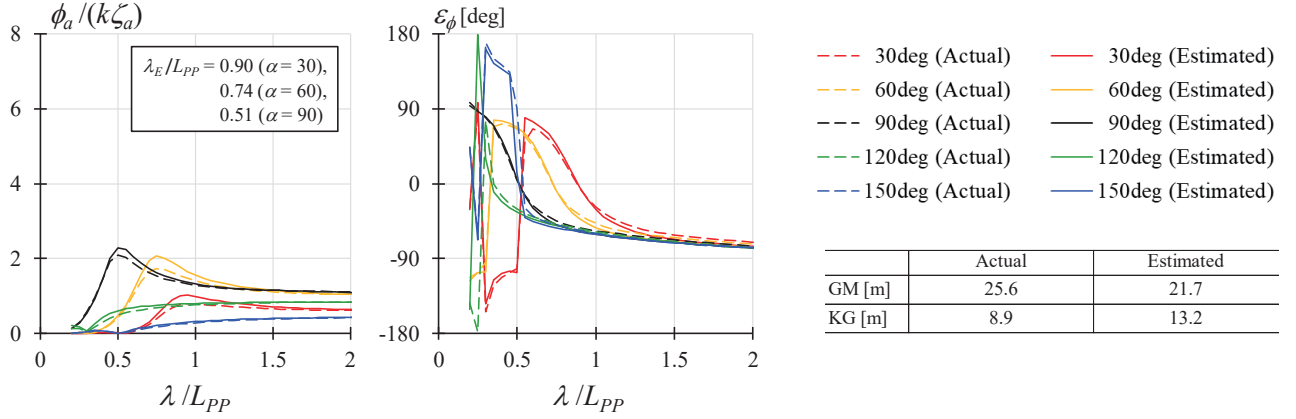


Figure 10 Frequency response of roll motion in regular waves. (TK25-bal)

3.2 Roll variance in short crested irregular waves

This section addresses the calculation of roll motion in short crested irregular waves and especially deals with roll variance in the irregular waves whose primary direction is 90deg. The roll variance can be calculated using the frequency response function in regular waves and wave directional spectrum as shown in Equation (10).

$$\frac{\sigma_{\phi_{cw}}^2}{H^2} = 2 \int_0^{2\pi} \int_0^\infty \left| \frac{\phi_a}{k\zeta_a} \right|^2 k^2 E(\omega, \alpha) d\omega d\alpha \quad (10)$$

In general, the restoring term and damping term in the roll motion equation has nonlinearity and can have impacts on the calculation of ship motion in irregular waves by the linear superposition. In this study, such the impacts are not considered since the calculation does not address the roll motion in the extreme sea conditions.

The directional spectrum is dealt with as a product of frequency spectrum and spreading function, which is expressed in Equation (11). This study adopts IACS-type spectrum¹⁰⁾, which is one of modified Pierson-Moskowitz spectrum, and spreading function of cosine square type, which are defined in Equation (12) to Equation (16).

$$E(\omega, \alpha; H, T, \theta) = S(\omega; H, T) D(\alpha; \theta) \quad (11)$$

$$S(\omega) = \frac{A}{\omega^5} e^{-\frac{B}{\omega^4}} \quad (12)$$

$$A = \frac{1}{4\pi} \left(\frac{2\pi}{T_{02}} \right)^4 H^2 \quad (13)$$

$$B = \frac{1}{\pi} \left(\frac{2\pi}{T_{02}} \right)^4 \quad (14)$$

$$T_{02} = \frac{\Gamma(3/4)}{\pi^{1/4}} T \approx 0.9204 T \quad (15)$$

$$D(\alpha; \theta) = \frac{2}{\pi} \cos^2(\alpha - \theta) \quad \text{for } -\frac{\pi}{2} \leq (\alpha - \theta) \leq \frac{\pi}{2} \quad (16)$$

Γ in Equation (14) is the Gamma function.

Roll variance in short crested irregular waves using the actual and estimated GM and KG is calculated as shown in Figure 11 to Figure 16. The figures shows that, as a whole, the roll variance using the estimated GM and KG matches that using the actual GM and KG. In the cases in this study, using the estimated GM and KG underestimates the roll variance for the container ship and the vehicle carrier while it overestimates for the bulk carrier and the tanker. For the container ship and the vehicle carrier, the underestimation of the variance value of the roll motion should be taken into consideration for assessing the safety of ships. The maximum difference between the roll variance using the estimated GM and KG and that using the actual GM and KG during the entire mean wave period is less than 0.2 range, which can conclude that the simplified estimation method of GM and KG proposed in this paper has practical accuracy for predicting roll variance in short crested irregular waves.

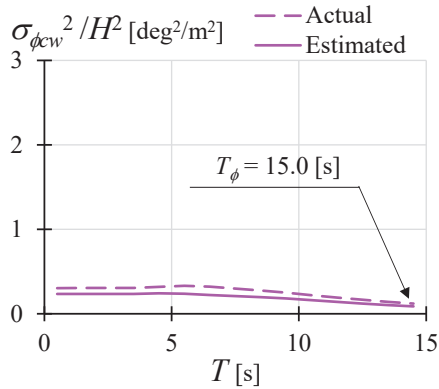


Figure 11 Roll variance in short crested irregular waves (CS23-full)

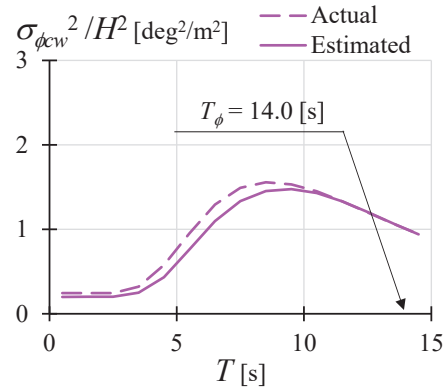


Figure 12 Roll variance in short crested irregular waves (VC6-full)

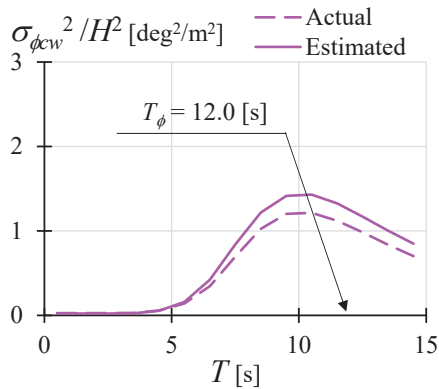


Figure 13 Roll variance in short crested irregular waves (BC16-full)

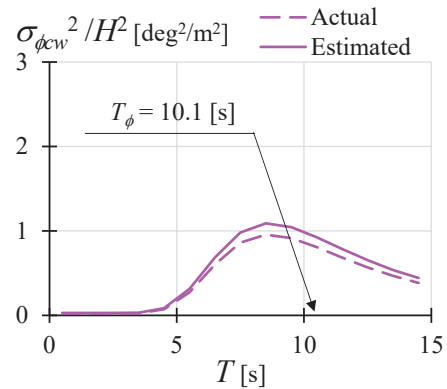


Figure 14 Roll variance in short crested irregular waves (BC16-bal)

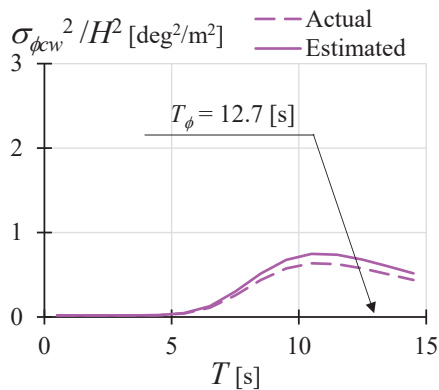


Figure 15 Roll variance in short crested irregular waves (TK25-full)

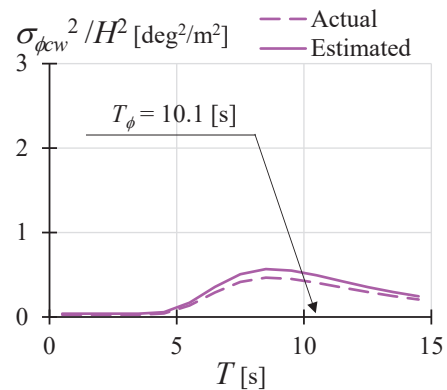


Figure 16 Roll variance in short crested irregular waves (TK25-bal)

4. Conclusions

In this study, a simple method for estimating the transverse metacenter height (GM) and the center of gravity height from the keel (KG), which are necessary for predicting the roll motion in actual seas, is proposed. This estimation method is based on data from existing ships and is characterized by the fact that it can be estimated only from data on the ship's length, breadth, draft, and displacement. The accuracy of the estimation method has been shown to be about 1.2m and 1.0m in the root mean square error for GM and KG.

Furthermore, the effect of the accuracy of the estimation method on the roll prediction in the actual seas is investigated. Specifically, the frequency response of roll in regular waves is obtained using the actual and estimated GM and KG, respectively, and roll variance in short crested irregular waves is calculated based on the obtained frequency response. The comparison between the roll variance using the actual and estimated GM and KG clarifies that the difference in the variance nondimensionalized by the significant wave height is less than 0.2. Accordingly, it is concluded that the simplified estimation method of GM and KG proposed in this paper has practical accuracy for roll prediction in actual seas.

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