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A New Computation Formula for the Added Resistance and Connection with Lagally's Theorem

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Review of Computation Methods

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- ***Far-field method**, based on the momentum-conservation principle*
  - *Maruo's formula, using the Kochin function of ship-generated propagating waves*
  - *Integration over a control surface S_∞ at a large distance far from the body considered*
  - *Relatively easy to compute, once the Kochin function is given*
- ***Near-field method**, based on the direct pressure integration*
  - *Integration over the wetted surface of the body S_B, suitable for Rankine panel method*

    (1) Square of relative wave elevation on the body-hull surface (which is dominant)
    Line integral along the intersection C_B between the body and free surfaces on $z = 0$
    (2) Surface integral over S_B of the 2nd-order dynamic pressure of fluid-velocity squared
  - *Relatively difficult to keep sufficient accuracy, if the constant-panel method is used*
- ***Middle-field method**, using the momentum-conservation principle*
  - *Applied at a relatively short distance from the body , suitable for Rankine panel method*
- ***Application of Lagally's theorem***
  - *Integration on the body surface S_B, in terms of hydrodynamic singularities representing the ship geometry and fluid-velocity field at the singularity points*
  - *Simple and compact in form, but basically cumbersome in numerical computations*

Review of Momentum-Conservation Principle at Zero Speed

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$$\frac{dM_i}{dt} = \frac{d}{dt}\iiint_{V(t)} \rho v_i \, dV = \rho \iiint_V \frac{\partial v_i}{\partial t}\, dV + \rho \iint_S v_i U_n \, dS$$

$$= -\rho \iiint_V \left[\frac{\partial}{\partial x_i}\left(\frac{p}{\rho} + gz\right) + \frac{\partial}{\partial x_j}(v_i v_j) \right] dV + \rho \iint_S v_i U_n \, dS$$

$$\longrightarrow \quad \frac{dM_i}{dt} = \frac{d}{dt}\iiint_{V(t)} \rho v_i \, dV = -\iint_S \Big[p n_i + \rho v_i (v_n - U_n) \Big] dS \quad \longleftarrow \quad S = S_B + S_F + S_\infty$$

$$\overline{\frac{dM_i}{dt}} = \overline{\frac{d}{dt}\iiint_{V(t)} \rho v_i \, dV} = 0$$

$$\overline{\iint_{S_B+S_F+S_\infty} \Big[p n_i + \rho v_i (v_n - U_n) \Big] dS} = 0$$

where

$$\left.\begin{aligned} p &= -\rho\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi + gz\right) \equiv p^{(1)} + p^{(2)} \\ p^{(1)} &= -\rho\left(\frac{\partial \Phi}{\partial t} + gz\right), \quad p^{(2)} = -\rho\,\frac{1}{2}\nabla\Phi\cdot\nabla\Phi \end{aligned}\right\}$$

$$\Phi = \Re\big[\phi\, e^{i\omega t}\big], \quad \zeta_w = -\frac{1}{g}\frac{\partial \Phi}{\partial t}\Big|_{z=0} = \Re\Big[-\frac{i\omega}{g}\phi\, e^{i\omega t}\Big]_{z=0}$$

Review of Momentum-Conservation Principle at Zero Speed

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Analysis with nonlinear boundary of $S_B + S_F$

$$\left.\begin{array}{l} v_n = U_n \quad \text{on } S_B + S_F \\ p = 0 \quad \text{on } S_F, \quad U_n = 0 \quad \text{on } S_\infty \end{array}\right\}$$

$$\Longrightarrow \quad \overline{F_x} \equiv \overline{\iint_{S_B} p n_x \, dS} = -\overline{\iint_{S_\infty} \left\{ p n_x + \rho \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial n} \right\} dS}$$

$$\longrightarrow \quad \overline{F_x} = -\int_{-\infty}^{0} dz \int_{C_\infty} \overline{\left\{ p^{(2)} n_x + \rho \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial n} \right\}} d\ell - \overline{\int_0^{\zeta_w} dz \int_{C_\infty} p^{(1)} n_x \, d\ell} + O(\Phi^3)$$

$$\int_0^{\zeta_w} p^{(1)} \, dz = -\rho \int_0^{\zeta_w} \left(\frac{\partial \Phi}{\partial t} + gz \right) dz \simeq -\rho \left(\frac{\partial \Phi}{\partial t} \zeta_w + \frac{1}{2} g \zeta_w^2 \right)_{z=0} = \frac{\rho}{2g} \left(\frac{\partial \Phi}{\partial t} \right)^2_{z=0}$$

$$\overline{F_x} = -\int_{-\infty}^{0} dz \int_{C_\infty} \overline{\left\{ p^{(2)} n_x + \rho \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial n} \right\}} d\ell - \frac{\rho}{2g} \int_{C_\infty} \overline{\left(\frac{\partial \Phi}{\partial t} \right)^2_{z=0}} n_x \, d\ell$$

$$= \underbrace{\frac{\rho}{2} \Re \int_{-\infty}^{0} dz \int_{C_\infty} \left\{ \frac{1}{2} \nabla \phi \cdot \nabla \phi^* n_x - \frac{\partial \phi}{\partial x} \frac{\partial \phi^*}{\partial n} \right\} d\ell}_{\textit{2nd-order dynamic pressure integration}} - \underbrace{\frac{\rho}{4} K \int_{C_\infty} \phi \phi^* \Big|_{z=0} n_x \, d\ell}_{\textit{Square of relative wave elevation}}$$

Review of Momentum-Conservation Principle at Zero Speed

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Analysis with linear (flat) boundary of S_F

$$\text{on } S_F \quad U_n = 0\,,\; v_n = \frac{\partial \Phi}{\partial n} = \frac{\partial \Phi}{\partial z} = -\frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2} \quad \longrightarrow \quad \frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial z} = K\phi \quad \text{on } z=0$$

$$\begin{aligned}
\overline{F_x} &\equiv \overline{\iint_{S_{B0}} p^{(2)} n_x \, dS} \\
&= -\rho \iint_{S_{F0}} \overline{\frac{\partial \Phi}{\partial x}\frac{\partial \Phi}{\partial n}}\, dS + \rho \iint_{S_\infty} \overline{\left\{ \frac{1}{2}\nabla\Phi\cdot\nabla\Phi n_x - \frac{\partial \Phi}{\partial x}\frac{\partial \Phi}{\partial n} \right\}}\, dS \\
&= \underbrace{-\frac{\rho}{4}\iint_{S_{F0}} \left(\frac{\partial \phi}{\partial x}\frac{\partial \phi^*}{\partial z} + \frac{\partial \phi^*}{\partial x}\frac{\partial \phi}{\partial z} \right) dxdy}_{} + \frac{\rho}{2}\Re \iint_{S_\infty} \left\{ \frac{1}{2}\nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial \phi}{\partial x}\frac{\partial \phi^*}{\partial n} \right\} dS
\end{aligned}$$

$$\begin{aligned}
\mathcal{F} &= -\frac{\rho}{4}\iint_{S_{F0}} \left(\frac{\partial \phi}{\partial x} K\phi^* + \frac{\partial \phi^*}{\partial x} K\phi \right)_{z=0} dxdy \\
&= -\frac{\rho}{4}K \iint_{S_{F0}} \frac{d}{dx}\Big[\phi\phi^*\Big]_{z=0} dxdy = -\frac{\rho}{4}K \left[\int_{C_\infty} + \int_{C_B} \right] \phi\phi^*\Big|_{z=0} n_x \, d\ell
\end{aligned}$$

Near-field method

Far-field method

$$\begin{aligned}
&\overline{\iint_{S_{B0}} p^{(2)} n_x \, dS} + \frac{\rho}{4}K \int_{C_B} \phi\phi^*\Big|_{z=0} n_x \, d\ell \\
&\qquad = \frac{\rho}{2}\Re \iint_{S_\infty} \left\{ \frac{1}{2}\nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial \phi}{\partial x}\frac{\partial \phi^*}{\partial n} \right\} dS - \frac{\rho}{4}K \int_{C_\infty} \phi\phi^*\Big|_{z=0} n_x \, d\ell
\end{aligned}$$

Review of Momentum-Conservation Principle at Zero Speed

- *Derivation of Tsubogo's formula, equivalent to Lagally's theorem*

$$\overline{F_x} = \underbrace{\frac{\rho}{2}\Re\int_{-\infty}^{0} dz \int_{C_\infty} \underbrace{\left\{ \frac{1}{2}\nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial n} \right\}} d\ell}_{\text{2nd-order dynamic pressure integration}} \underbrace{- \frac{\rho}{4}K\int_{C_\infty} \phi\,\phi^*\Big|_{z=0} n_x\, d\ell}_{\text{Square of relative wave elevation}}$$

$$\begin{aligned}
\mathcal{I} &\equiv \frac{1}{2}\nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial n} \\
&= \underbrace{\frac{1}{2}\left\{ \nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial n} - \phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right) \right\}} + \frac{1}{2}\left\{ \phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right) - \frac{\partial\phi^*}{\partial n}\frac{\partial\phi}{\partial x} \right\}
\end{aligned}$$

$$\begin{aligned}
&\nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial n} - \phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right) \\
&\quad = n_x\left(\phi_x\phi_x^* + \phi_y\phi_y^* + \phi_z\phi_z^*\right) - \phi_x\left(\phi_x^* n_x + \phi_y^* n_y + \phi_z^* n_z\right) - \phi\left(\phi_{xx}^* n_x + \phi_{xy}^* n_y + \phi_{xz}^* n_z\right) \\
&\quad = n_x\left(\phi_y\phi_y^* + \phi_z\phi_z^* - \phi\phi_{xx}^*\right) - n_y\left(\phi_x\phi_y^* + \phi\phi_{xy}^*\right) - n_z\left(\phi_x\phi_z^* + \phi\phi_{xz}^*\right) \\
&\quad = \boldsymbol{n}\cdot\left(\nabla\times\boldsymbol{A}\right)
\end{aligned}$$

where $\boldsymbol{A} = \left(0,\ -\phi\frac{\partial\phi^*}{\partial z},\ \phi\frac{\partial\phi^*}{\partial y}\right)$

Review of Momentum-Conservation Principle at Zero Speed

Derivation of Tsubogo's formula, equivalent to Lagally's theorem

$$\overline{F_x} = \underline{\frac{\rho}{4}\Re\iint_{S_\infty} \boldsymbol{n}\cdot(\nabla\times\boldsymbol{A})\,dS}$$
$$+\frac{\rho}{4}\Re\iint_{S_\infty}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS \underline{- \frac{\rho}{4}K\int_{C_\infty}\phi\,\phi^*\Big|_{z=0} n_x\,d\ell}$$

Stokes' formula $$\iint_{S_\infty}\boldsymbol{n}\cdot(\nabla\times\boldsymbol{A})\,dS = -\int_{C_\infty}\boldsymbol{A}\cdot d\boldsymbol{r} \longleftarrow \boldsymbol{A}=\left(0,\ -\phi\frac{\partial\phi^*}{\partial z},\ \phi\frac{\partial\phi^*}{\partial y}\right)$$

$$\iint_{S_\infty}\boldsymbol{n}\cdot(\nabla\times\boldsymbol{A})\,dS = -\int_{C_\infty}\left(-\phi\frac{\partial\phi^*}{\partial z}\right)n_x\,d\ell = \underline{K\int_{C_\infty}\phi\,\phi^*\Big|_{z=0} n_x\,d\ell} \longleftarrow \frac{\partial\phi}{\partial z}=K\phi \quad \text{on } z=0$$

$$\Longrightarrow \quad \overline{F_x} = \underline{\frac{\rho}{4}K\int_{C_\infty}\phi\,\phi^*\Big|_{z=0} n_x\,d\ell}$$
$$+\frac{\rho}{4}\Re\iint_{S_\infty}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS \underline{- \frac{\rho}{4}K\int_{C_\infty}\phi\,\phi^*\Big|_{z=0} n_x\,d\ell}$$
$$=\frac{\rho}{4}\Re\iint_{S_\infty}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS$$

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Review of Momentum-Conservation Principle at Zero Speed

Applying Green's second identity,

$$\iint_{S_B+S_\infty}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS=0$$

$$\Longrightarrow\quad \overline{F_x}=\frac{\rho}{4}\,\Re\iint_{S_\infty}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS$$
$$=-\frac{\rho}{4}\,\Re\iint_{S_B}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS$$

• • • • which is the formula derived by Tsubogo in 2007

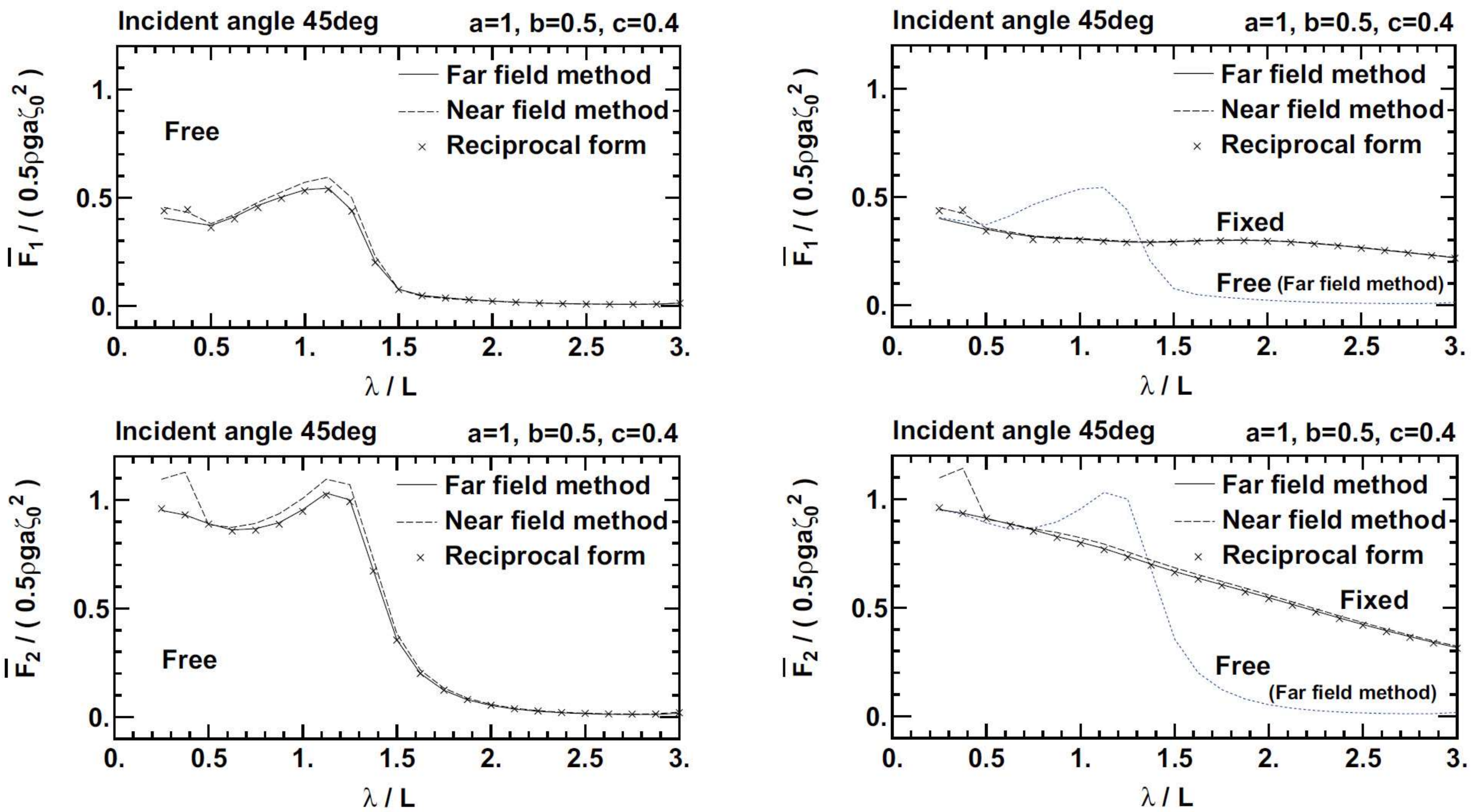
- ***Lagally's theorem***

$$F_i=\rho\frac{d}{dt}\iint_{S_B}\phi n_i\,dS-4\pi\rho\iiint_V\sigma v_i\,dV-4\pi\rho\sum\left[m v_i'+\mu_k\frac{\partial v_i'}{\partial x_k}-\frac{4\pi}{3}\sigma\mu_i\right]$$

$$\Longrightarrow\quad \overline{F_x}=-4\pi\rho\frac{1}{2}\,\mathrm{Re}\left[\sum\left\{\sigma(P)\frac{\partial\phi^*}{\partial x}+\mu(P)\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)\right\}\right]$$
$$=-2\pi\rho\,\Re\iint_{S_B}\left\{\sigma(P)\frac{\partial\phi^*}{\partial x}+\mu(P)\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)\right\}dS$$

Review of Momentum-Conservation Principle at Zero Speed

Derivation of Tsubogo's formula, equivalent to Lagally's theorem



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Extension to the Forward-Speed Case

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Momentum-conservation principle

$$\overline{F_x} = \overline{\iint_{S_B} p n_x \, dS} = -\overline{\iint_{S_\infty} \left\{ p n_x + \rho \frac{\partial \Phi}{\partial x}\left(\frac{\partial \Phi}{\partial n} - U\, n_x \right) \right\} dS}$$

$$p = -\rho \left(\frac{\partial \Phi}{\partial t} - U \frac{\partial \Phi}{\partial x} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + g z \right)$$

$$\left. \begin{aligned} \zeta_w &= -\frac{1}{g}\left(\frac{\partial \Phi}{\partial t} - U \frac{\partial \Phi}{\partial x} \right)_{z=0} = \Re \left[-\frac{1}{g}\left(i\omega_e - U \frac{\partial}{\partial x} \right) \phi \, e^{i\omega_e t} \right]_{z=0} \\ \omega_e &= \omega - \frac{\omega^2}{g} U \cos\beta = \omega + \frac{\omega^2}{g} U \quad (\text{for } \beta = \pi) \end{aligned} \right\}$$

$$\begin{aligned} \overline{F_x} &= \rho \int_{-\infty}^{0} dz \int_{C_\infty} \overline{\left\{ \frac{1}{2} \nabla \Phi \cdot \nabla \Phi n_x - \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial n} \right\}} d\ell + \rho U \int_{C_\infty} \overline{\frac{\partial \Phi}{\partial x} \zeta_w} \, n_x \, d\ell + \rho \int_{C_\infty} \overline{\left\{ \left(\frac{\partial \Phi}{\partial t} - U \frac{\partial \Phi}{\partial x} \right) \zeta_w + \frac{1}{2} g \, \zeta_w^2 \right\}_{z=0}} n_x \, d\ell \\ &= \rho \int_{-\infty}^{0} dz \int_{C_\infty} \overline{\left\{ \frac{1}{2} \nabla \Phi \cdot \nabla \Phi n_x - \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial n} \right\}} d\ell - \frac{\rho}{2g} \int_{C_\infty} \overline{\left(\frac{\partial \Phi}{\partial t} - U \frac{\partial \Phi}{\partial x} \right)\left(\frac{\partial \Phi}{\partial t} + U \frac{\partial \Phi}{\partial x} \right)_{z=0}} n_x \, d\ell \\ &= \underbrace{\frac{\rho}{2} \Re \int_{-\infty}^{0} dz \int_{C_\infty} \left\{ \frac{1}{2} \nabla \phi \cdot \nabla \phi^* n_x - \frac{\partial \phi^*}{\partial x} \frac{\partial \phi}{\partial n} \right\} d\ell}_{\text{2nd-order dynamic pressure integration}} \; \underbrace{- \frac{\rho}{4} \int_{C_\infty} \left(K_e \phi \, \phi^* - \frac{1}{K_0} \frac{\partial \phi}{\partial x} \frac{\partial \phi^*}{\partial x} \right)_{z=0} n_x \, d\ell}_{\text{Square of relative wave elevation}} \end{aligned}$$

Extension to the Forward-Speed Case

Transformation using Stokes' theorem

$$\frac{1}{2}\nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial n} = \frac{1}{2}\boldsymbol{n}\cdot(\nabla\times\boldsymbol{A}) + \frac{1}{2}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right) - \frac{\partial\phi^*}{\partial n}\frac{\partial\phi}{\partial x}\right\}$$

where $\boldsymbol{A} = \left(0,\ -\phi\frac{\partial\phi^*}{\partial z},\ \phi\frac{\partial\phi^*}{\partial y}\right)$

$$\begin{aligned}\overline{F_x} &= \frac{\rho}{2}\Re\int_{-\infty}^{0}dz\int_{C_\infty}\left\{\frac{1}{2}\nabla\phi\cdot\nabla\phi^* n_x - \frac{\partial\phi^*}{\partial x}\frac{\partial\phi}{\partial n}\right\}d\ell - \frac{\rho}{4}\int_{C_\infty}\left(K_e\phi\,\phi^* - \frac{1}{K_0}\frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial x}\right)_{z=0} n_x\,d\ell \\ &= \frac{\rho}{4}\Re\iint_{S_\infty}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right) - \frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS \\ &\quad + \frac{\rho}{4}\Re\boxed{\iint_{S_\infty}\boldsymbol{n}\cdot(\nabla\times\boldsymbol{A})\,dS} - \frac{\rho}{4}\int_{C_\infty}\left(K_e\phi\phi^* - \frac{1}{K_0}\frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial x}\right)_{z=0} n_x\,d\ell\end{aligned}$$

$$\boxed{\frac{\partial\phi^*}{\partial z}} = K_e\phi^* - i2\tau\frac{\partial\phi^*}{\partial x} - \frac{1}{K_0}\frac{\partial^2\phi^*}{\partial x^2} \quad \text{on } z=0$$

$$\begin{aligned}\iint_{S_\infty}\boldsymbol{n}\cdot(\nabla\times\boldsymbol{A})\,dS &= -\int_{C_\infty}\boldsymbol{A}\cdot d\boldsymbol{r} = \int_{C_\infty}\phi\boxed{\frac{\partial\phi^*}{\partial z}}dy \quad \longleftarrow \quad dy = n_x\,d\ell \\ &= \int_{C_\infty}\phi\left\{K_e\phi^* - i2\tau\phi\phi_x^* - \frac{1}{K_0}\phi\phi_{xx}^*\right\}_{z=0} n_x\,d\ell\end{aligned}$$

Extension to the Forward-Speed Case

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- *Applying Green's second identity,*

$$\iint_{S_\infty}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS=-\iint_{S_B+S_F}\left\{\phi\frac{\partial}{\partial n}\left(\frac{\partial\phi^*}{\partial x}\right)-\frac{\partial\phi}{\partial n}\frac{\partial\phi^*}{\partial x}\right\}dS$$

$$\overline{F_x}=-\frac{\rho}{4}\Re\iint_{S_B+S_F}\left\{\phi\frac{\partial\phi_x^*}{\partial n}-\frac{\partial\phi}{\partial n}\phi_x^*\right\}dS+\frac{\rho}{4}\Re\underline{\int_{C_\infty}\left\{-i2\tau\phi\phi_x^*+\frac{1}{K_0}\left(\phi_x\phi_x^*-\phi\phi_{xx}^*\right)\right\}_{z=0}n_x\,d\ell}$$

- *Case of Neumann-Kelvin formulation*

$$\frac{\partial\phi_x^*}{\partial z}=K_e\phi_x^*-i2\tau\phi_{xx}^*-\frac{1}{K_0}\phi_{xxx}^*,\quad\frac{\partial\phi}{\partial z}=K_e\phi+i2\tau\phi_x-\frac{1}{K_0}\phi_{xx}\quad\text{on } z=0$$

$$\begin{aligned}\mathcal{I}_f&\equiv-\iint_{S_F}\left\{\phi\frac{\partial\phi_x^*}{\partial n}-\frac{\partial\phi}{\partial n}\phi_x^*\right\}dS=-\iint_{S_F}\left\{\phi\frac{\partial\phi_x^*}{\partial z}-\frac{\partial\phi}{\partial z}\phi_x^*\right\}dxdy\\&=-\iint_{S_F}\left[\phi\left\{K_e\phi_x^*-i2\tau\phi_{xx}^*-\frac{1}{K_0}\phi_{xxx}^*\right\}-\left\{K_e\phi+i2\tau\phi_x-\frac{1}{K_0}\phi_{xx}\right\}\phi_x^*\right]dxdy\\&=\iint_{S_F}\left[i2\tau\frac{d}{dx}\left(\phi\phi_x^*\right)-\frac{1}{K_0}\frac{d}{dx}\left(\phi_x\phi_x^*-\phi\phi_{xx}^*\right)\right]dxdy\\&=\left[\int_{C_B}+\underline{\int_{C_\infty}\right]\left\{i2\tau\phi\phi_x^*-\frac{1}{K_0}\left(\phi_x\phi_x^*-\phi\phi_{xx}^*\right)\right\}_{z=0}n_x\,d\ell}\end{aligned}$$

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Extension to the Forward-Speed Case

- *Case of Neumann-Kelvin formulation*

$$\overline{F_x} = -\frac{\rho}{4}\Re\iint_{S_B}\left\{\phi\frac{\partial\phi_x^*}{\partial n}-\frac{\partial\phi}{\partial n}\phi_x^*\right\}dS+\underline{\frac{\rho}{4}\Re\int_{C_B}\left\{i2\tau\phi\phi_x^*-\frac{1}{K_0}\left(\phi_x\phi_x^*-\phi\phi_{xx}^*\right)\right\}_{z=0}n_x\,d\ell}$$

which is zero for a submerged body even at forward speed

- *Case of double-body-flow formulation*

$$\frac{\partial\phi}{\partial z} = K_e\phi - i2\tau\nabla\Phi_S\cdot\nabla\phi - \frac{1}{K_0}\nabla\Phi_S\cdot\nabla\left(\nabla\Phi_S\cdot\nabla\phi\right)$$
$$-\frac{1}{K_0}\nabla\left(\frac{1}{2}\nabla\Phi_S\cdot\nabla\Phi_S\right)\cdot\nabla\phi - \nabla^2\Phi_S\left(i\tau\phi+\frac{1}{K_0}\nabla\Phi_S\cdot\nabla\phi\right)\quad\text{on } z=0$$

where $K_e=\omega_e^2/g,\ \tau=U\omega_e/g,\ \text{and } K_0=g/U^2$

$$\left.\begin{aligned}\Phi(\boldsymbol{x},t) &= U\Phi_S(\boldsymbol{x})+\Re\left[\phi(\boldsymbol{x})\,e^{i\omega_e t}\right]\\ \Phi_S(\boldsymbol{x}) &= -x+\varphi_S(\boldsymbol{x})\end{aligned}\right\}\qquad \frac{\partial\Phi_S}{\partial n}=0\quad\left(\text{or } \frac{\partial\varphi_S}{\partial n}=n_x\right)\quad\text{on } S_B$$

Extension to the Forward-Speed Case

Case of double-body-flow formulation

$$\frac{\partial\phi}{\partial z}=K_e\phi-i2\tau\left\{\frac{\partial\Phi_S}{\partial x_j}\frac{\partial\phi}{\partial x_j}+\frac{1}{2}\frac{\partial^2\Phi_S}{\partial x_j^2}\phi\right\}$$
$$-\frac{1}{K_0}\left[\frac{\partial\Phi_S}{\partial x_j}\frac{\partial}{\partial x_j}\left(\frac{\partial\Phi_S}{\partial x_k}\frac{\partial\phi}{\partial x_k}\right)+\frac{\partial^2\Phi_S}{\partial x_j^2}\frac{\partial\Phi_S}{\partial x_k}\frac{\partial\phi}{\partial x_k}+\frac{1}{2}\frac{\partial}{\partial x_j}\left(\frac{\partial\Phi_S}{\partial x_k}\frac{\partial\Phi_S}{\partial x_k}\right)\frac{\partial\phi}{\partial x_j}\right]\quad \text{on } z=0$$

$$-\left\{\phi\frac{\partial\phi_x^*}{\partial z}-\frac{\partial\phi}{\partial z}\phi_x^*\right\}=-i2\tau\frac{\partial}{\partial x_j}\left[\frac{\partial\Phi_S}{\partial x_j}\phi\phi_x^*\right]-\frac{1}{K_0}\frac{\partial}{\partial x_j}\left[\frac{\partial\Phi_S}{\partial x_j}\frac{\partial\Phi_S}{\partial x_k}\left\{\phi_x^*\frac{\partial\phi}{\partial x_k}-\phi\frac{\partial\phi_x^*}{\partial x_k}\right\}\right]-\frac{1}{K_0}\frac{\partial\Phi_S}{\partial x_k}\frac{\partial^2\Phi_S}{\partial x_k\partial x_j}\left\{\frac{\partial\phi}{\partial x_j}\phi_x^*-\frac{\partial\phi_x^*}{\partial x_j}\phi\right\}$$

$$\mathcal{I}_f\equiv-\iint_{S_F}\left\{\phi\frac{\partial\phi_x^*}{\partial n}-\frac{\partial\phi}{\partial n}\phi_x^*\right\}dS$$
$$=-i2\tau\left[\int_{C_B}+\int_{C_\infty}\right]\frac{\partial\Phi_S}{\partial n}\phi\phi_x^*\,d\ell-\frac{1}{K_0}\left[\int_{C_B}+\int_{C_\infty}\right]\frac{\partial\Phi_S}{\partial n}\frac{\partial\Phi_S}{\partial x_k}\left\{\phi_x^*\frac{\partial\phi}{\partial x_k}-\phi\frac{\partial\phi_x^*}{\partial x_k}\right\}d\ell \quad \longleftarrow \quad \frac{\partial\Phi_S}{\partial n}=0\quad \text{on } S_B$$
$$-\frac{1}{K_0}\iint_{S_F}\frac{\partial\Phi_S}{\partial x_k}\frac{\partial^2\Phi_S}{\partial x_k\partial x_j}\left\{\frac{\partial\phi}{\partial x_j}\phi_x^*-\frac{\partial\phi_x^*}{\partial x_j}\phi\right\}dxdy$$
$$=\int_{C_\infty}\left\{i2\tau\phi\phi_x^*-\frac{1}{K_0}\left(\phi_x\phi_x^*-\phi\phi_{xx}^*\right)\right\}_{z=0}n_x\,d\ell-\frac{1}{K_0}\iint_{S_F}\nabla\left(\frac{1}{2}\nabla\Phi_S\cdot\nabla\Phi_S\right)\left\{\phi_x^*\nabla\phi-\phi\nabla\phi_x^*\right\}dxdy$$

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Extension to the Forward-Speed Case

- *Case of double-body-flow formulation*

$$\mathcal{I}_f \equiv -\iint_{S_F}\left\{\phi\frac{\partial\phi_x^*}{\partial n}-\frac{\partial\phi}{\partial n}\phi_x^*\right\}dS$$

$$= -i2\tau\left[\int_{C_B}+\int_{C_\infty}\right]\frac{\partial\Phi_S}{\partial n}\phi\phi_x^*\,d\ell-\frac{1}{K_0}\left[\int_{C_B}+\int_{C_\infty}\right]\frac{\partial\Phi_S}{\partial n}\frac{\partial\Phi_S}{\partial x_k}\left\{\phi_x^*\frac{\partial\phi}{\partial x_k}-\phi\frac{\partial\phi_x^*}{\partial x_k}\right\}d\ell \quad \longleftarrow \quad \frac{\partial\Phi_S}{\partial n}=0 \quad \text{on } S_B$$

$$-\frac{1}{K_0}\iint_{S_F}\frac{\partial\Phi_S}{\partial x_k}\frac{\partial^2\Phi_S}{\partial x_k\partial x_j}\left\{\frac{\partial\phi}{\partial x_j}\phi_x^*-\frac{\partial\phi_x^*}{\partial x_j}\phi\right\}dxdy$$

$$= \underline{\int_{C_\infty}\left\{i2\tau\phi\phi_x^*-\frac{1}{K_0}(\phi_x\phi_x^*-\phi\phi_{xx}^*)\right\}_{z=0}n_x\,d\ell}-\frac{1}{K_0}\iint_{S_F}\nabla\left(\frac{1}{2}\nabla\Phi_S\cdot\nabla\Phi_S\right)\left\{\phi_x^*\nabla\phi-\phi\nabla\phi_x^*\right\}dxdy$$

$$\overline{F_x} = -\frac{\rho}{4}\Re\iint_{S_B+S_F}\left\{\phi\frac{\partial\phi_x^*}{\partial n}-\frac{\partial\phi}{\partial n}\phi_x^*\right\}dS+\frac{\rho}{4}\Re\underline{\int_{C_\infty}\left\{-i2\tau\phi\phi_x^*+\frac{1}{K_0}(\phi_x\phi_x^*-\phi\phi_{xx}^*)\right\}_{z=0}n_x\,d\ell}$$

$$\boxed{\overline{F_x} = -\frac{\rho}{4}\Re\iint_{S_B}\left\{\phi\frac{\partial\phi_x^*}{\partial n}-\frac{\partial\phi}{\partial n}\phi_x^*\right\}dS-\frac{\rho}{4K_0}\Re\iint_{S_F}\frac{\partial\Phi_S}{\partial x_k}\frac{\partial^2\Phi_S}{\partial x_k\partial x_j}\left\{\frac{\partial\phi}{\partial x_j}\phi_x^*-\frac{\partial\phi_x^*}{\partial x_j}\phi\right\}dxdy}$$